POSSIBLE DECAYS OF NEW MESONS

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Evaluations of the probabilities of the decays of unstable mesons are given. It is shown that in certain cases the small decay widths may be accounted for by purely kinematic factors. The small pion-decay widths may make radiative decays (with the emission of photons) significant.

1. INTRODUCTION

In our earlier papers [1,2] we discussed the properties of hypothetical neutral mesons with isotropic spin equal to zero: the vector meson (vecton) and pseudoscalar (σ) mesons. Recently the existence of a neutral vector meson with T = 0 was experimentally demonstrated [3-6]. The experimenters who discovered this particle called it the ω^0 meson.1) No σ^0 mesons have been observed so far. In most recent times the properties of the ω^0 and σ^0 were the subject of several theoretical papers [7-9] 2). Certain physical considerations and estimates contained in these papers seem to us to be unconvincing.

In the present paper we analyze the estimates of the probabilities of various ω^0- and σ^0-meson decays. We start from phenomenological matrix elements, and assume a normalization in which the probability of the n-particle decay $m_0 \rightarrow m_1 + m_2 + \ldots + m_n$ is expressed as

$$w = \int \frac{1}{2m_0} \overline{|M|^2} (2\pi)^4 \delta^4 (\Sigma k_i - k_0) \prod_{i=1}^{n} \frac{d\mathbf{k}_i}{2E_i (2\pi)^3}. \quad (1)$$

Here $\overline{|M|^2}$ is the square of the modulus of the matrix element, averaged over the initial polarization and summed over the final polarization.

The order of magnitude of the constants entering into the matrix element M will be estimated from general considerations, perturbation theory, and a comparison with known decays. In spite of the considerable uncertainty, the results obtained disclose several interesting properties of the ω^0 and σ^0 mesons. In particular, it is found that in many cases the photon decays of the new mesons can compete successfully with their pion decays.

2. DECAYS OF ω^0 VECTOR MESON

As is well known, the ω^0 meson is observed as a narrow resonant state ($\Gamma \sim 10$ MeV) of a three-pion system in the reactions

$$\bar{p} + p \rightarrow 2\pi^+ + 2\pi^- + \pi^0 \; [3,4], \quad (2)$$

$$\bar{p} + p \rightarrow 3\pi^+ + 3\pi^- + \pi^0 \; [5], \quad (2')$$

$$\pi^+ + d \rightarrow 2p + \pi^+ + \pi^- + \pi^0 \; [6] \quad . \quad (3)$$

Some authors consider the width of this resonance to be anomalously small and discuss various hypothetical hinderances which can lead to such low values. It can be seen, however, that the small width of the ω^0 meson is perfectly natural. Indeed, let the matrix element of the decay

$$\omega^0 \rightarrow \pi^+ + \pi^- + \pi^0 \quad (4)$$

have the form

$$M = L^3 \varepsilon_{\alpha\beta\gamma\delta} k_{1\alpha} k_{2\beta} k_{3\gamma} \varphi_\delta \varphi_1 \varphi_2 \varphi_3, \quad (5)$$

where L is a quantity with dimension of length, k_1, k_2, k_3 and φ_1, φ_2, φ_3 are the four-momenta and wave functions of the pions, and φ_δ is the wave function of the ω^0 meson. Then the probability of the decay (4) is

$$w = \frac{L^6 m_\omega^7}{2^{12} 90 \pi^3} f, \quad (6)$$

where the coefficient f = 0.23 accounts for the pion mass ($f \rightarrow 1$ as $m_\pi \rightarrow 0$). When $L = 1/m_\pi$, formula (6) yields $w \approx 0.5$ MeV; at $L = 1/2m_\pi$ we have $w = 7 \times 10^{-3}$ MeV; the perturbation-theory

1)Although all the quantum numbers of the ω^0 meson and of the vecton are the same, we have no confirmation whatever, that the ω^0 meson is indeed that fundamental vecton, whose strong interaction with three baryon fields (p, n, and Λ) is the primary strong interaction, as is proposed in the vecton model[1]. It is possible that the ω^0 meson is a compound vector meson[10,11].

2)A list of earlier papers on σ and ω mesons is contained in [1,2].

calculation given in the Appendix yields $L \sim 1/m_\pi$.

An objection to the foregoing estimate is that L cannot be regarded as a constant in (5), inasmuch as the pion momenta in the decay (4) are not small compared with L^{-1}. This effect, however, can only be expected to decrease the decay probability. In addition, this effect is apparently small, since the distribution plotted from the experimental data [4] on the Dalitz diagram for the decay (4), is in good agreement with the matrix element (5) with L constant.

Thus, the small width of the ω^0 meson can be explained without additional hindrance and may be completely due to ordinary kinematic factors.

The small width of the decay (4) gives grounds for assuming that the two-particle radiative decay

$$\omega^0 \to \pi^0 + \gamma \qquad (7)$$

can successfully compete with it. The matrix element of this decay has the form

$$M = \sqrt{\alpha} L \varepsilon_{\alpha\beta\gamma\delta} k_{1\alpha} k_{2\beta} e_\gamma \varphi_\delta, \qquad (8)$$

where L is a quantity with dimension of length, $\alpha = 1/137$, k_1 and k_2 are the four-momenta of the π^0 meson and of the photon, e_γ is the photon wave function, and φ_δ is the wave function of the ω^0 meson. The probability of decay (7) is equal to

$$w = \alpha L^2 m_\omega^3 / 96\pi. \qquad (9)$$

When $L = 1/m_\pi$, formula (9) yields $w \approx 0.5$ MeV, and when $L = 1/2m_\pi$ we get $w \approx 0.1$ MeV. (The perturbation-theory estimate given in the Appendix yields $L \sim 1/1.6\, m_\pi$ [3].) The ratio of the probabilities (6) and (9) is $R = (Lm_\omega)^4 \times 0.23/\alpha \times 2^7 \times 30\pi^2$, if we assume that the values of L are the same in these formulas. We see that when $L \sim 1/m_\pi$ the probabilities of decays (4) and (7) are comparable. We note in this connection that Duerr and Heisenberg [7] state that the decay (4) should exceed decay (7) by approximately four orders of magnitude.

The decay of the ω^0 meson into two pions

$$\omega^0 \to \pi^+ + \pi^- \qquad (10)$$

can result only from nonconservation of the isotopic spin. A measurement of its probability can yield valuable information on the magnitude of the isotopically invariant interaction. The matrix element of decay (10) has the form

$$\beta\, (k_1 - k_2)_\alpha\, \varphi_\alpha \varphi_1 \varphi_2, \qquad (11)$$

where β is a dimensionless quantity, and the probability is

$$w = \beta^2 p^3 / 6\pi m_\omega^2, \qquad (12)$$

where p is the momentum of the decay products. When $\beta = \sqrt{4\pi}\,\alpha$ formula (12) yields $w = 3 \times 10^{-3}$ MeV. It may turn out, however, that $\beta \gg \alpha$: perturbation-theory calculation (see the Appendix) yields $\beta \sim 20\alpha$. Decay (10) can compete successfully with decays (4) and (7) only if β is anomalously large.

Fubini [8] mentions that the existence of two narrow ρ^0 mesons was established at Berkeley. One of them possibly coincides with the ω^0 meson and represents decay (10) [4]. As shown above, juggling the numbers could make such a decay relatively probable. It would be more strange were it to be confirmed [5] that the lighter ρ^0 meson is also narrow. In this case it would be necessary to ascertain whether these ρ mesons are produced paired with some other particles, so as to "decouple" the interactions responsible for the decay and production of these particles.

Pevsner and Block [6] report that along with the ω^0 meson there exists in reaction (3) still another resonant state of three pions with mass 550 MeV. This resonance is attributed to a neutral vector meson with T = 0, which they called η^0 meson. This η^0 meson differs from the ω^0 meson only in the mass. It is easy to see that for the η^0 meson the ratio of the probabilities of decays

$$\eta^0 \to \pi^0 + \gamma \qquad (7')$$

and

$$\eta^0 \to \pi^0 + \pi^+ + \pi^- \qquad (4')$$

should be appreciably larger than the same ratio for the ω^0 meson. The corresponding probabilities for the η^0 meson are

$$w_{\pi\gamma} = \alpha L_{\pi\gamma}^2\, m_\eta^3 / 96\pi, \qquad (9')$$

$$w_{3\pi} = L_{3\pi}^6 m_\eta^3 (m_\eta - 3m_\pi)^4 / 2^8 \cdot 3^{11/2} \pi^2. \qquad (6')$$

[3] For an estimate of L in the decay (7) see also the paper by Gell-Mann and Zachariasen [12].

[4] This possibility was pointed out by V. A. Lyubimov (private communication).

[5] Expression (12) describes also the probability of the decays $\rho \to 2\pi$ and $K' \to K + \pi$, if ρ and K' are vector mesons. If we put $\beta = \sqrt{4\pi}\, g_\rho$, then (12) yields $w \approx 60$ [MeV]$\cdot g_\rho^2$. If the ρ meson interacts with the isovector current, then $g_\rho = 2g_{\rho N}$, where $g_{\rho N}$ is the constant of interaction between the ρ meson and the nucleon. Therefore, within the framework of such a scheme, it would be impossible to reconcile the small width of the ρ meson ($\Gamma \ll 100$ MeV) with the intensive production of this particle.

For each of the K' decays, (12) yields $w_{K'} \approx 18$ MeV $g_{K'}^2$. Unitary symmetry requires that the constants of the decays $\rho^+ \to \pi^+ + \pi^0$, $K'^+ \to \pi^+ + K^0$, and $K'^+ \to K^+ + \pi^0$ be related as $2 : \sqrt{2} : 1$. One can therefore expect K'^+ to be approximately four or five times "narrower" than ρ^+.

When $L_{3\pi} = L_{\pi\gamma} = 1/m_\pi$ we find that for the η meson

$$w_{\pi\gamma}/w_{3\pi} \sim 25.$$

Searches for the radiative decay (7′) are of great interest.

3. DECAYS OF PSEUDOSCALAR MESON σ^0

We have previously considered [2] the following σ^0-meson decays

$$\sigma^0 \to 2\pi^+ + 2\pi^-, \quad (13)$$

$$\sigma^0 \to 2\pi^0 + \pi^+ + \pi^-, \quad (14)$$

$$\sigma^0 \to 4\pi^0, \quad (15)$$

$$\sigma^0 \to \pi^+ + \pi^- + \gamma, \quad (16)$$

$$\sigma^0 \to 2\gamma. \quad (17)$$

We present below some additional considerations pertaining to these decays.

We note first that isotopic invariance leads to the following relation between the decays (13), (14), and (15):

$$2w(2\pi^+ + 2\pi^-) = w(\pi^+ + \pi^- + 2\pi^0) + 4w\,(4\pi^0). \quad (18)$$

This relation is obtained directly if we stipulate that the four pions in the decay of the σ^0 meson be isotopically unpolarized [13]. Inasmuch as the decay (15) should be suppressed, [2] it follows from (18) that [6)]

$$2w\,(2\pi^+ + 2\pi^-) \approx w\,(\pi^+ + \pi^- + 2\pi^0). \quad (19)$$

We now consider decays (16) and (17). The matrix element of the decay $\sigma \to 2\pi + \gamma$ has the form

$$\sqrt{\alpha}\, L^3 \varepsilon_{\alpha\beta\gamma\delta} k_{1\alpha} k_{2\beta} k_{3\gamma} e_\delta \varphi_1 \varphi_2 \varphi_3, \quad (20)$$

where k_1, k_2, and k_3 are the four-momenta of the pions and the photon, while e_δ is the wave function of the latter. The probability of this decay is

$$w = \frac{\alpha L^6 m_\sigma^7}{2^{12} 30\, \pi^3} f, \quad (21)$$

where the coefficient f takes into account the pion masses: $f \approx 0.3$ when $m_\sigma = 700$ MeV. When $L = 1/m_\pi$, (21) yields $w \sim 6 \times 10^{-3}$ MeV. Perturbation theory calculation yields $L \sim 1/2m_\pi$, corresponding to $w \sim 1 \times 10^{-4}$ MeV. The matrix element of the decay $\sigma \to 2\gamma$ has the form

$$\alpha L \varepsilon_{\alpha\beta\gamma\delta} k_{1\alpha} k_{2\beta} e_{1\gamma} e_{2\delta} \varphi. \quad (22)$$

The probability of this decay is

$$w = \alpha^2 L^2 m_\sigma^3 / 64\, \pi. \quad (23)$$

Perturbation theory yields $L = 2g_\sigma/\sqrt{\pi}\, m_N$, from which follows a formula similar to the well known formula for the π^0-meson decay probability

$$w = \frac{\alpha^2 g_\sigma^2}{16\pi^2} \left(\frac{m_\sigma}{m_N}\right)^2 m_\sigma. \quad (24)$$

For the π^0 meson formula (24) yields $w \approx 12$ eV, or $\tau \approx 5 \times 10^{-17}$ sec; experiment yields $\tau = (2.2 \pm 0.8) \times 10^{-16}$. If unitary symmetry holds, then $g_\sigma^2 \approx 14/3$ in (24). Then formula (24) yields $w \approx 6 \times 10^{-4}$ MeV for $m_\sigma = 700$ MeV.

Assuming that L has the same value in (21) and in (23), we obtain for the ratio of the probabilities (16) and (17)

$$R = (Lm_\sigma)^4 \cdot 0.3/\alpha 2^6 \cdot 30\pi^2. \quad (25)$$

Formula (25) shows that within a rather wide range of the values of m and L the decay into two photons should predominate. This was first noted by Sawada and Yonezawa [15].

Searches for the σ^0 meson in the xenon chamber of the Institute of Theoretical and Experimental Physics (U.S.S.R. Academy of Sciences) have shown that the cross section of the single production (without pions) of a σ^0 meson decaying into two photons is less than 0.1 mb per nucleon for pions with momentum 2.8 BeV/c (private communication from A. I. Alikhanov and Ya. Ya. Shalamov).

Comparison of two-particle and three-particle decays for mesons with mass 700—800 MeV shows that when $L \sim 1/m_\pi - 1/2m_\pi$ these decays have comparable probabilities if the two-particle decay contains additional electromagnetic interaction. The decay

$$\sigma^0 \to \pi^+ + \pi^- + \pi^0 \quad (26)$$

can result only from nonconservation of isotopic spin. If we write the matrix element of this decay in the form

$$\beta \varphi_\sigma \varphi_1 \varphi_2 \varphi_3, \quad (27)$$

then the probability of decay is

$$w = \frac{\beta^2 m_\sigma}{2^9 \pi^3} f, \quad (28)$$

where $f \sim 0.37$ when $m_\sigma = 700$ MeV. If we put in (28) $\beta = \alpha = 1/137$, then the ratio of the probabilities (21) and (28) is

$$w_{3\pi}/w_{2\pi+\gamma} \approx 240\, \alpha\, /\, (Lm_\sigma)^6. \quad (29)$$

When $L \sim 1/m_\pi$ and $\beta \sim \alpha$, this ratio is much smaller than unity and the $\sigma \to 3\pi$ decay cannot compete with the $\sigma \to 2\pi + \gamma$ decay. In order for the $\sigma \to 3\pi$ decay to become appreciable, we must

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[6)] We analyzed the isotopic properties of the decays (13), (14), and (15) jointly with I. I. Gurevich. Relation (19) is contained also in the paper by Gell-Mann [14].

have $\beta \gg \alpha$. This takes place in perturbation theory, which yields $\beta \sim 8\alpha g^4 (\ln \Lambda/m_N - 1)$, where Λ is the cutoff momentum. This estimate, however, can be regarded as too high. We note that Duerr and Heisenberg [7] suggest that the $\sigma \rightarrow 3\pi$ decay will be the principal one for a matrix element of order α. From (29) it follows that this calls for $L < 1/m_\sigma$. For this value of L, however, the principal decay, as followed from (25), would be $\sigma \rightarrow 2\gamma$.

4. CONCLUSION

The foregoing estimates show that the experimentally observed narrowness of the resonances of some new mesons (in particular, the ω meson) can be explained in natural fashion by means of ordinary kinematic factors and so far needs no hindrances for its explanation.

The relatively low decay probability makes it possible in the case of new mesons, to obtain decays with participation of photons (real or virtual) and with probabilities comparable with the probability of pure pion decays. The photon decays can be observed either directly (for example in a xenon chamber) or by the "departing mass" method, as, for example, in the case of the $\sigma \rightarrow \pi^+ + \pi^- + \gamma$ decay, if the momenta of all the charged particles participating in the reaction are known. Searches for photon decays of new mesons and a determination of their widths are problems of prime importance.

The foregoing estimates are very uncertain; we do not know essentially what is the natural normalization for the different amplitudes. Yet a change in dimensions of the effective region by a factor of 2 changes the probability 64 times! Some information on the effective dimensions of particles can be obtained by comparing the probabilities of the $K_{\mu 2}$ and K_{e3} decays, and also the τ and θ decays. The θ and τ decays indicate that the effective dimension of the K meson is a quantity $\sim 1/m_\pi$. Comparison of the $K_{\mu 2}$ and K_{e3} decays also yields an effective dimension of order $1/m_\pi$. By effective dimension we mean the factor, with dimensionality of length, by which one must multiply the normalization of the matrix element for the decay into n mesons in order to obtain the normalization of the matrix element for the decay into n + 1 mesons. Such an estimate is confirmed by perturbation-theory calculations when the corresponding integrals converge. In spite of these considerations, we have no convincing proof that the constants L introduced by us should always be of order $1/m_\pi - 1/2m_\pi$ and do not contain some large numerical coefficients. To clarify this question it is necessary to determine L directly from the experimental lifetimes (widths) of the unstable mesons.

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APPENDIX

Let us calculate by perturbation theory some of the decays considered in the article. In these calculations we assume that the momenta of the "external" particles are appreciably smaller than the momenta and the masses of the virtual particles, and take only the first nonvanishing terms in the expansions of the corresponding integrals in terms of the external momenta. Since the masses of the heavy mesons are not small, this approximation is quite crude, but for an order of magnitude estimate this is satisfactory.

1. The decay (4) is described by the following diagrams (Fig. 1). The contributions of diagrams b and b′, c and c′ cancel each other, and the contributions of diagrams a and a′ add up. The matrix element corresponding to diagram a is

$$M = (4\pi)^2 2g^3 g_\omega \int \mathrm{Sp}\, \gamma_\alpha \frac{1}{\hat{p} - m} \hat{k}_1 \frac{1}{\hat{p} - m} \gamma_5 \frac{1}{\hat{p} - m} (\hat{k}_1 + \hat{k}_2)$$
$$\times \frac{1}{\hat{p} - m} \gamma_5 \frac{1}{\hat{p} - m} (\hat{k}_1 + \hat{k}_2 + \hat{k}_3) \frac{1}{\hat{p} - m} \gamma_5 \frac{1}{\hat{p} - m}$$
$$\times \frac{d^4p}{(2\pi)^4} \varphi_1\varphi_2\varphi_3\varphi_\alpha = \frac{2g^3 g_\omega}{\pi^2} \int \frac{d^4p}{(p^2 - m^2)^4} \mathrm{Sp}\, \gamma_5 \gamma_\alpha$$
$$\times (\hat{p} + m) \hat{k}_1 (\hat{k}_1 + \hat{k}_2)(\hat{k}_1 + \hat{k}_2 + \hat{k}_3) \varphi_1\varphi_2\varphi_3\varphi_\alpha,$$

where m is the nucleon mass. Recognizing that

$$\int \frac{d^4p}{(p^2 - m^2)^4} = \frac{\pi^2}{6m^3},$$

we obtain for the sum of a and a′

$$\frac{16\, g^3 g_\omega}{6m^3} \varepsilon_{\alpha\beta\gamma\delta} k_{1\beta} k_{2\gamma} k_{3\delta} \varphi_\alpha \varphi_1\varphi_2\varphi_3.$$

Consequently, $L^3 = 8g^3 g_\omega/3m^3$. For $g_\omega = 1$ and $g^2 = 14$ we have $L = 1/1.2\, m_\pi$.

2. We calculate decay (16) in similar fashion. In this case only the diagram of Fig. 2 contributes. The corresponding value of L^3, determined by formula (20), is

$$L^3 = 4g^2 g_\sigma / 3m^3.$$

In the unitary symmetry theory $g_\sigma = g/\sqrt{3}$, and we obtain $L^3 = 1/7m_\pi^3$.

3. Decay (14) corresponds to the diagrams of Fig. 3. The corresponding matrix element is

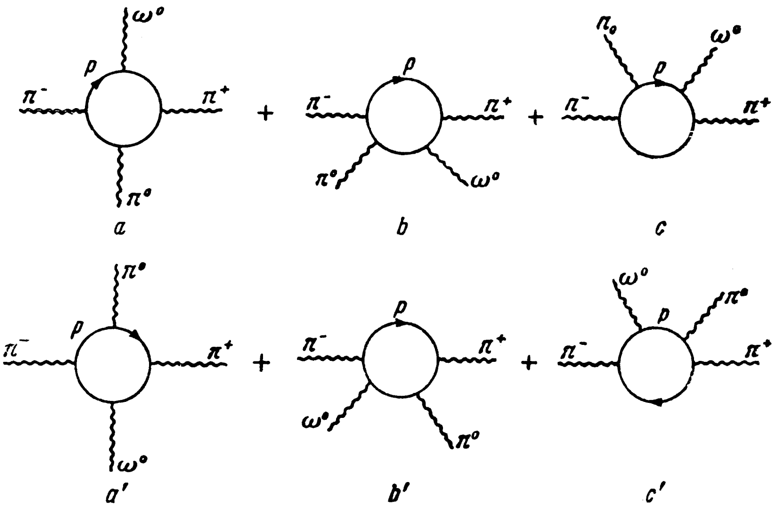

FIG. 1

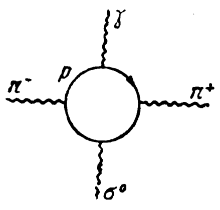

FIG. 2

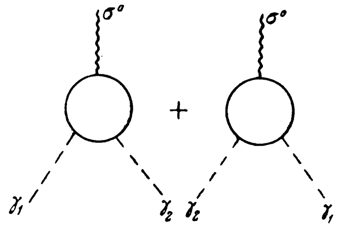

FIG. 3

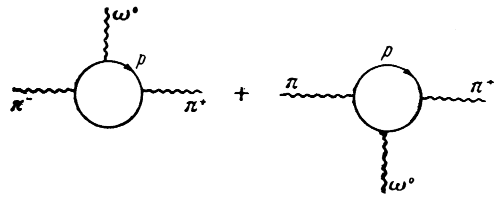

FIG. 5

$$M = 2\alpha (4\pi)^{3/2} g_\sigma \int \mathrm{Sp}\, \gamma_5 \frac{1}{\hat{p}-m} \hat{k}_1 \frac{1}{\hat{p}-m} \hat{e}_1 \frac{1}{\hat{p}-m} \hat{e}_2 \times \frac{1}{\hat{p}-m} \hat{k}_2 \frac{1}{\hat{p}-m} \frac{d^4p}{(2\pi)^4}$$

$$= -\frac{\alpha g_\sigma}{\pi^{5/2}} m\, \mathrm{Sp}\, \gamma_5 \hat{k}_2 \hat{k}_1 \hat{e}_1 \hat{e}_2 \int \frac{d^4p}{(p^2-m^2)^3}.$$

Recognizing that

$$\int \frac{d^4p}{(p^2-m^2)^3} = \pi^2/2m^2,$$

we have $M = 2\alpha g_\sigma/\sqrt{\pi}\, m$.

The diagrams of decay (7) have a similar form (Fig. 4). The corresponding matrix element is

$$M = \frac{2\sqrt{\alpha}\, g_\omega g}{\sqrt{\pi}\, m} \varepsilon_{\alpha\beta\gamma\delta} k_{1\alpha} k_{2\beta} e_{1\gamma} e_{2\delta}.$$

If $g_\omega = 1$ and $g = \sqrt{15}$, then $L = 1/1.6\, m_\pi$.

5. Decay (10) is described by a sum of two diagrams (Fig. 5). If $g_{\omega p} = g_{\omega n} = g_\omega$, then the contributions of these diagrams cancel exactly. Owing to violation of isotopic symmetry, $\Delta = g_{\omega p} - g_{\omega n} \neq 0$; we can expect that $\Delta \sim \alpha g_\omega$. Then the amplitude of the $\omega \to \pi^+ + \pi^-$ decay has the form

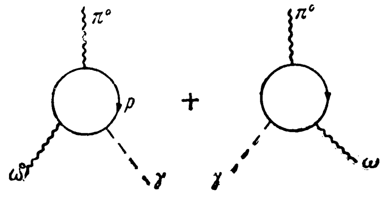

FIG. 4

$$M = (4\pi)^{3/2} 2g^2 \Delta \int \mathrm{Sp}\Big[-\gamma_5 \frac{1}{\hat{p}-m} \hat{k}_1 \frac{1}{\hat{p}-m} \gamma_\alpha \frac{1}{\hat{p}-m} \gamma_5 \frac{1}{\hat{p}-m} + \gamma_5 \frac{1}{\hat{p}-m} \gamma_\alpha \frac{1}{\hat{p}-m} \hat{k}_2 \frac{1}{\hat{p}-m} \gamma_5 \frac{1}{\hat{p}-m}\Big] \frac{d^4p}{(2\pi)^4} \varphi_1 \varphi_2 \varphi_\alpha$$

$$= \frac{2}{\sqrt{\pi}} \Delta g^2 \left(\ln \frac{\Lambda^2}{m^2} - \frac{1}{2}\right) (k_1 - k_2)_\alpha \varphi_1 \varphi_2 \varphi_\alpha.$$

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