CERTAIN INTERFERENCE PHENOMENA IN $K^0\bar{K}^0$ *SYSTEMS*

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The nature of the Pais-Piccioni type of beats in decays of a $K^0\bar{K}^0$ pair is studied. These beats turn out to depend essentially on the relative weights and phase difference of the states of the $K^0\bar{K}^0$ system with even and with odd orbital angular momentum.

INGLIS [1] and Day [2] have remarked on certain peculiarities of the Pais-Piccioni process for $K\bar{K}$ pairs.¹⁾ The present work deals with a further development of this problem, the discussion following closely the results of a previous paper, [3] in which the connection was pointed out between the orbital angular momentum of the $K\bar{K}$ pair and the allowed modes of decay (see also [4]).

Let us suppose that in the proper frame of referance of the $K\bar{K}$ pair its angular momentum is odd. Then the wave function of the system at the time of production is antisymmetric. It can be written in the form

$$\psi_a = -i2^{-1/2}\{K_1(\mathbf{p})K_2(\mathbf{q})\exp[-i(m_1\tau+m_2\theta)-\lambda_1\tau/2-\lambda_2\theta/2]-K_2(\mathbf{p})K_1(\mathbf{q})\exp[-i(m_1\theta+m_2\tau)-\lambda_1\theta/2-\lambda_2\tau/2]\}, \quad (1)$$

where $\mathbf{p}$ and $\mathbf{q}$ are the momenta of the particles considered, τ and θ are their proper times, m_1 and m_2 are the masses of the K_1 and K_2 particles, and λ_1 and λ_2 are their decay constants.

If K_1 and K_2 are expressed in terms of K and $\bar{K}$ then it is an easy matter to obtain with the help of Eq. (1) the probability for, for example, one of the particles to be at the instant τ in the state $K(\mathbf{p})$ [or $\bar{K}(\mathbf{p})$] together with the other particle to be at the instant θ in the state, say, $\bar{K}(\mathbf{q})$ [or $K(\mathbf{q})$]. The indicated probabilities are given by [2]

$$w\{K(\mathbf{p}),K(\mathbf{q});\tau,\theta\}=w\{\bar{K}(\mathbf{p}),\bar{K}(\mathbf{q});\tau,\theta\} = \tfrac{1}{8}\{\exp(-\lambda_1\tau-\lambda_2\theta)+\exp(-\lambda_1\theta-\lambda_2\tau) - 2\exp[-(\lambda_1+\lambda_2)(\tau+\theta)/2]\cos[\Delta m(\tau-\theta)]\}, \quad (2)$$

$$w\{K(\mathbf{p}),\bar{K}(\mathbf{q});\tau,\theta\}=w\{\bar{K}(\mathbf{p}),K(\mathbf{q}),\tau,\theta\} = \tfrac{1}{8}\{\exp(-\lambda_1\tau-\lambda_2\theta)+\exp(-\lambda_1\theta-\lambda_2\tau) + 2\exp[-(\lambda_1+\lambda_2)(\tau+\theta)/2]\cos[\Delta m(\tau-\theta)]\}. \quad (2')$$

Thus in this case as well beats are present, analogous to the beats in the conventional Pais-Piccioni process.

If we are interested in the probability for one of the particles to be in the state $K(\mathbf{p})$ [or $\bar{K}(\mathbf{p})$] regardless of what state the other particle might be in, then the corresponding probabilities will be of the form

$$w\{K(\mathbf{p}),K(\mathbf{q});\tau,\theta\}+w\{K(\mathbf{p}),\bar{K}(\mathbf{q});\tau,\theta\} = w\{\bar{K}(\mathbf{p}),K(\mathbf{q});\tau,\theta\}+w\{\bar{K}(\mathbf{p}),\bar{K}(\mathbf{q});\tau,\theta\} = \tfrac{1}{4}\{\exp(-\lambda_1\tau-\lambda_2\theta)+\exp(-\lambda_1\theta-\lambda_2\tau)\}. \quad (3)$$

We see that in this case the beats disappear.

In the case when the orbital angular momentum of the $K\bar{K}$ system is even, the wave function is symmetric and analogous relations are valid, namely [2]

$$\psi_c = 2^{-1/2}\{K_1(\mathbf{p})K_1(\mathbf{q})\exp[-im_1(\tau+\theta)-\lambda_1(\tau+\theta)/2] + K_2(\mathbf{p})K_2(\mathbf{q})\exp[-im_2(\tau+\theta)-\lambda_2(\tau+\theta)/2]\}, \quad (4)$$

$$w\{K(\mathbf{p}),K(\mathbf{q});\tau,\theta\}=w\{\bar{K}(\mathbf{p}),\bar{K}(\mathbf{q});\tau,\theta\} = \tfrac{1}{8}\{\exp[-\lambda_1(\tau+\theta)]+\exp[-\lambda_2(\tau+\theta)] - 2\exp[-(\lambda_1+\lambda_2)(\tau+\theta)/2]\cos[\Delta m(\tau+\theta)]\}, \quad (5)$$

$$w\{K(\mathbf{p}),\bar{K}(\mathbf{q});\tau,\theta\}=w\{\bar{K}(\mathbf{p}),K(\mathbf{q});\tau,\theta\} = \tfrac{1}{8}\{\exp[-\lambda_1(\tau+\theta)]+\exp[-\lambda_2(\tau+\theta)] + 2\exp[-(\lambda_1+\lambda_2)(\tau+\theta)/2]\cos[\Delta m(\tau+\theta)]\} \quad (5')$$

$$w\{K(\mathbf{p}),K(\mathbf{q});\tau,\theta\}+w\{K(\mathbf{p}),\bar{K}(\mathbf{q}),\tau,\theta\} = w\{\bar{K}(\mathbf{p}),K(\mathbf{q});\tau,\theta\}+w\{\bar{K}(\mathbf{p}),\bar{K}(\mathbf{q}),\tau,\theta\} = \tfrac{1}{4}\{\exp[-\lambda_1(\tau+\theta)]+\exp[-\lambda_2(\tau+\theta)]\}. \quad (5'')$$

Let us consider the general case when the wave function is of the form

$$\Psi = \alpha\psi_c+\beta\psi_a, \qquad |\alpha|^2+|\beta|^2=1. \quad (6)$$

¹⁾Here, and in the following, we consider only neutral K mesons.

In what follows we put

$$\alpha = ae^{iA}, \qquad \beta = be^{iB}, \qquad C = A - B,$$

where a, b, A, and B are real quantities dependent on, generally speaking, **p** and **q** as well as on the momenta of the remaining particles produced together with the $K\bar{K}$ pair.

Calculations fully analogous to those carried out above show that in this case $w(K, K) \neq w(\bar{K}, \bar{K})$ and $w(K\bar{K}) \neq w(\bar{K}K)$. Thus, for example,

$$w\{K(\mathbf{p}), K(\mathbf{q}); \tau, \theta\} = \tfrac{1}{8}a^2\{\exp[-\lambda_1(\tau+\theta)] + \exp[-\lambda_2(\tau+\theta)] - 2\exp[-(\lambda_1+\lambda_2)(\tau+\theta)/2] \times \cos[\Delta m(\tau+\theta)]\} + \tfrac{1}{8}b^2\{\exp(-\lambda_1\tau-\lambda_2\theta) + \exp(-\lambda_1\theta-\lambda_2\tau) - 2\exp[-(\lambda_1+\lambda_2) \times (\tau+\theta)/2]\cos[\Delta m(\tau-\theta)]\} + \tfrac{1}{4}ab\{\exp[-\lambda_1\theta-(\lambda_1+\lambda_2)\tau/2]\cos(C-\Delta m\tau) + \exp[-\lambda_1\tau-(\lambda_1+\lambda_2)\theta/2]\cos(C+\Delta m\tau) - \exp[-\lambda_1\tau-(\lambda_1+\lambda_2)\theta/2]\cos(C-\Delta m\theta) - \exp[-\lambda_2\tau-(\lambda_1+\lambda_2)\theta/2]\cos(C+\Delta m\theta)\} \quad (7)$$

whereas the expression for $w\{\bar{K}(\mathbf{p}), \bar{K}(\mathbf{q}); \tau, \theta\}$ differs from Eq. (7) in the sign of the term involving the product ab.

One may also calculate the probability for one particle to be in a given state regardless of the state of the other particle. In this case one obtains, for example,

$$w\{K(\mathbf{p}), K(\mathbf{q}); \tau, \theta\} + w\{K(\mathbf{p}), \bar{K}(\mathbf{q}); \tau, \theta\} = \tfrac{1}{4}a^2\{\exp[-\lambda_1(\tau+\theta)] + \exp[-\lambda_2(\tau+\theta)]\} + \tfrac{1}{4}b^2\{\exp(-\lambda_1\tau-\lambda_2\theta) + \exp(-\lambda_2\tau+\lambda_1\theta)\} + 2ab\{\exp[-\lambda_1\theta-(\lambda_1+\lambda_2)\tau/2]\cos(C-\Delta m\tau) + \exp[-\lambda_1\tau-(\lambda_1+\lambda_2)\theta/2]\cos(C+\Delta m\tau) + \exp[-\lambda_1\theta-(\lambda_1+\lambda_2)\tau/2]\cos(C-m\theta) + \exp[-\lambda_2\theta-(\lambda_1+\lambda_2)\tau/2]\cos(C+\Delta m\tau)\}. \quad (8)$$

Thus in this case there persist beats connected with interference between states corresponding to orbital angular momenta of different parity.

The beats manifest themselves in a cleanest way if the problem is formulated somewhat differently; this formulation also leads to a situation which is easier to realize experimentally. Suppose that the state of one of the particles is classified in terms of K_1 and K_2, and the state of the other in terms of K and $\bar{K}$. In order to obtain the corresponding probabilities we must substitute into Eq. (6) the expressions (1) and (4), express $K_1(\mathbf{p})$ and $K_2(\mathbf{p})$ in terms of $K(\mathbf{p})$ and $\bar{K}(\mathbf{p})$, and calculate the modulus squared of the coefficient of products of the type $K(\mathbf{p})K_1(\mathbf{q})$, $\bar{K}(\mathbf{p})K_1(\mathbf{q})$, etc.

We list, as an example, the following two expressions:

$$w\{K(\mathbf{p}), K_1(\mathbf{q}); \tau, \theta\} = \tfrac{1}{4}\exp(-\lambda_1\theta)\{a^2\exp(-\lambda_1\tau) + b^2\exp(-\lambda_2\tau) + 2ab\exp[-(\lambda_1+\lambda_2)\tau/2]\cos(C-\Delta m\tau)\}, \quad (9)$$

$$w\{\bar{K}(\mathbf{p}), K_1(\mathbf{q}); \tau, \theta\} = \tfrac{1}{4}\exp(-\lambda_1\theta)\{a^2\exp(-\lambda_1\tau) + b^2\exp(-\lambda_2\tau) - 2ab \times \exp[-(\lambda_1+\lambda_2)\tau/2]\cos(C-\Delta m\tau)\}. \quad (9')$$

We note that in Eqs. (9) and (9′) the variables τ and θ are "separable." This makes it possible to utilize, while studying the beats in τ, all events without regard to the instant θ at which the decay of the second particle takes place.

In conclusion we wish to emphasize that the determination of the magnitude and sign of C is essential for a complete analysis of the $K\bar{K}$ interaction. The relations (9) and (9′) permit the determination of the phase difference C if the magnitude and sign of Δm is known. The inverse problem could also be posed (the determination of the magnitude and sign of Δm), if an independent method could be devised for the determination of the phase difference C.

[1] D. R. Inglis, Revs. Modern Phys. **33**, 1 (1961).
[2] T. B. Day, Phys. Rev. **121**, 1204 (1961).
[3] Ogievetskiĭ, Okonov, and Podgoretskiĭ, JETP **43**, 720 (1962), Soviet Phys. JETP **16**, 511 (1962).
[4] Ogievetskiĭ, Okonov, and Podgoretskiĭ, Preprint Joint Inst. Nuc. Res., R-960 (1962).

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235