

CASCADE COULOMB EXCITATION OF  $4^+$  AND  $6^+$  ROTATIONAL LEVELS

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Cascade Coulomb excitation of rotational levels of Sm, Gd, Er, and W isotopes was studied. Targets of enriched isotopes were irradiated with 50 MeV  $N^{14} 5^+$  ions from a cyclotron. Coincidences were recorded between the  $\gamma$  quanta emitted as a result of the Coulomb excitation and the inelastically scattered ions. The ions were recorded with a silicon p-n detector. Cascade Coulomb excitation was observed of rotational levels with spin and parity  $4^+$  in  $Sm^{154}$ ,  $Gd^{154}$ ,  $Gd^{156}$ ,  $Gd^{158}$ ,  $Gd^{160}$ ,  $Er^{164}$ ,  $Er^{166}$ ,  $Er^{168}$ ,  $Er^{170}$ ,  $W^{182}$ ,  $W^{184}$ ,  $W^{186}$ , and with spin and parity  $6^+$  in  $Sm^{154}$  and  $Gd^{160}$ . The results were compared with the theory of cascade Coulomb excitation developed by Alder and Winther. For most of the nuclei there is good agreement with theory. Exceptions are the tungsten isotopes and  $Gd^{154}$ , for which the observed yield is less than the theoretical value.

THE even-even deformed nuclei are characterized by the familiar system of rotational levels with the sequence of spin and parity  $0^+$ ,  $2^+$ ,  $4^+$ ,  $6^+$ , etc. The Coulomb excitation of the first levels in these nuclei has been studied in considerable detail. In the lowest approximation of perturbation theory, which is valid if the probability of excitation in an individual collision is much less than unity, the cross section for Coulomb excitation of the second  $4^+$  levels (E4 transition) is several orders of magnitude less than that for exciting the first excited levels (E2 transition).

For large energies of the bombarding particles, which can be used when one has heavy multicharged ions, the probability of excitation increases markedly and the role of second order processes increases. One of these processes is cascade Coulomb excitation, which can be crudely described as a series of successive E2 transitions occurring in a single collision. As a result of this process it is possible to have Coulomb excitation of  $4^+$ ,  $6^+$ , etc, rotational levels.

The theory of cascade Coulomb excitation has been given in a paper of Alder and Winther.<sup>[2]</sup> This theory is based on the assumption that during the time of the collision the shape of the nucleus and its orientation in space remain unchanged. This condition is satisfied if the Coulomb excitation parameter  $\xi \rightarrow 0$ . Then the probability of Coulomb excitation of any of the levels of the rotational band ( $P_N$ ) is a function of a parameter  $q$ , which depends both on the internal properties of the nucleus and on the conditions of excitation:

$$q = 7,6241 \frac{A_1^{1/2} Q_0}{(1 + A_1/A_2)^2 Z_1 Z_2^2} E^{3/2}, \quad (1)$$

where  $A_1$  and  $Z_1$  are the mass number and charge of the bombarding particle;  $Q_0$  is the intrinsic quadrupole moment of the nucleus in units of  $e \cdot 10^{-24} \text{ cm}^2$ ;  $E$  is the energy of the bombarding particle (in MeV).

The differential cross section for excitation ( $d\sigma$ ) is related to the probability of excitation by the formula

$$d\sigma = P_N d\sigma_R, \quad (2)$$

where  $d\sigma_R$  is the differential cross section for Rutherford scattering.

For sufficiently large values of the parameter  $q$  (of the order of several units) in deformed nuclei, one can have Coulomb excitation of a series of levels of the ground rotational band. The study of cascade Coulomb excitation is of interest for two reasons. First it makes it possible to check the theory of cascade Coulomb excitation.<sup>[2]</sup> Secondly, from the values of the excitation cross sections one can get some information about the reduced transition probabilities for excited states.

Cascade Coulomb excitation has been studied in several experiments. Newton and Stephens<sup>[3]</sup> observed double Coulomb excitation when irradiating a normal mixture of tungsten isotopes with oxygen ions having energies from 40 to 80 MeV. One of the present authors investigated double Coulomb excitation of the  $4^+$  level in separated isotopes of tungsten.<sup>[4]</sup> Nathan and Popov<sup>[5]</sup>

cited several cases of double Coulomb excitation from irradiation of rare earth elements with 17-MeV  $\alpha$  particles. Finally, in the irradiation of  $U^{238}$  and  $Th^{232}$  with 190-MeV  $Ar^{40}$  ions, cascade Coulomb excitation of a whole series of rotational levels was observed (up to  $12^+$  in  $U^{238}$  and  $10^+$  in  $Th^{232}$ ).

In our work we investigated the cascade Coulomb excitation of the even-even isotopes of Gd, Er, W, and  $Sm^{154}$ . Targets of enriched isotopes of these elements were irradiated with  $N^{14}$   $5^+$ -ions accelerated to 50 MeV in a cyclotron. The rare-earth elements were in the form of the oxide ( $Sm_2O_3$ ,  $Gd_2O_3$ ,  $Er_2O_3$ ), while the tungsten isotopes were the pure metal. We studied the spectrum of  $\gamma$  rays emitted after Coulomb excitation and the spectrum of coincidences of  $\gamma$  quanta with inelastically scattered ions.

A block diagram of the experimental arrangement is shown in Fig. 1. The beam of accelerated ions struck a target fixed to the bottom of the Faraday cup, which served to measure the charge transferred by the ion beam. The  $\gamma$  radiation emitted by the target nuclei was detected by a scintillation spectrometer with a  $4 \times 4$  cm<sup>2</sup> NaI(Tl) crystal.

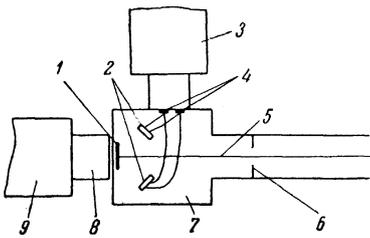


FIG. 1. Block diagram of experimental arrangement: 1) target; 2) p-n detectors; 3) preamplifier; 4) vacuum leads; 5) trajectory of ion beam; 6) diaphragm; 7) Faraday cup; 8) NaI (Tl) crystal; 9) photomultiplier.

The inelastically scattered  $N^{14}$  ions were recorded by silicon p-n detectors, placed at an angle of  $135^\circ$  to the direction of the beam. To increase the solid angle we used four detectors connected in parallel, each having an area of  $5 \times 5$  mm<sup>2</sup>. The voltage on the detectors was chosen so that the whole range of the  $N^{14}$  ions was in the sensitive layer. When this is done,  $\alpha$  particles and protons lose only a small part of their energy in the layer. Consequently the amplitudes of pulses from  $\alpha$  particles and protons emitted in nuclear reactions with oxygen were much smaller than the amplitude of pulses from  $N^{14}$  ions and could easily be distinguished.

The spectrum of charged particles recorded by the p-n detector is shown in Fig. 2. The rise at low amplitudes is due to  $\alpha$  particles and protons emitted from reactions on oxygen. The electronic circuits in the equipment were described earlier.<sup>[7]</sup> The scintillation spectrometer and the p-n detectors were connected in a fast-slow coincidence

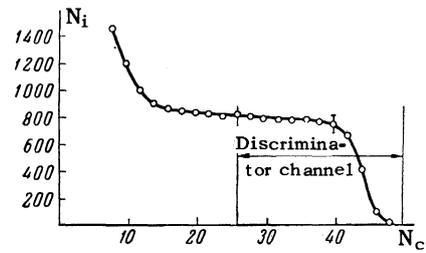


FIG. 2. Spectrum of charged particles from irradiation of a  $Gd_2^{54}O_3$  target with  $N^{14}$  ions.

circuit with a resolving time of  $4 \times 10^{-8}$  sec. By means of a single-channel discriminator we selected and counted in coincidence with the  $\gamma$  rays only the pulses corresponding to  $N^{14}$  ions. In recording the spectrum of  $\gamma$ N-coincidences we were thus able to eliminate the  $\gamma$  ray background from nuclear reactions on oxygen, since the background  $\gamma$  rays are emitted along with light particles.

The  $\gamma$ -ray spectra and the spectra of coincidences of  $\gamma$  quanta with inelastically scattered  $N^{14}$  ions ( $\gamma$ N-coincidences) are shown in Figs. 3-5. In all the spectra one observes  $\gamma$  rays associated with the excitation of the first and second levels. In the spectra of the direct  $\gamma$  radiation,

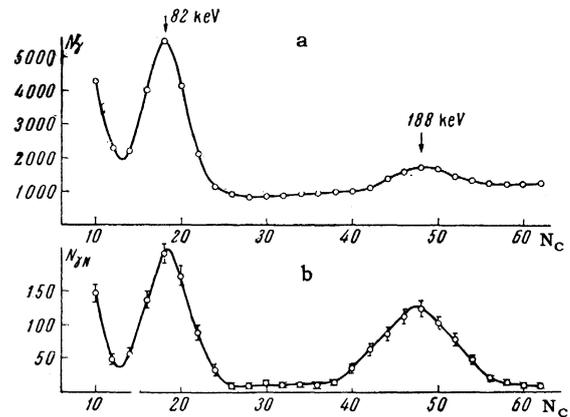


FIG. 3.  $Sm_2^{54}O_3 + N^{14}$  reaction: a)  $\gamma$  spectrum; b)  $\gamma$ N-coincidence spectrum.

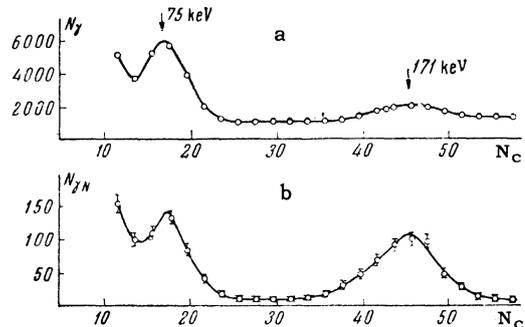


FIG. 4.  $Gd_2^{60}O_3 + N^{14}$  reaction: a)  $\gamma$  spectrum; b)  $\gamma$ N-coincidence spectrum.

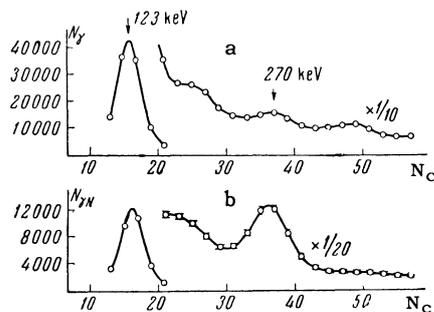


FIG. 5.  $W^{186} + N^{14}$  reaction: a)  $\gamma$  spectrum; b)  $\gamma N$ -coincidence spectrum. The curves on the right are expanded by 10:1 and 20:1.

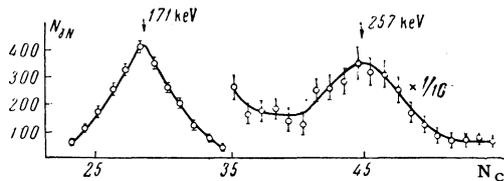


FIG. 6.  $Gd_2^{160}O_3 + N^{14}$  reaction. Spectrum of  $\gamma N$ -coincidences.

the  $\gamma$  lines are observed on a large background caused by nuclear reactions on oxygen. Coincidences with inelastically scattered ions enable one to reduce this background considerably. From Figs. 3–5 we see that the relative contribution of the background radiation to the coincidence spectra is much less than in the direct spectra. The low level of the background enabled us to observe the  $\gamma$  lines associated with Coulomb excitation of the third  $6^+$  level in these even-even nuclei. Such a spectrum is shown in Fig. 6. The  $\gamma$  rays seen here are associated with cascade excitation of the second and third ( $4^+$  and  $6^+$ ) levels of  $Gd^{160}$ .

Table I gives the energies of the rotational levels obtained from our experiments on cascade Coulomb excitation. The error in the energy determination does not exceed 2%. These levels are known for most of the nuclei. In all such cases the

values of the level energies found by us are in good agreement with the known values. In some cases, (second and third levels of  $Sm^{154}$ , second level of  $Er^{170}$ , third level of  $Gd^{160}$ ) the levels observed by us were not previously known. From the unified model of the nucleus, the ratio of the energies of the second and first levels in even-even deformed nuclei should be 3.33. For most of the nuclei, the experimentally determined ratio is close to this value. For nuclei which are close to the boundary of the deformed region ( $Gd^{154}$ ,  $W^{184}$ ,  $W^{186}$ ) this ratio has a somewhat lower value.

From the spectra we can determine the ratio of the yields associated with excitation of the first and second states in the various nuclei. These ratios were determined by starting from the areas under the peaks and taking account of the efficiency of recording  $\gamma$  rays of a given energy and the values of the conversion coefficients. In addition, in determining the yield of  $\gamma$  radiation emitted during deexcitation of the first level one must take into account the contribution from the cascade transition from the second level to the first. The values found for the ratio of the yields are given in Table II. The errors in determining the ratios are about 20%.

The yield ratios found experimentally can be compared with those calculated from the theory of cascade excitation. As already stated earlier, the probability of excitation of a level of a given spin and, consequently, the differential excitation cross section, are functions of the parameter  $q$ . The values of  $q$  computed using formula (1) for a bombarding energy of 50 MeV are shown in Table II. The value of the quadrupole moment  $Q_0$  which appears in formula (1) was found from the relation

$$eQ_0 = 4\sqrt{1/5\pi} B(E2, 0 \rightarrow 2). \quad (3)$$

The values  $B(E2, 0 \rightarrow 2)$  of the reduced probabilities of E2 transitions for isotopes of the rare

Table I

Isotope	$\Delta E(0 \rightarrow 2)$ , keV	$\Delta E(0 \rightarrow 4)$ , keV	$\Delta E(0 \rightarrow 6)$ , keV
$Sm^{154}$	82	270	534
$Gd^{154}$	123	370	
$Gd^{156}$	89	285	
$Gd^{158}$	79	260	
$Gd^{160}$	75	246	503
$Er^{164}$	90	290	
$Er^{166}$	81	266	
$Er^{168}$	80	263	
$Er^{170}$	79	261	
$W^{182}$	100	326	
$W^{184}$	111	357	
$W^{186}$	123	393	

Table II

Isotope	$q$	$Y(0 \rightarrow 2)/Y(0 \rightarrow 4)$		$Y(0 \rightarrow 4)/Y(0 \rightarrow 6)$	
		expt.	theor.	expt.	theor.
$Sm^{154}$	2.13	5.85	5.02	16.2	14.9
$Gd^{154}$	1.72	12.60	8.25		
$Gd^{156}$	1.99	5.12	6.17		
$Gd^{158}$	2.18	6.40	4.92		
$Gd^{160}$	2.25	4.25	4.61	11.4	13.9
$Er^{164}$	1.87	7.00	6.86		
$Er^{166}$	1.99	8.10	6.17		
$Er^{168}$	2.00	6.67	6.11		
$Er^{170}$	1.96	7.40	6.39		
$W^{182}$	1.52	16.67	11.35		
$W^{184}$	1.49	20.7	11.70		
$W^{186}$	1.35	28.9	14.65		

earth elements were taken from [8] and those for the tungsten isotopes, from [9]. In the coincidence spectra, the observed yield of  $\gamma$  radiation is related only to ions scattered through  $135^\circ$ . Therefore, for comparison with yields found from the coincidence spectra, we calculated the differential cross section for excitation during scattering through  $135^\circ$ , using formula (2). The values of  $P_N$  were taken from the tables given in [2].

Since our experiments did not determine the ratio of the cross sections but rather the ratio of the yields in a given interval of energy of the bombarding particles, it was necessary to compute the yield for this same energy interval. The yields were computed by numerical integration of the excitation cross section over the effective thickness of the target. The effective target thickness was determined from the threshold of the single channel discriminator and was equivalent to an 8 MeV energy loss of the bombarding particles. In computing the excitation cross sections, corrections were made for the finite value of the parameter  $\xi$ . The excitation probability for  $\xi \neq 0$  ( $P_N(\xi)$ ) was found from the formula

$$P_N(\xi) = P_N + \Lambda_N \xi. \quad (4)$$

The correction  $\Lambda_N$  because of the finite value of  $\xi$  is a function of the parameter  $q$ . The values of  $\Lambda_N$  were taken from Alder and Winther. [2] The computed yield ratios are given in Table II. This table also gives the yield ratios for excitation of the second and third levels in  $\text{Sm}^{154}$  and  $\text{Gd}^{160}$ , found experimentally and computed from the theory of cascade excitation. The treatment of the experimental data and the computation of the theoretical yield ratios were done in these cases in the same way as for the data given in the third and fourth columns of Table II. The error of the computed values of the yield ratios is 10% and is related to the errors in determining the bombarding energy and the values of  $B(E2, 0 \rightarrow 2)$ . From Table II we see that for most of the nuclei the experimental values of the yield ratios agree with the calculated values within the limits of error. But for some of the nuclei ( $\text{Gd}^{154}$  and the tungsten isotopes) there are differences exceeding the experimental errors.

Alder and Winther [2] assumed that the values of the reduced probabilities for transition from a level  $I_i$  to level  $I_f$  are given by the well-known expression of the unified nuclear model:

$$B(E2I_i \rightarrow I_f) = \frac{5}{16\pi} e^2 Q_0^2 \langle I_i 2K0 | I_i 2I_f K \rangle^2. \quad (5)$$

We may assume that for those nuclei where the

ratios found experimentally and computed from the results of [2] agree within the limits of error, the values of  $B(E2, I_i \rightarrow I_f)$  satisfy (5). In this case

$$\begin{aligned} B(E2, 2 \rightarrow 4) &= (18/35) B(E2, 0 \rightarrow 2), \\ B(E2, 4 \rightarrow 6) &= (175/198) B(E2, 2 \rightarrow 4). \end{aligned} \quad (6)$$

Since the values of  $B(E2, 0 \rightarrow 2)$  are known for all the nuclei which we studied, we can, from the ratios given above, find the values of  $B(E2, 2 \rightarrow 4)$  and  $B(E2, 4 \rightarrow 6)$ . Using the values of  $B(E2)$ , we can determine the lifetimes of the levels. For the  $\text{Gd}^{156}$  nucleus, the lifetime of the second level is known from delayed coincidence measurements [10] and is equal to  $(1.5 \pm 0.3) \times 10^{-10}$  sec. This lifetime is in good agreement with our value,  $(1.65 \times 10^{-10}$  sec).

For  $\text{Gd}^{154}$ ,  $\text{W}^{182}$ ,  $\text{W}^{184}$ , and  $\text{W}^{186}$ , the experimental yield ratios are smaller than calculated. This may mean that for these nuclei the values of the reduced transition probabilities  $B(E2, 2 \rightarrow 4)$  from the first to the second level are less than those used in the unified model. It was shown in [11] that for the osmium isotopes ( $\text{Os}^{188}$  and  $\text{Os}^{190}$ ), which are at the edge of the region of deformed nuclei, the ratio  $B(E2, 2 \rightarrow 4)/B(E2, 0 \rightarrow 2)$  is less than the value from the uniform model. Our measurements indicate that the same situation apparently exists also for  $\text{Gd}^{154}$ ,  $\text{W}^{182}$ ,  $\text{W}^{184}$ , and  $\text{W}^{186}$ .

The state of the theory of cascade excitation at present is such that for the nuclei cited we must limit ourselves to the qualitative conclusions presented above. The theory presented by Alder and Winther [2] does not allow one to compute values of  $B(E2)$  for individual transitions if experiment shows a deviation of the  $B(E2)$  values from the dependence (5). In this theory the probability of excitation of any one level depends on the probability of excitation of all the other levels and, consequently, depends on the unknown values of the reduced probabilities for transition between them. To calculate the values of  $B(E2, 2 \rightarrow 4)$  one could in principle use computations in second order perturbation theory. Such computations are given in [1]. These computations are valid only for small values of the parameter  $q$ , which is not the case in our experiments. For small values of  $q$ , the conditions of the experiment become complicated because of the low value of the cascade excitation cross section.

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