TRIPLE SCATTERING OF 660-MeV PROTONS
III. ANGULAR DEPENDENCE OF THE PARAMETER R

Yu. P. KUMEKIN, M. G. MESHCHERYAKOV, S. B. NURUSHEV, and G. D. STOLETOV

Joint Institute for Nuclear Research

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In accordance with the program of reconstructing the pp scattering matrix at ~ 660 MeV, further experiments on the triple scattering of protons have been carried out. The experimental arrangement is described and results of the measurement of the parameter R at 54, 72, 90, 108, and 126° in the c.m.s. are presented. The results are interpreted in terms of the pp scattering matrix amplitudes. Values of the moduli and relative phase angles of the matrix elements are found for the angle 90°.

1. INTRODUCTION

In experiments on triple scattering, the change in the polarization of the primary beam $\mathbf{P}_1$ in proton-proton scattering is studied. This change is described by the polarization tensor of the scattered particles D_{ip} and the polarization tensor of the recoil particles K_{iq}:[1]

$$P_{2p} = \frac{P_{2p}^{(0)} + D_{ip}P_{1i}}{1 + \mathbf{P}_2^{(0)}\mathbf{P}_1}, \quad P_{2q} = \frac{P_{2q}^{(0)} + K_{iq}P_{1i}}{1 + \mathbf{P}_2^{(0)}\mathbf{P}_1}. \qquad (1)$$

Here the index i refers to the initial polarization component of the incident particles; p and q refer to the measured polarization component of the scattered and recoil particles, respectively; $\mathbf{P}^{(0)}$ is the polarization arising in the scattering of the unpolarized beam. In what follows we employ the notation and coordinate system introduced by Wolfenstein.[2]

In pp scattering, the tensor D_{ip} (and similarly, the tensor K_{iq}) has the form

$$\|D_{ip}\| = \begin{matrix} n \\ P \\ K \end{matrix}\begin{Vmatrix} D_{nn} & 0 & 0 \\ 0 & X & Z \\ 0 & -Z & Y \end{Vmatrix}. \qquad (2)$$

In our previous experiments on triple proton scattering[3,4] at 660 MeV, we already measured the angular dependence of the depolarization parameter D $[D(\theta) = D_{nn}(\theta),\ D(180° - \theta) = K_{nn}(\theta),\ 0 \le \theta \le 90°]$, which determines the change in the polarization component normal to the scattering plane and is measured in the experiment when all three scatterings occur in one plane.

The change in the polarization component lying in the scattering plane is described by the two-dimensional tensor

$$\begin{Vmatrix} X & Z \\ -Z & Y \end{Vmatrix}. \qquad (3)$$

The relation between the elements X, Y, and Z of this tensor and the directly measured Wolfenstein parameters R, R′, A, and A′, which are defined by the relations

$$\mathbf{P}_2\mathbf{s}_2 = R\mathbf{P}_1[\mathbf{n}_2\mathbf{k}_2] + A\mathbf{P}_1\mathbf{k}_2, \qquad (4)^*$$

$$\mathbf{P}_2\mathbf{k}_2' = R'\mathbf{P}_1[\mathbf{n}_2\mathbf{k}_2] + A'\mathbf{P}_1\mathbf{k}_2, \qquad (5)$$

have the following form in the nonrelativistic case:

$$R = Z\sin(\theta/2) + Y\cos(\theta/2),\ A = Z\cos(\theta/2) - Y\sin(\theta/2);$$
$$R' = -Z\cos(\theta/2) + X\sin(\theta/2),\ A' = Z\sin(\theta/2) + X\cos(\theta/2). \qquad (6)$$

As was shown by Zastavenko et al[5] and Marshak,[6] a unique pp scattering matrix can be reconstructed only when at least one scattering experiment is carried out for scattering in different planes. The simplest experiment to determine the parameter R is to study triple scattering, where the three scattering planes are mutually perpendicular. The geometry of such an event is shown in Fig. 1, which represents the case $\theta_2 = 90°$.

As is seen from Fig. 1, the polarization $\mathbf{P}_1$ is directed along the vector $\mathbf{n}_2 \times \mathbf{k}_2$, so that the polarization $\mathbf{P}_2$ will have the components RP_1 and $R'P_1$ directed along the vectors $\mathbf{s}_2$ and $\mathbf{k}_2'$, respectively, in the plane of the second scattering π_2. Henceforth we shall omit the index 2 for the second scattering if it does not lead to confusion. In the measurement of the parameter R, the third scat-

*$[\mathbf{n}_2\mathbf{k}_2] = \mathbf{n}_2 \times \mathbf{k}_2$.

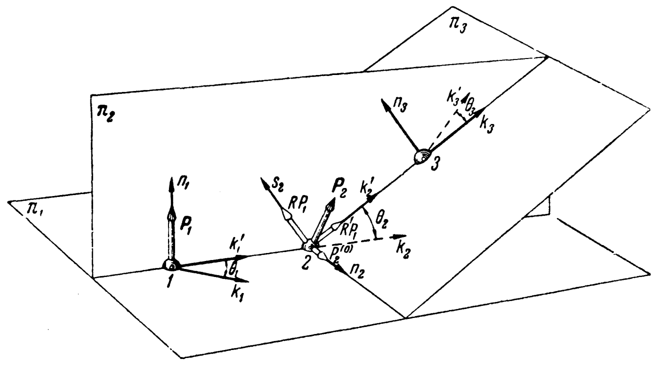

FIG. 1. Geometry of the experiment on triple scattering of protons: measurement of the parameter R.

tering should have either $\mathbf{n}_3 = \mathbf{s}_2$ (third scattering to the left) or $\mathbf{n}_3 = -\mathbf{s}_2$ (third scattering to the right).

From the corresponding counting rates N_L and N_R, we calculate the asymmetry

$$\varepsilon_{3s} = (N_L - N_R) / (N_L + N_R), \tag{7}$$

which is connected with the parameter R by a relation which has the simplest form when $\theta_2 = 90°$ (second scattering is up) or $\theta = 270°$ (second scattering is down):

$$\varepsilon_{3s} = RP_1P_3 \sin\varphi_2. \tag{8}$$

The product P_1P_3 is determined in the calibrating experiment from double scattering in which the beam with polarization P_1 is slowed down to an energy equal to the energy of the double scattering of protons; then the asymmetry

$$\varepsilon_3 = P_1P_3, \tag{9}$$

characterizing the analyzing power of the third target is measured. These two asymmetries determine the value of the parameter R:

$$R = \varepsilon_{3s} / (\varepsilon_3 \sin\varphi_2). \tag{10}$$

2. EXPERIMENTAL ARRANGEMENT AND RESULTS

In this experiment, we used a proton beam with a polarization $P_1 = 0.58 \pm 0.03$ and energy 640 ± 12 MeV extracted from the six-meter synchrocyclotron of the Joint Institute for Nuclear Research. [7]

The arrangement for measuring the parameter R consisted of a rigid frame which could be rotated in the vertical plane π_2 (see Fig. 1) about the horizontal axis perpendicular to the incident beam direction and passing through the center 2 of the second scatterer (cylindrical vessel of 12-cm dia. filled with liquid hydrogen). Scintillation counters 41 and 42 were mounted on one arm of this arrangement and counters 31, 32, 3A, and the third scatterer T_3 (graphite block) were mounted on the other arm, as is shown in Fig. 2. The axis of rotation of the two additional arms supporting counters 11, 12, 13, and 21, 22, 23, respectively, passed through the center 3 of the third scatterer (see Fig. 1). This axis lay in the vertical plane π_2 and was perpendicular to the line 2—3 (Fig. 1).

Counters 11, 12, 13; 21, 22, 23; 31, 32; 31, 32, 3A; and 41, 42 formed telescopes 1, 2, 3, 3A, and 4. The angle between telescopes 3 and 4 corresponded to the kinematics of pp elastic scattering. The resolving times in the coincidence circuits were 1.8×10^{-8} sec for telescopes 1 and 2 and 1.0×10^{-8} sec for telescopes 3, 3A, and 4. Moreover, the pulses from telescopes 1, 3, 4; 2, 3, 4; and 4, 3A were connected in coincidence. The counting rates N_{134A} and N_{234A} of anticoincidences corresponding to (1+3+4) − (3A+4) and (2+3+4) − (3A+4) were recorded simultaneously. For the calibration measurements of the asymmetry ϵ_3, telescope 4 was disconnected and the counting rates N_{13A} and N_{23A} were recorded.

During the measurements, telescopes 1 and 2 were moved to the left and to the right by an angle

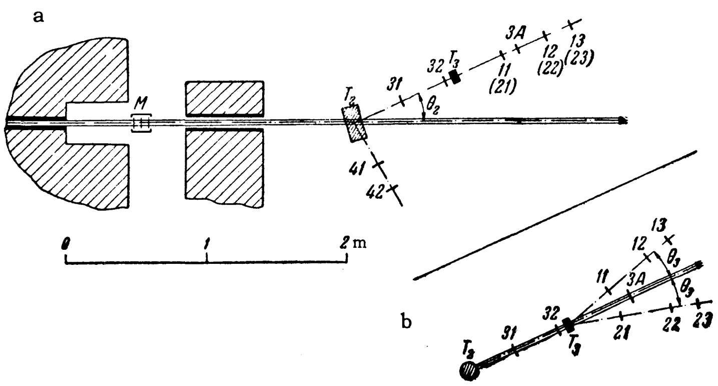

FIG. 2. Arrangement of scatterers and the detecting apparatus: a – in the vertical plane, b – in the plane of the third scattering. M – monitor (ionization chamber), T_2 – second scatterer (vessel with liquid hydrogen), T_3 – third scatterer (graphite block 80 × 60 mm, 50 mm thick); 11, 12, 13, 21, 22, 23, 31, 32, 3A, 41, 42 – scintillation counters of size (height by width) 60 × 40, 70 × 40, 80 × 50, 60 × 40, 70 × 40, 80 × 50, 50 × 40, 50 × 40, 90 × 80, 100 × 70, and 100 × 75 mm, 6 mm thick. For the measurement of the parameter R(126°), a third scatterer of 80 × 60-mm cross section, 20 mm thick was used.

$\theta_3 = 12°$ relative to the beam of the doubly scattered protons; the angular resolution was $\Delta\theta_3 = \pm 3°$. The value of θ_3 was chosen as a compromise between the requirements to obtain, on the one hand, the values of the differential cross section for pC scattering and the proton polarization in this process and the need to determine, on the other hand, the values of the effects due to the contribution of protons not scattered in carbon; these latter effects increase with decreasing θ_3 and increase with the relative error of the angle θ_3.

The recorded counting rates N_{134A} and N_{234A} were corrected to take into account: a) the background observed in the absence of the third scatterer and caused almost entirely by the scattering of protons in the scintillator of counter 32; b) the effect observed in the absence of liquid hydrogen in the vessel; c) the effect of random coincidences. For all angles of observation, the counting rates were negligible in the absence of the second and third scatterers and also when in the presence of the second and third scatterer, the angle between telescopes 3 and 4 did not correspond to the kinematics of pp elastic collisions.

For each angle of observation of the parameter R, we made five to ten independent repeated measurements of the asymmetry ϵ_{3S}. Before beginning each measurement, we carefully determined the beam "profile" for the doubly scattered protons in order to eliminate the possible appearance of a spurious asymmetry. In most cases, the results given by channels 134A and 234A were in agreement with each other, within the limits of error, and also were in agreement with the results of the repeated measurements. For the angle 54°, the measurements were made in the same way as for $\theta_2 = 90°$ and $\theta_2 = 270°$.

During the calibration measurements, the intensity of the primary beam was reduced by a factor of 10—100. In the calibration for the angle $\theta_2 = 126°$, we made direct measurements of the energy of the scattered and slowed-down beam, since in this case the value of the calibration asymmetry ϵ_3 quite strongly depends on the thickness of the slowing-down filter.

The table shows the values found for the asymmetries ϵ_{3S} and ϵ_3 with their statistical errors and the values of the parameter R calculated from formula (10). The values of the parameter D found earlier [3,4] are also shown.

θ, deg	$\epsilon_{3S} \pm \Delta\epsilon_{3S}$, %	$\epsilon_3 \pm \Delta\epsilon_3$, %	$R \pm \Delta R$	$D \pm \Delta D$
54	4.9±0.9	10.9±0.3	0.45±0.08	0.99±0.25
72	6.8±1.0	13.8±0.7	0.49±0.08	0.69±0.20
90	5.5±1.4	21.1±1.3	0.26±0.07	0.93±0.17
108	6.9±1.1	20.5±1.1	0.32±0.06	0.28±0.16
126	4.9±1.3	10.2±0.5	0.49±0.13	0.57±0.20

3. DISCUSSION

The last problem of the experiment on pp scattering was the carrying out of a phase-shift analysis or, at least, a direct reconstruction of the scattering matrix. In the latter case, the matrix amplitudes were determined for several selected values of the scattering angle from the results of various independent experiments made at the same energy.

A. Scattering angle $\theta = 90°$. For the scattering angle $\theta = 90°$, only three complex amplitudes of the pp scattering matrix M_{pp} are different from zero. With an accuracy to a phase factor and the uncertainty arising from the bilinearity of the corresponding expressions, these amplitudes can be determined directly from the data of five independent experiments. For an energy of ~ 640 MeV, the following measurements have been made:

1) differential cross section $\sigma_0(90°) = 2{,}07\pm0.03$ mb/sr, [8],
2) depolarization parameter $D(90°) = 0{,}93\pm0{,}17$ [3],
3) correlation coefficient $C_{KP}(90°) = 0{,}22\pm0{,}18$ [9],
4) correlation coefficient $C_{nn}(90°) = 0{,}93\pm0{,}20$ [10],
5) parameter $R(90°) = 0{,}26\pm0{,}07$ (present experiment).

(11)

The elements of the M_{pp} matrix written in the singlet-triplet representation [11] are most simply expressed in terms of the data. The correlation data determine the moduli of the matrix elements, while the parameters of the triple scattering determine the relative phases:

$$|M_{ss}|^2 = 2\sigma_0(1 - C_{nn}),$$

$$|M_{01}|^2 = \frac{1}{2}\sigma_0\left[1 + C_{nn} + 2C_{KP} - (1 - C_{nn})\frac{E_{lab}}{4mc^2 + E_{lab}}\right],$$

$$|M_{10}|^2 = \frac{1}{2}\sigma_0\left[1 + C_{nn} - 2C_{KP} + (1 - C_{nn})\frac{E_{lab}}{4mc^2 + E_{lab}}\right],$$

$$|M_{01}||M_{10}|\cos\varphi_{01,10} = -\sigma_0 D,$$

$$|M_{01}||M_{ss}|\cos\varphi_{01,ss} = \sigma_0 R\left[2\frac{4mc^2 + E_{lab}}{2mc^2 + E_{lab}}\right]^{1/2}. \quad (12)$$

Here m is the proton mass, c is the velocity of light, E_{lab} is the kinetic energy of the incident proton in the l.s.

Relativistic corrections [12] were introduced in formulas (12). The results of the calculations give the following values for the moduli of the matrix elements:

$$|M_{ss}| = (0.24 \pm 0.11) \cdot 10^{-13} \text{ cm},$$
$$|M_{01}| = (0.51 \pm 0.05) \cdot 10^{-13} \text{ cm},$$
$$|M_{10}| = (0.40 \pm 0.06) \cdot 10^{-13} \text{ cm} \quad (13)$$

and the cosines of the relative phases of these elements

$$\cos \varphi_{01,10} = -0.96 \pm 0.24,$$
$$\cos \varphi_{01,ss} = 0.84 \pm 0.42. \quad (14)$$

The values of $M_{SS}/\sqrt{2}$, $M_{01}/\sqrt{2}$, and $M_{10}/\sqrt{2}$ determined from the data (13) and (14) are shown in Fig. 3 on the complex plane. The circle of radius $\sqrt{\sigma_0}$ corresponds to the maximum possible value of the moduli of these quantities; the absolute phase of the element M_{01} was taken equal to 180° (see below).

The energy dependence of the moduli of the matrix elements at $\theta = 90°$ is shown in Fig. 4. For energies up to 310 MeV, we used the results of the phase-shift analysis in [13] (first set).

The energy dependence of the phases of the matrix elements is shown in Fig. 5, where the absolute phase of the element M_{01} at an energy 640 MeV was taken equal to 180° in analogy with the data at the lower energies. The ambiguity as

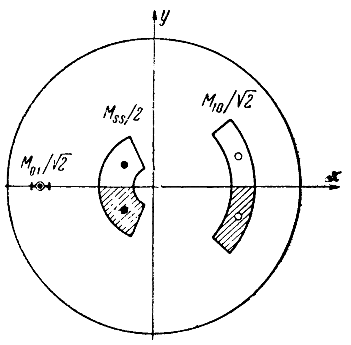

FIG. 3. Position of the elements of the matrix M_{pp}(640 MeV, 90°) on the complex plane. The absolute phase shift of the element M_{01} was taken equal to 180°. The ambiguity of the phase shifts of the elements M_{SS} and M_{10} (shaded region) arises as a result of the insufficient number of experiments made at $\theta = 90°$.

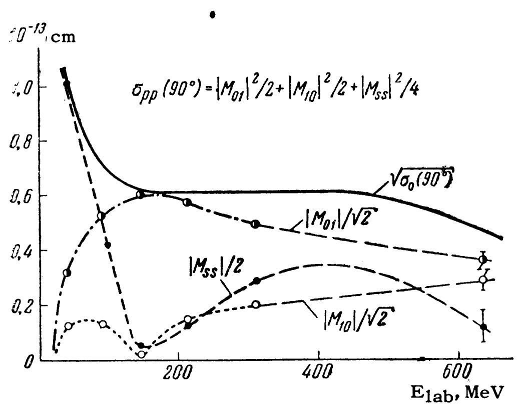

FIG. 4. Energy dependence of the moduli of the elements for the matrix $M_{pp}(90°)$. Between 310 and 640 MeV the behavior of the moduli of the elements M_{01}, M_{10}, and M_{SS} are represented by free curves.

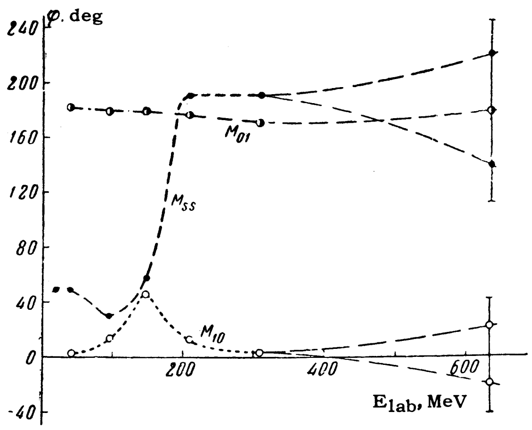

FIG. 5. Energy dependence of the phases of the elements for the matrix $M_{pp}(90°)$. The absolute phases of the element M_{01} (640 MeV, 90°) was taken equal to 180°. Between 310 and 640 MeV, the behavior of the matrix elements is shown as free curves.

regards the phases of elements M_{10} and M_{SS} arises as a result of the ambiguity of the arc-cosines and clearly indicates that for a unique reconstruction of the matrix $M_{pp}(90°)$ further experiments are necessary (in particular, a measurement of the parameter A).

B. Reconstruction of the matrix M_{pp} in the general case. In the case $\theta \neq 90°$, the scattering matrix M_{pp} can be reconstructed if, in addition to experiments (11), a number of more complex independent experiments are made.[14] Since at the present time no measurements even of the correlation coefficients C_{nn} and C_{KP} (at angles not equal to 90°) have been made, the matrix M_{pp} for these angles cannot be reconstructed.

In conclusion, we note that from the data of the phase-shift analysis [13] and the results of experiments at 640 MeV it follows that the values of the modulus $|C|$ of the spin-orbit amplitude $C = i(M_{10} - M_{01})/2\sqrt{2}$ for the scattering angle $\theta = 90°$ is approximately the same for 40, 95, 140, 210, 310, and 640 MeV. It is difficult to reconcile this energy dependence of $|C(90°)|$ with the hypothesis of Sakurai,[15] according to which this amplitude can be completely explained by the contribution from a diagram corresponding to the exchange of only one vector meson.

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Note added in proof: Recently, Hoshizaki and Machida (preprint RIFP-21, 1962, Kyoto University, Japan) carried out a phase-shift analysis of data obtained in our laboratory on the differential cross sections for polarization and depolarization in pp elastic scattering at 660 MeV. The best of these data are in agreement with the following set of the real parts of the phase-shifts obtained by them: $\delta(^1S_0) = -48°$; $\delta(^1D_2) = 9°$; $\delta(^1G_4) = 4°$; $\delta(^3P_0) = -38°$; $\delta(^3P_1) = -42°$; $\delta(^3P_2) = 20°$; $\delta(^3F_2) = 2°$; $\delta(^3F_3) = -3°$; $\delta(^3F_4) = -3.5°$. The imaginary parts of the phase shifts were estimated on the basis of the resonance model of pion production in pp collisions. The set of phase shifts given here is in agreement with the values of the parameter R obtained in our experiment.

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289