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ELECTRON-PHOTON SHOWERS PRODUCED BY HIGH ENERGY MUONS

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Integral spectra of shower electrons and photons in thick absorbers are calculated by taking into account direct formation of electron-positron pairs and bremsstrahlung radiation by high-energy μ mesons.

To study the spectrum of muons with energy above 10^{12} eV the installations usually employed consist of ionization chambers screened with a thick layer of a high-Z substance. The energy spectrum of the muons can be reconstituted from the large bursts generated by the muons as a result of bremsstrahlung. However, in view of the smallness of the differential cross section of the bremsstrahlung and the drooping character of the energy spectrum of the muons, the measurement of the spectrum at $E_\mu > 10^{12}$ eV becomes difficult.

In 1959 Alekseev and Zatsepin [1] pointed out the possibility of recording and measuring the energy of muons with $E_\mu > 10^{12}$ eV with the aid of a multi-layer system of ionization chambers, owing to the appreciable role that is played in this range of energies by the process of direct production of electron-positron pairs by the muon. Installations of this type should pick out the small bursts produced essentially by the direct pair production. To measure the energy of an individual muon with the aid of a multi-layer system of ionization chambers, it is necessary to know the dependence of the number of charged particles in the burst on the muon energy.

In the present paper we calculate the integral spectra of the electrons and photons in showers generated by high-energy muons ($E_\mu > 10^{12}$ eV), with account of the direct production of electron-positron pairs.

1. Let us consider first a shower generated by a single muon with energy E_μ.

The muon passing through the matter is a "source" of electrons and photons. Let $S_p(E, E_\mu, t)dE\,dt$ be the number of electrons, and let $S_\Gamma(E, E_\mu, t)dE\,dt$ be the number of photons with energies $(E, E+dE)$, generated by the muon in a layer $(t, t+dt)$ (t is measured in electron radiation units of length t_0). Then the equations of the cascade theory are written in the form

$$\frac{\partial}{\partial t} P(E, E_\mu, t) = L_1[P, \Gamma] + \beta \frac{\partial}{\partial E} P(E, E_\mu, t) + S_p(E, E_\mu, t),$$
$$\frac{\partial}{\partial t} \Gamma(E, E_\mu, t) = L_2[P, \Gamma] + S_\Gamma(E, E_\mu, t). \qquad (1)$$

Here $P(E, E_\mu, t)$ and $\Gamma(E, E_\mu, t)$ are functions which yield the average numbers of the electrons and photons respectively in the energy interval $(E, E+dE)$ at a depth t of the layer of substance under consideration. $L_1[P, \Gamma]$ and $L_2[P, \Gamma]$ are linear integral operators, which take into account the processes of radiative slowing down and formation of electron pairs [2,3]. If the layer of matter is not very thick ($t < 10^5$ g/cm^2), and the muon energy is $E \sim 10^{11}$ eV and higher, then we can neglect the change in energy due to the passage of the muon through the substance and the muon can be regarded as a constant source of electrons and photons, that is,

$$S_p(E, E_\mu, t) = S_p(E, E_\mu), \quad S_\Gamma(E, E_\mu, t) = S_\Gamma(E, E_\mu).$$

Let us assume that the muon generates electrons and photons only by direct creation of pairs and bremsstrahlung and let us assume for $S_p(E, E_\mu)$ the expression obtained by Ternovskiĭ [4] for the cross section in the case of total screening

$$S_p(E, E_\mu) = \frac{4\alpha}{3\pi} m^2 a w(E, E_\mu) \Big/ \ln\frac{183m}{Z^{1/3}};$$

$$w(E, E_\mu) = \int_0^{E_\mu - E} \frac{d\varepsilon}{(E+\varepsilon)^2} \ln\left(\frac{l\sqrt{1+x}}{\alpha Z^{1/3}}\right)\left\{\frac{E_\mu^2 + (E_\mu - E - \varepsilon)^2}{E_\mu^2}\left[A(x) + 2\frac{E^2+\varepsilon^2}{(E+\varepsilon)^2}B(x)\right] + \frac{(\varepsilon+E)^2}{E_\mu^2}\left[C(x) + 2\frac{E^2+\varepsilon^2}{(E+\varepsilon)^2}D(x)\right] + 8\frac{E\varepsilon}{(E+\varepsilon)^2}\frac{E_\mu - E - \varepsilon}{E_\mu(1+x)}\right\}, \qquad (2)$$

where

$$x = m^2 E\varepsilon / E_\mu (E_\mu - \varepsilon - E),$$

$$A(x) = (1+2x)\ln(1+1/x) - 2,$$

$$B(x) = (1+x)\ln(1+1/x) - 1,$$
$$C(x) = (1+2x)/(1+x) - 2x\ln(1+1/x),$$
$$D(x) = 1 - x\ln(1+1/x).$$

Here m is the ratio of the muon mass to the electron mass, E is the electron energy, α the fine-structure constant, Z the nuclear charge, a the ratio of the electron radiation length unit to the muonic one, and l a constant of order unity.

For $S_\Gamma(E, E_\mu)$ we have also in the case of total screening[5]

$$S_\Gamma(E, E_\mu) = \frac{a}{E}\left[1 + \left(1 - \frac{E}{E_\mu}\right)^2 + \frac{2}{3}\left(1 - \frac{E}{E_\mu}\right) + \frac{b}{9}\left(1 - \frac{E}{E_\mu}\right)\right];$$
$$b = 1/\ln(183\, m/Z^{1/3}). \tag{3}$$

If one muon of energy E_μ is incident on the boundary of the matter, then the boundary conditions have the form

$$P(E, E_\mu, 0) = \Gamma(E, E_\mu, 0) = 0. \tag{4}$$

Solving the system (1) with boundary conditions (4) we obtain for the integral spectra of the electrons $N_p(E, E_\mu, t)$ when $E < \beta$ and $t > 1$[2,3,6]

$$N_p(E, E_\mu, t) = N_p^{(1)}(E, E_\mu, t) + N_p^{(2)}(E, E_\mu, t), \tag{5}$$

where $N_p^{(1)}(E, E_\mu, t)$ is the contribution of the shower generated by direct pair production, and $N_p^{(2)}(E, E_\mu, t)$ is the contribution of the shower generated by the muon bremsstrahlung. When $t < y_1 = \ln(E_\mu/m\beta)$ we have

$$N_p^{(1)}(E, E_\mu, t) = \frac{4\alpha}{3\pi}\,\frac{m^2 a \ln(1/\alpha Z^{1/3})}{\ln(183\, m/Z^{1/3})}\,\frac{H_1(s)\,D(s)\,G(s, \varepsilon)\,\mathscr{E}_1(s)}{s\sqrt{2\pi\lambda_1''(s)\,t}} \times \frac{\exp(y_1 s + \lambda_1(s)\,t)}{\lambda_1(s)}, \tag{6}$$

where s is determined by the equation

$$\lambda_1'(s)\,t + y_1 = 0.$$

For $t \geq y_1$ we have

$$N_p^{(1)}(E, E_\mu, t) = \frac{4\alpha}{3\pi}\,\frac{ma\ln(1/\alpha Z^{1/3})}{\ln(183\, m/Z^{1/3})}\,\frac{H_1(1)\,D(1)\,G(1, \varepsilon)}{|\lambda_1'(1)|} \times \mathscr{E}_1(1)\,(E_\mu/\beta)\,\{1 + \Phi[(t-y)/\sqrt{2\lambda_1''(1)\,y_1}]\},$$
$$\Phi(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt. \tag{6a}$$

For $N_p^{(2)}(E, E_\mu, t)$ we obtain $[t < y = \ln(E_\mu/\beta)]$

$$N_p^{(2)}(E, E_\mu, t) = \frac{aH_1(s)\,D(s)\,G(s, \varepsilon)\,B^2(s)}{s(\lambda_1(s) + \sigma_0)\sqrt{2\pi\lambda_1''(s)\,t}}\,\frac{\exp(ys + \lambda_1(s)\,t)}{\lambda_1(s)}, \tag{7}$$

where s is determined from the equation

$$\lambda_1'(s)\,t + y = 0$$

and when $t \geq y$ we have

$$N_p^{(2)}(E, E_\mu, t) = \frac{a}{2}\,\frac{H_1(1)\,D(1)\,G(1, \varepsilon)\,B^2(1)}{\sigma_0\,|\lambda_1'(1)|}\,\frac{E_\mu}{\beta} \times \{1 + \Phi(|t-y|/\sqrt{2\lambda_1''(1)\,y})\}. \tag{7a}$$

For $N_\Gamma(E, E_\mu, t)$ we obtain

$$N_\Gamma(E, E_\mu, t) = \frac{H_1'(s)\,G_2(s, \varepsilon)}{H_1(s)\,G_1(s, \varepsilon)}\,N_p(E, E_\mu, t). \tag{8}$$

Here $\epsilon = Ef(\lambda_1(s))/\beta$, where β is the critical energy, $H_1(s)$, $H_1'(s)$, $D(s)$, $G(s, \epsilon)$, $G_2(s, \epsilon)$, $B(s)$, $f(\lambda)$, and $\lambda_1(s)$ and its derivatives are the known functions of cascade shower theory (see, for example [3]),

$$\mathscr{E}_1(s) = m^s\int_0^{E_\mu} dE\,E^s w(E, E_\mu). \tag{9}$$

It is seen from (6) and (6a) that in the first stage of its development the shower develops as if it were caused by an electron with energy E_μ/m (the smallness of the pair-production cross section is manifest only in the magnitude of the average number of charged particles). However, at depths $\sim 2y$ the shower is already in practical equilibrium with the radiation that generates it. For the energy $E_\mu/\beta \sim 10^6 - 10^8$ eV we have $y_1 \approx 11-16$, so that the depth at which equilibrium is established is $\sim 22-32\, t_0$. Analogously, a shower generated by muon bremsstrahlung reaches equilibrium at $(25-35)\, t_0$.

It must be noted that an account of the variation of the muon energy with depth introduces practically no changes and reduces only to multiplying (6a) and (7a) by $\exp(-Lt)$, where L is the fraction of the muon energy lost in one t_0 unit.

From (2) and (9) we obtain for $\mathscr{E}_1(s)$, accurate to terms of the next order in m^{-1}, for $0 < s < 2$,

$$\mathscr{E}_1(s) = \frac{\pi^2(1+\cos\pi s)}{\sin^2\pi s}\left[\left(\frac{6}{s} - \frac{4}{s+2}\right)s - \left(\frac{2}{s} - 1 - \frac{1}{s+2}\right)s(s+2) + \frac{1}{6}\left(1 - \frac{1}{s+2}\right)(s+4)(s+2)\,s\right]$$
$$+ \frac{\pi(1+\cos\pi s)}{\sin\pi s}\left[2\left(\frac{2}{s} - 1 - \frac{1}{s+2}\right) \times (s+1) - \left(\frac{6}{s} - \frac{4}{s+2}\right) - \frac{1}{6}\left(1 - \frac{1}{s+2}\right)((s+4)(s+2) + 2s(s+3)) + \left(\frac{2}{s} + \frac{10}{3} - \frac{10}{3(s+2)}\right) + \frac{1}{2}\left(1 - \frac{1}{s+2}\right)(s+2)\,s\right]. \tag{10}$$

For depths $t > 2y$, the total number of charged particles with $E_\mu/\beta \sim 10^6$ is $N_e \sim 200$ in the ground ($A = 20$, $Z = 10$) and $N_e \sim 100$ in lead.[1)]

2. We have neglected the contribution of the production of delta electrons by the muon. An account of the contribution of this process reduces to adding to (5) a term $N_p^{(3)}(E, E_\mu, t)$, which has the following form when $t < y$:

$$N_p^{(3)}(E, E_\mu, t) = 0{,}3\,\frac{Z}{A}\,\frac{m_e}{\beta}\,t_0\,\frac{H_1(s)\,D(s)\,G(s,\varepsilon)}{\lambda_1(s)\,s\sqrt{2\pi\lambda_1''(s)\,t}}$$
$$\times\, g(s)\exp\left[y(s-1)+\lambda_1(s)\,t\right], \qquad (11)$$

where $\lambda'(s)t + y = 0$

$$g(s) \approx \frac{1}{s-1}\left[1-\left(\frac{e^2}{m_e^2}\,\frac{NA}{ZE_\mu}\right)^{s-1}\right] \qquad (11a)$$

and when $t > y$

$$N_p^{(3)}(E, E_\mu, t) = 0{,}3\,\frac{Z}{A}\,\frac{m_e}{\beta}\,t_0\,\frac{H_1(1)\,D(1)\,G(1,\varepsilon)}{|\lambda_1'(1)|}$$
$$\times \ln\left(\frac{m_e^2}{e^2}\,\frac{ZE_\mu}{NA}\right)\{1+\Phi((t-y)/\sqrt{2\lambda_1''(1)\,y})\}. \qquad (12)$$

When $E_\mu > 10^{12}$ eV the contribution of this term is small.

3. The expressions presented above describe only the average picture of the shower. A very important role may be played in the measurement of the muon energy by fluctuations of two types: the uncertainty in the measurement of the energy, connected with the "decreasing" character of the energy spectrum of the cosmic muons, and the fluctuations in the development of the cascade showers. It is easy to see that the fluctuations of the former type introduce no essential changes but only to the appearance of an additional factor $q = \gamma/(\gamma-1)$ in (6a) and (7a), where γ is the index of the integral energy spectrum of the muons.

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[1)]Strictly speaking, the present analysis does not apply to heavy substances, since we have not taken into consideration the scattering of the particles and the dependence of the total absorption coefficient of the photons on the energy. However, calculations based on (5) give the correct order of magnitude of the total number of charged particles in the shower in heavy substances.

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