RADIATION REACTION IN THE CASE OF MEDIA WITH NEGATIVE ABSORPTION

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With the vibrations of an oscillator in a medium with negative absorption as an example, it is shown that the nature of the reaction depends on the initial conditions, and that oscillations can also build up. It may be important to take into account this circumstance in the study of radio-frequency and optical quantum generators.

If a radiator (for example a moving charge) is situated in a medium, then the radiation reaction may differ greatly from radiation in vacuum. The foregoing is well known on the basis of many examples, while a general analysis of the reaction force produced when a charge moves in an arbitrary transparent medium without spatial dispersion has already been carried out by us [1]. It is of interest to generalize these results to allow for absorption and spatial dispersion by the medium. By the same token, we take into consideration the reaction connected with the radiation of longitudinal (plasma) waves, which is particularly significant in a plasma [2,3]. In addition, all the ionization losses are taken into account in this way automatically (see, for example, the paper by Tsytovich [4]).

Without dwelling on this group of problems, here we touch only on one unique case, namely the radiation reaction for media with negative absorption. It is precisely such media that are dealt with when waves are amplified or generated (this pertains in particular to quantum generators—masers and lasers).

We assume, for simplicity, that the medium is isotropic and describe it by means of the complex dielectric constant

$$\varepsilon'(\omega) = \varepsilon_1 - i\varepsilon_2 = \varepsilon(\omega) - i4\pi\sigma(\omega)/\omega.$$

Then $\sigma > 0$ in an absorbing medium and $\sigma = -|\sigma| < 0$ in a medium with negative absorption [1)]. For plane waves

$$\mathbf{E} = \mathbf{E}_0 e^{-i(\omega t - \omega nz/c)} e^{-\omega\varkappa z/c};$$

in such a medium

$$(n - i\varkappa)^2 = \varepsilon', \quad n^2 - \varkappa^2 = \varepsilon$$

and

$$n\varkappa = -2\pi|\sigma|/\omega,$$

i.e., the signs of n and κ are different. The sign of ϵ can be arbitrary, but only when $\epsilon > 0$ does the index $\kappa \to 0$ when $\sigma \to 0$. We assume throughout that $\epsilon > 0$.

Let us calculate now the work done by the field on the particle

$$A = \int_0^T e\mathbf{v}\mathbf{E}dt$$

where v(t) is the velocity of a particle with charge e. In the absence of an external field, the work A is equal to the change in the particle energy due to the radiation of transverse electromagnetic waves and due to polarization losses; if spatial dispersion is taken into account, the latter loss is none other than the energy of the radiated longitudinal waves (we are dealing here with an isotropic medium; the losses connected with "short-range" collisions are not discussed here).

In an absorbing medium (when $\sigma > 0$) the work is $A < 0$, i.e., the particle slows down. For example, for an oscillator (a charge moving with velocity $v = v_0 \cos \nu_0 t$) the work of the radiated transverse field over the time T is in the dipole approximation

$$A = -e^2 v_0^2 \nu_0^2 \sqrt{\varepsilon} T/3c^3, \tag{1}$$

where it is assumed that the medium is weakly absorbing, i.e.,

$$4\pi|\sigma|/\omega \ll \varepsilon \tag{2}$$

(for an absorbing medium $\sigma > 0$, but we have used the absolute-value sign in (2) to be able to use this inequality later on also when $\sigma < 0$; the frequency ω is assumed positive throughout, $\omega > 0$).

If we now use the standard procedure under condition (2), but with $\sigma < 0$, (see, for example, [5]) we obtain the result (1) with the sign reversed. The same (the reversal of the sign of A) occurs in the case of Cerenkov radiation [2)] and for any other

[1)]We can say, using a different terminology, that the medium with $\sigma < 0$ employed here is absolutely unstable. We note also that the inequality $\sigma(\omega) < 0$ is assumed satisfied only in a finite frequency interval.

[2)]This was noted earlier by V. P. Silin.

motion. This result, namely the build-up of oscillations in a medium with negative absorption, cannot be regarded as correct a priori. In fact, when $\sigma < 0$ the waves in the medium become more intense, and it is clear from this alone that the medium has a reserve of energy. Therefore upon radiation the source could build up at the expense of the energy reserves in the medium.

There is no doubt, however, that when $\sigma < 0$ the question of the radiation and the reaction force must be reexamined. It is sufficient to state that when $\sigma < 0$ the radiation field in the medium builds up even in the absence of a source, and by the same token the problem can depend essentially on the initial conditions. We shall show that this is really the case.

The purpose of the present article is to clarify the character of the radiation reaction [3] when $\sigma < 0$ only to the extent of understanding the physical picture of the process. We therefore confine ourselves to an examination of one very simple problem, radiation from an oscillator. In this case the simplest and clearest calculation is carried out by the so-called Hamiltonian method. We represent the vector potential of the field in the form

$$\mathbf{A} = \sum_{\lambda,\, i=1,\,2} q_{\lambda i}(t)\, \mathbf{A}_{\lambda i}, \qquad \mathbf{A}_{\lambda i} = \sqrt{4\pi c^2}\, \mathbf{a}_{\lambda i} e^{-i\mathbf{k}_\lambda \mathbf{r}},$$

$$\operatorname{div} \mathbf{A}_{\lambda i} = 0, \qquad \mathbf{k}_\lambda \mathbf{a}_{\lambda i} = 0,$$

$$\mathbf{a}_{\lambda 1}\mathbf{a}_{\lambda 2} = 0, \qquad \int \mathbf{A}_{\lambda i}\mathbf{A}^*_{\mu j} dv = 4\pi c^2 \delta_{\lambda\mu}\delta_{ij}. \tag{3}$$

The equation for $\mathbf{A}$ has the form

$$\Delta \mathbf{A} - \frac{4\pi\hat{\sigma}}{c^2}\frac{\partial \mathbf{A}}{\partial t} - \frac{1}{c^2}\frac{\partial}{\partial t}\hat{\varepsilon}\frac{\partial \mathbf{A}}{\partial t}$$

$$= -\frac{4\pi}{c}\dot{\mathbf{j}}_{ext} + \frac{4\pi\hat{\sigma}}{c}\nabla\varphi + \frac{1}{c}\frac{\partial}{\partial t}\hat{\varepsilon}\nabla\varphi, \tag{4}$$

where φ is the scalar potential and, in connection with the presence of dispersion, $\hat{\epsilon}$ and $\hat{\sigma}$ are operators. It will be shown in the appendix how the problem must be solved in the presence of dispersion. For the time being we put $\hat{\sigma} = \sigma(\nu_0)$ and $\hat{\epsilon} = \epsilon(\nu_0)$, and also

$$\frac{1}{c}\int \mathbf{j}_{ext}\, \mathbf{A}^*_{\lambda i} dV = \sqrt{4\pi}\mathbf{a}_{\lambda i}\mathbf{v}_0 \cos\nu_0 t$$

(the dipole approximation for the oscillator). Then, after substituting (3), in (4), multiplying by $\mathbf{A}^*_{\lambda j}$, and integrating over the volume, we get

$$\ddot{q}_\lambda + 2\mu_\lambda \dot{q}_\lambda + \nu_\lambda^2 q_\lambda = \frac{1}{\varepsilon}\sqrt{4\pi}e \sin\theta v_0 \cos\nu_0 t$$

$$= f_0 \cos\nu_0 t = f(t);$$

$$\mu \equiv \mu_\lambda = 2\pi\sigma/\varepsilon, \quad \nu_\lambda^2 = c^2 k_\lambda^2/\varepsilon,$$

$$q_\lambda \equiv q_{\lambda 1}, \quad \mathbf{a}_{\lambda 1}\mathbf{v}_0 = v_0 \sin\theta, \quad \mathbf{a}_{\lambda 2}\mathbf{v}_0 = 0. \tag{5}$$

The solution of the system (5) under arbitrary initial conditions is elementary. The forced solution of Eq. (5) is in this case

$$\dot{q}_{\lambda f}(t) = \frac{f_0}{(\nu_\lambda^2 - \nu_0^2)^2 + 4\mu^2\nu_0^2}$$

$$\times \{-\nu_0(\nu_\lambda^2 - \nu_0^2)\sin\nu_0 t + 2\mu\nu_0^2 \cos\nu_0 t\}$$

and leads to the already mentioned formula for the radiation reaction A [subject to condition (2)]

$$A = -\tfrac{1}{3} e^2 v_0^2 \nu_0^2 \sqrt{\varepsilon} T c^{-3} \operatorname{sign}\mu. \tag{6}$$

This means that when $\sigma < 0$ oscillations build up. Physically this is quite obvious, inasmuch as Eq. (5) is the oscillation equation for the field oscillators q_λ, with the oscillation of q_λ when $\mu > 0$ always "retarded" relative to the external force f; on the other hand, if $\mu < 0$, the oscillation of q_λ is "advanced" relative to the external force. Therefore the work done by the force f on the field oscillator depends on the sign of μ even as $\mu \to 0$. On the other hand, if the oscillator starts radiating at a certain instant of time $t = 0$, then, of course, the value of A remains the same as in the case of forced oscillations, provided only that the field existing in the medium at the instant when the oscillator was turned on was identical with the field of the forced oscillations [this means that $q_\lambda(0) = q_{\lambda,f}(0)$ and $\dot{q}_\lambda(0) = \dot{q}_{\lambda,f}(0)$].

Let us consider now in somewhat greater detail the character of the radiation reaction in a medium with $\sigma < 0$ in the case when all the fields were equal to zero prior to turning on the oscillator, i.e., $q_\lambda(0) = \dot{q}_\lambda(0) = 0$. Then we obtain in the usual fashion

$$A = \int_0^T e\mathbf{v}\mathbf{E}\, dt = -\frac{e^2 v_0 \sqrt{\varepsilon}}{3\pi c^3}\int_{\nu=0}^{\infty} d\nu \int_{t=0}^{T} \frac{\nu^2 \cos\nu_0 t}{\nu^2 - \nu_0^2 - 2i\mu\nu_0}$$

$$\times \left\{ i\nu_0 e^{i\nu_0 t} - \frac{(i\nu_0 - r_2) r_1}{2\sqrt{\mu^2 - \nu^2}} e^{r_1 t} + \frac{(i\nu_0 - r_1) r_2}{2\sqrt{\mu^2 - \nu^2}} e^{r_2 t} \right\} dt + \text{c.c.};$$

$$\nu \equiv \nu_\lambda, \; r_1 = -\mu + \sqrt{\mu^2 - \nu^2}, \; r_2 = -\mu - \sqrt{\mu^2 - \nu^2},$$

$$\mathbf{E} = \dot{\mathbf{E}}(t, \mathbf{r} = 0) = -4\pi \sum_\lambda \mathbf{a}_{\lambda 1}\dot{q}_{\lambda 1}(t). \tag{7}$$

We recall that the first term in the curly bracket of the expression (6) $i\nu_0 \exp(i\nu_0 t)$ corresponds

[3] Inasmuch as we take into consideration the effect produced on a particle not only by the field radiated by it but also by the field in the medium, the term "radiation reaction" assumes, of course, a somewhat arbitrary character.

to forced oscillation, and the other two correspond to natural oscillations of the system (5). When $\mu > 0$ and $\mu T \gg 1$ the terms corresponding to the natural oscillations in (7) are inessential and, for example, when $\mu \ll \nu_0$ [see (2)], using the formula

$$\lim_{\mu\to 0}\frac{\mu\nu^2}{(\nu^2-\nu_0^2)^2+4\mu^2\nu_0^2}=\frac{\pi}{4}\delta(\nu-\nu_0)\,\mathrm{sign}\,\mu, \tag{8}$$

we obtain the expression (1).

To analyze the case $\mu < 0$ it is convenient to rewrite (7) in the form

$$A=-\frac{4e^2v_0^2\sqrt{\varepsilon}}{3\pi c^3}\int_{t=0}^{T}dt\left\{\int_{\nu=0}^{\infty}\frac{\nu^2 d\nu\cos\nu_0 t}{\zeta}J_1(\nu,t)+\int_{\nu=0}^{\mu}\frac{\nu^2 d\nu\cos\nu_0 t}{\zeta}J_2(\nu,t)+\int_{\nu=\mu}^{\infty}\frac{\nu^2 d\nu\cos\nu_0 t}{\zeta}J_3(\nu,t)\right\};$$

$$\zeta=(\nu^2-\nu_0^2)^2+4\mu^2\nu_0^2,$$

$$J_1=\nu_0\left[-(\nu^2-\nu_0^2)\sin\nu_0 t+2\mu\nu_0\cos\nu_0 t\right],$$

$$J_2=(\mu^2-\nu^2)^{-1/2}\{[-2\mu\nu_0^2 r_1+(\nu^2-\nu_0^2)r_1 r_2]e^{r_1 t}+[2\mu\nu_0^2 r_2-r_1 r_2(\nu^2-\nu_0^2)]e^{r_2 t}\},$$

$$J_3=e^{-\mu t}\{[\nu^2(\nu^2-\nu_0^2)+2\mu^2\nu_0^2]\sin(\sqrt{\nu^2-\mu^2}t)/\sqrt{\nu^2-\mu}-2\mu\nu_0^2\cos\sqrt{\nu^2-\mu^2}t\}. \tag{9}$$

When integrating with respect to ν in (9) it is necessary to exercise a certain caution. This is connected, in particular, with the fact that formula (8) under the integral sign can be used only in the case when the integrand $f(\nu, t, \mu)$ as a function of ν varies little in the interval $|\nu - \nu_0| \approx |\mu|$ for all values of $t \leq T$. If we take this circumstance into account, then it turns out that the character of the behavior of the expression for the work of the field A depends essentially on the value of the parameter $|\mu|T$ (we take into consideration below only time intervals $\nu_0 T \gg 1$). Indeed, when $|\mu|T \ll 1$ and $\nu_0 T \gg 1$ the main contributions to the value of A [see (8)] are made by the terms with J_1 and J_3, which contain for values of ν close to ν_0 the least rapidly oscillating expression for Φ:

$$\Phi=2\mu\nu_0^2\cos\nu_0 t\,[\cos\nu_0 t-e^{-\mu t}\cos(\sqrt{\nu^2-\mu^2}t)]$$

or, leaving only the difference frequencies,

$$\Phi\approx\mu\nu_0^2[1-e^{-\mu t}\cos[(\nu_0-\sqrt{\nu^2-\mu^2})\,t]].$$

Thus, under the assumption made that $|\mu|T \ll 1$, the value of Φ changes little in the frequency interval $|\nu - \nu_0| \approx |\mu|$ and we can employ formula (8) (see above). As a result we have

$$A(T)=-\frac{e^2v_0^2\nu_0^2\sqrt{\varepsilon}}{3c^3}T\frac{|\mu|T}{2},\quad T\nu_0\gg 1,\quad |\mu|T\ll 1. \tag{10}$$

It follows therefore that in the time interval $1/|\mu| \gg T \gg 1/\nu_0$ the radiation reaction corresponds to friction independently of the sign of μ. In both cases the oscillator consumes an equal amount of energy for the excitation of the electromagnetic oscillations in the medium. The picture changes, however, quite radically at time intervals T so large that the following condition is satisfied

$$|\mu|T\gg 1. \tag{11}$$

Unlike the case $\mu > 0$, where the radiation reaction is characterized by expression (1), the values of A when $\mu < 0$ and condition (11) is satisfied execute exponentially-growing oscillations even for small values of $|\mu|$. This is connected with the fact that the term J_1 in (9), corresponding to the forced oscillations, increases linearly, while J_2 and J_3 contain expressions that increase exponentially with increasing T.

A direct estimate of the integral containing J_2 [see (9)] yields for $|\mu| \ll \nu_0$

$$A^{(J_2)}(T)\approx\frac{e^2v_0^2\sqrt{\varepsilon}}{3c^3}\frac{\mu^4}{\nu_0^3}e^{2|\mu|T}\sin\nu_0 T,$$

$$|\mu|T\gg 1,\qquad |\mu|\ll\nu_0,\ \mu<0. \tag{12}$$

Thus, the value of $A^{(J2)}$, roughly speaking, oscillates with frequency ν_0, and the amplitude of the oscillations of $A^{(J2)}$ increases exponentially.

Let us proceed to estimate the third term in (9). It is easy to see that when $|\mu| \to 0$ and $\mu < 0$ the main contribution to the expression $\Phi_1 = J_3 \times \cos(\nu_0 t)/\zeta$ is made by the region of frequencies close to ν_0 ($|\nu - \nu_0| \approx |\mu|$), but formula (8) becomes inapplicable by virtue of condition (11). Indeed, even the term in Φ_1 with the slowest oscillations has the form $\mu\nu_0^2\zeta^{-1}[\exp(-\mu t)]\cos\Omega t$, where the difference frequency is $\Omega = \nu_0 - \sqrt{\nu^2 - \mu^2}$. In view of this, $\Phi_1(\nu)$ subject to condition (11) is an oscillating function in an appreciable interval of the values of ν (when $|\nu - \nu_0| \approx |\mu|$). In particular, if $\mu T \cdot \mu/\nu_0 \ll 1$, then we can write $\cos\Omega t = \cos\chi t$, where the new variable is $\chi = \nu - \nu_0$.

Then, carrying out elementary integration [see (9)] with respect to t and with respect to χ within the limits of the width of the resonance curve $\zeta(\nu)$ $-|\mu| < \chi < |\mu|$, we obtain approximately (we are writing out only the term with the slowest oscillations)

$$A^{(J_3)}(T) = \frac{e^2 v_0^2 \nu_0^2 \sqrt{\varepsilon} T}{3c^3} \frac{e^{|\mu| T}}{(T\mu)^2} (\sin |\mu| T + \varphi);$$

$$|\mu| T \gg 1, \mu < 0, \mu T \cdot \mu / \nu_0 \ll 1, \quad (13)$$

where φ is a certain constant phase with order of magnitude unity. It follows from (13) that the quantity $A^{(J_3)}(T)$ oscillates in this case with a frequency equal to $|\mu|$, and the amplitude of the oscillations increases as $T[\exp(|\mu| T)]/(\mu T)^2$. The oscillation frequency of the term $A^{(J_3)}$ is much smaller than $A^{(J_2)}$ when $|\mu| \ll \nu_0$. We note that the character of the behavior of $A^{(J_3)}$ remains the same even at such large values of T that the condition $T\mu^2/\nu_0 \ll 1$ is not satisfied.

Finally, the forced oscillations are characterized by a term $A^{(J_1)}$ which, as can be readily verified, coincides with expression (6) when $T\nu_0 \gg 1$ and $|\mu| \ll \nu_0$. It follows from (6), (12), and (13) that when $|\mu| \ll \nu_0$ the radiation reaction first ($|\mu| T \ll 1$) hinders the oscillations of the oscillator (i.e., $A < 0$) regardless of the sign of μ, and then, when $|\mu| T \gg 1$ and $\mu < 0$, the work done by the field on the oscillator becomes an oscillating quantity with a minimum oscillation frequency of the order of $|\mu|$. Then the amplitude of the oscillations of A increases exponentially. Thus, for the case of media with negative absorption ($\sigma < 0$) for $|\mu| T \gg 1$, generally speaking, there is no particular sense in speaking of the sign of the radiation friction, for it varies periodically in time. Further, the character of the radiation reaction depends on the initial conditions. For example, if at the instant when the oscillator is turned on there is present in the medium a field coinciding with the field of the forced oscillations, the radiation reaction corresponds to the buildup of oscillations [see (6)]. On the other hand, in the case of zero initial conditions (i.e., in the case when the field in the medium is zero when $t = 0$), the character of the behavior of the radiation reaction with time is more complicated, as was already discussed [see (10)—(13)].

In the present article we attempted only to clarify the character of the radiation reaction in the case of media with negative absorption (amplification). It is also natural to raise the question of the possible value of this reaction in various systems containing amplifying media. We refer specifically, above all, to quantum generators operating in the radio band (masers) and in the optical regions (lasers). In such cases it is necessary to take into account the fact that the medium is bounded in space (this is equivalent to a certain degree to a limitation on the time T), and also go beyond the limits of linear electrodynamics (obviously the medium was regarded as linear in the discussion above). It is also quite clear that the results can be extended to include the case of media with spatial dispersion (in this case it is interesting to consider systems not only with absolute instability but also with convective instability). In addition, it is necessary to emphasize that the oscillator oscillations were assumed specified above, yet the necessity may arise of taking into account the influence of the radiation reaction on the motion of the radiating system (the oscillator). Finally, it is advantageous to carry out a quantum analysis of the question of radiation reaction for systems with two and more levels in the presence of a medium with $\mu < 0$.

APPENDIX

The use of the Hamiltonian method, i.e., the use of an equation of the type (5), is very convenient in the solution of many electrodynamic problems (particularly in the case of an anisotropic medium, where methods other than the Hamiltonian or the related Fourier method, are either lacking or are poorly developed). This, however, raises the question of consistent account of dispersion. It was indicated earlier [6] that in a transparent medium it is possible to obtain the correct result by ignoring the dispersion in the calculation, but by assuming in the final result that the refractive index depends on the frequency. The same follows from [1,7], and also from the calculations of V. V. Zheleznyakov. However, recognizing that this aspect was never discussed in the literature with sufficient detail and is at the same time quite important, particularly for media with negative absorption, we present here the corresponding analysis.

From Eq. (4) we can readily obtain in analogy with the transition to the system (5)

$$\hat{\varepsilon}\ddot{q}_\lambda + 4\pi\hat{\sigma}\dot{q}_\lambda + k_\lambda^2 c^2 q_\lambda = f(t);$$

$$\hat{\varepsilon} e^{i\omega t} = \varepsilon(\omega) e^{i\omega t}, \qquad \hat{\sigma} e^{i\omega t} = \sigma(\omega) e^{i\omega t}. \quad (A.1)$$

The forced solution (A.1) is written in the form

$$q_\lambda(t) = \int_{-\infty}^{\infty} e^{i\omega t} q_{B\lambda\omega}\, d\omega, \quad f(t) = \int_{-\infty}^{\infty} e^{i\omega t} f_\omega d\omega; \quad (A.2)$$

$$q_{B\lambda\omega} = f_\omega [-\varepsilon(\omega)\omega^2 + 4\pi i\omega\sigma(\omega) + k_\lambda^2 c^2]^{-1}. \quad (A.3)$$

To find the solution of the homogeneous equation

$$\hat{\varepsilon}\ddot{q}_\lambda + 4\pi\hat{\sigma}\dot{q}_\lambda + k_\lambda^2 c^2 q_\lambda = 0 \quad (A.4)$$

we proceed in the usual fashion. Putting

$$q_\lambda = q_{\lambda 0} e^{rt}, \; r = i\omega,$$

we obtain the dispersion relation

$$\varepsilon(-ir)\, r^2 + 4\pi\sigma(-ir)\, r + k_\lambda^2 c^2 = 0. \qquad (A.5)$$

This equation has, generally speaking, several roots $r_j = i\omega_j$ which depend on k. If there are two such roots (j = 1, 2) then the general solution of (A.1) has the form

$$q_\lambda(t) = \int_{-\infty}^{\infty} q_{B\lambda\omega} e^{i\omega t} d\omega + q_{\lambda 1} e^{r_1 t} + q_{\lambda 2} e^{r_2 t}, \qquad (A.6)$$

where the amplitudes of the natural oscillations are determined from the initial conditions.

Thus for the case considered above, when $q_\lambda(0) = \dot{q}_\lambda(0) = 0$ and $f(t) = f_0 \cos \nu_0 t$, we obtain in lieu of (7)

$$A = -\frac{e^2 v_0^2 \sqrt{\varepsilon_0}}{3\pi c^3} \int_{\nu=0}^{\infty} d\nu \int_{t=0}^{T} \frac{\nu^2 \cos \nu_0 t}{\nu^2 - \nu_0^2 - 2i\mu\nu_0}$$

$$\times \left\{ i\nu_0 e^{i\nu_0 t} + \frac{(i\nu_0 - r_2)\, r_1}{r_2 - r_1} e^{r_1 t} + \frac{(i\nu_0 - r_1)\, r_2}{r_1 - r_2} e^{r_2 t} \right\} dt + \text{c.c.},$$

$$\nu = kc / \sqrt{\varepsilon(\nu_0)}, \quad \mu = \mu(\nu_0) \doteq 2\pi\sigma(\nu_0) / \varepsilon(\nu_0), \ \varepsilon_0 \equiv \varepsilon(\nu_0). \qquad (A.7)$$

It is thus easy to see that if dispersion is taken into account it is necessary to put in formulas (10) and (13) $\mu = \mu(\nu_0)$ while in expression (12), the value of which is determined by the small values of ν, we must put

$$\mu = \mu(\nu) \approx \mu(0) = 2\pi\sigma(0)/\varepsilon(0).$$

It was assumed above that $\epsilon(\omega) \neq 0$, or at any rate the possible occurrence of longitudinal waves was disregarded. In addition, if the dispersion condition (A.5) has more than two roots, it is necessary to specify for t = 0 higher derivatives of q(t). This circumstance reflects the fact that in the case of an arbitrary dispersive medium the initial-value problem is solved correctly only if account is taken of the field in the medium when t < 0 (see [8], Sec. 5).

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316