SOVIET PHYSICS JETP VOLUME 16, NUMBER 5 MAY, 1963

MOMENTS OF INERTIA OF A HEATED NUCLEUS AND ANGULAR ANISOTROPY OF FISSION FRAGMENTS

Yu. T. GRIN'

Submitted to JETP editor June 8, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **43**, 1880-1884 (November, 1962)

The moments of inertia of a nucleus at a temperature $T \neq 0$ are calculated as functions of the excitation energy of the nucleus. Good agreement between the theory and experiment is found for the angular anisotropy of the fission fragments.

THE angular anisotropy of fission fragments is determined by the moment of inertia $J_{eff} = (1/J_{\parallel} - 1/J_{\perp})^{-1}$[1], which by now has been sufficiently well determined by experiment [2,3]. It turns out that J_{eff} is several times smaller than that for a solid body. Halpern and Strutinskiĭ [1] proposed that this decrease is due to pair correlation of the nucleons in the nucleus [1].

It is therefore of interest to calculate J_{eff} theoretically as a function of the temperature and the nucleon excitation energy with account of pair-correlation effects.

The theory of the moment of inertia of a nucleus in the ground state has been sufficiently well developed [4,5]. It has been shown that $J_{\parallel}$—the moment of inertia about the symmetry axis of the nucleus—is equal to zero, while the moment of inertia $J_{\perp}$ about an axis perpendicular to the symmetry axis is several times smaller than the solid-body value J_0. If the pair correlation of the particles is disregarded, then $J_{\perp} = J_0$.

In the present paper a single method is used to obtain expressions for $J_{\perp}$ and $J_{\parallel}$ at $T \neq 0$, with $J_{\parallel}$ other than zero. At the phase-transition point with temperature $T = T_c$, when the superconducting state goes over into the normal state, and at higher temperatures, the moment of inertia of the nucleus is that of a solid body. The temperature can be readily related with the excitation energy of the nucleus by means of the formulas of superconductivity theory. The theoretical values of J_{eff} obtained for U^{234}, U^{236}, and Pu^{240} at excitation energies from 3 to 7 MeV are in very good agreement with the experimental data when the nucleon pair correlation energy is ~ 10 MeV. The deformation corresponds here to the fission barrier. Estimates obtained for deformations corresponding to neck-scission yield for J_{eff} values that are 20% less than for the fission barrier, and are in somewhat poorer agreement with experiment. It is difficult, however, to draw any unambiguous conclusion in this case, since the energy consumed in excitation as the nucleus descends from the barrier is unknown.

1. The moments of inertia can be readily obtained from the relation

$$J_i = \frac{\langle M_i \rangle}{\Omega_i} = \frac{\mathrm{Sp}\, M_i \rho}{\Omega_i}, \tag{1}$$

where $\langle M_i \rangle$ is the average momentum of the system, Ω_i is the angular velocity about the i axis, and ρ is the system density matrix. The mean value of M_i is calculated in the same manner as in Migdal's paper [4], except that G and F are replaced by the thermodynamically averaged Green's functions [6]

$$G(\varepsilon, \omega) = \frac{u^2(1-n)}{\omega - E + i\delta} + \frac{v^2 n}{\omega + E + i\delta} + \frac{v^2(1-n)}{\omega + E - i\delta} + \frac{u^2 n}{\omega - E - i\delta},$$

$$F(\varepsilon, \omega) = iuv\Big[\frac{1-n}{\omega - E + i\delta} - \frac{n}{\omega + E + i\delta} - \frac{1-n}{\omega + E - i\delta} + \frac{n}{\omega - E - i\delta}\Big];$$

$$u = \sqrt{\tfrac{1}{2}\left(1 + \tfrac{\varepsilon}{E}\right)}, \quad v = \sqrt{\tfrac{1}{2}\left(1 - \tfrac{\varepsilon}{E}\right)}, \quad E = \sqrt{\varepsilon^2 + \Delta^2},$$

$$n = \left[\exp\left(\frac{E}{T}\right) + 1\right]^{-1}, \tag{2}$$

Δ determines the pair correlation and depends on the temperature in a known fashion.

After simple calculations we obtain

$$J_i = -\sum_{12} |M_i|^2_{12} (G_1 G_2 - F_1 F_2) = \sum_{12} \Big[\frac{(u_1 v_2 - v_1 u_2)^2 \left(\mathrm{th}\frac{E_1}{2T} + \mathrm{th}\frac{E_2}{2T}\right)}{2(E_1 + E_2)} + \frac{(u_1 u_2 + v_1 v_2)^2 \left(\mathrm{th}\frac{E_1}{2T} - \mathrm{th}\frac{E_2}{2T}\right)}{2(E_1 - E_2)}\Big] |M_i|^2_{12}. \tag{3*}$$

Here $(M_i)_{12}$ is the matrix element of the operator

*th = tanh.

for the projection of the angular momentum along the i axis. In formula (3) we have left out for simplicity the part of the moment of inertia connected with the change in the pair correlation due to rotation. The matrix element M_z is diagonal, and therefore

$$J_{\parallel} = \sum_{12} |M_z|^2_{12} \frac{1}{2} \lim_{1\to 2} \frac{1}{E_1 - E_2}\left(\operatorname{th}\frac{E_1}{2T} - \operatorname{th}\frac{E_2}{2T}\right)$$

$$= \frac{1}{4T}\sum_{1} |M_z|^2_{11} \frac{1}{\operatorname{ch}^2(E_1/2T)}. \qquad (4)†$$

Changing from summation to integration and assuming that the matrix element is a smooth function on the Fermi surface we obtain

$$J_{\parallel} = \frac{1}{4T}\int_{-\infty}^{+\infty} \frac{d\varepsilon}{\operatorname{ch}^2(\sqrt{\varepsilon^2+\Delta^2}/2T)} \sum_{1} |M_z|^2_{11}\delta(\varepsilon_1)$$

$$= \frac{2\Delta}{T}\sum_{m=1}^{\infty}(-1)^{m+1} m K_1\left(\frac{m\Delta}{T}\right) J^0_{\parallel}. \qquad (5)$$

Here

$$J^0_{\parallel} = \sum_{1} |M_z|^2_{11}\delta(\varepsilon_1)$$

is the moment of inertia of the solid body about the z axis, and K_1 is a cylindrical function. Formulas (4) and (5) were first derived by Strutinskiĭ by a semiclassical method.

When $T \ll \Delta$ we need retain in formula (5) only the first term in the sum over n and, using the asymptotic expression for K_1, we get

$$J_{\parallel} = \sqrt{2\pi\Delta_0/T}\, e^{-\Delta_0/T} J^0_{\parallel}, \quad \Delta_0 = \Delta|_{T=0}. \qquad (6)$$

Thus $J_{\parallel}$ tends exponentially to zero when $T \ll \Delta$.

For T close to the transition temperature T_c we have $\Delta \to 0$ and $\Delta/2T \ll 1$. The significant region of integration with respect to $(\epsilon/2T)$ in formula (5) is of the order of unity. Expanding the $\cosh^2$ in the integrand of (5) in powers of Δ/T, we obtain

$$J_{\parallel} = \left[1 - \left(\frac{\Delta}{2T}\right)^2 \int_0^{\infty}\left(\frac{\operatorname{th} x}{x}\right)^2 dx\right] J^0_{=},$$

$$J_{\parallel} = \left[1 - 0.9\left(1 - \frac{T}{T_c}\right)\right] J^0_{\parallel} \qquad (7)$$

for $T = T_c$ and $J_{\parallel} = J^0_{\parallel}$. When $\Delta = 0$ it follows from (5) that $J_{\parallel} = J^0_{\parallel}$, i.e., the moment of inertia about the symmetry axis is equal to the solid-body moment of inertia, no matter how low the temperature, and vanishes abruptly when T = 0. Actually, however, the thermodynamic formulas for the nucleus can be used only when the excitation energy is much smaller than the distance between the levels of the system, i.e., $T \gg 1/\rho_0$, where ρ_0 is the level density on the Fermi surface. On the other hand, in the region $T \sim 1/\rho_0$ a thermodynamic analysis is incorrect and J changes from 0 to J^0. Let us now calculate $J_\perp$. The operator M_x has nondiagonal matrix elements. Introducing the notation $d = \epsilon_1 - \epsilon_2$, we can readily rewrite (3) in the form

$$J_\perp = \sum_{12}\left\{1 + \frac{\Delta^2}{d}\int_0^{\infty}\left[\frac{\operatorname{th}[\sqrt{\Delta^2+(\varepsilon+d/2)^2}/2T]}{\varepsilon\sqrt{\Delta^2+(\varepsilon+d/2)^2}} - \frac{\operatorname{th}[\sqrt{\Delta^2+(\varepsilon-d/2)^2}/2T]}{\varepsilon\sqrt{\Delta^2+(\varepsilon-d/2)^2}}\right] d\varepsilon\right\} |M_x|^2_{12}\delta(\varepsilon_1).$$

Inasmuch as $d/4T \gtrsim 1$ and the significant values of ϵ in the integrand are near zero, we can put $\epsilon = 0$ in the continuous function $\tanh[\sqrt{\Delta^2 + (\epsilon \pm d/2)^2}/2T]$ and obtain

$$J_\perp = \sum_{12}\left\{1 - g\left(\frac{d}{2\Delta}\right)\operatorname{th}\frac{\sqrt{\Delta^2+d^2/4}}{2T}\right\} |M_x|^2_{12}\delta(\varepsilon_1),$$

$$g(x) = \frac{\operatorname{arc\,sh} x}{x\sqrt{1+x^2}}. \qquad (8)^*$$

We carry out the calculations that follow for an oscillator potential. Calculating the sum (8) in the usual quasiclassical method [4], we obtain

$$J_\perp = \left\{1 - \frac{1}{d_1^2+d_2^2}\left[d_2^2 g\left(\frac{d_1}{2\Delta}\right)\operatorname{th}\frac{\sqrt{\Delta^2+d_1^2/4}}{2T} + d_1^2 g\left(\frac{d_2}{2\Delta}\right)\operatorname{th}\frac{\sqrt{\Delta^2+d_2^2/4}}{2T}\right]\right\} J^0_\perp,$$

$$J^0_\perp = \sum_{12} |M_x|^2_{12}\delta(\varepsilon_1), \quad d_1 = \omega_x - \omega_z, \quad d_2 = \omega_x + \omega_z. \qquad (9)$$

Here $J^0_\perp$ is the moment of inertia corresponding to the solid body, while ω_x and ω_z are the oscillator frequencies along the x and z axes. When $T \to 0$ the moment $J_\perp$ tends exponentially to the quanity

$$J_\perp = J^0_\perp\left[1 - \frac{d_2^2 g(d_1/2\Delta) + d_1^2 g(d_2/2\Delta)}{d_1^2+d_2^2}\right],$$

obtained by Migdal [4] for the moment of inertia of the ground state. As $T \to T_c$, $\Delta \to 0$, and $g \to 0$ we get $J_\perp \to J^0_\perp$.

2. The angular anisotropy of the fission fragments is determined by the value of the effective moment of inertia $J_{eff} = (1/J_{\parallel} - 1/J_\perp)^{-1}$. It is therefore interesting to know this value as a function of the excitation energy of the nucleus. The energy of the heated nucleus can be written in the form

$$E = E_0 - \frac{\rho_0\Delta^2}{4} + \frac{TS}{2},$$

where E_0 is a constant, ρ_0 is the density of the

†ch = cosh.

*arc sh = $\sinh^{-1}$.

single-particle levels on the Fermi surface, and S is the entropy. Then the excitation energy E* is

$$E^* = TS/2 + \rho_0 (\Delta_0^2 - \Delta^2)/4. \qquad (10)$$

For Δ and S we can use the ordinary formulas obtained in the theory of superconductivity

$$S = \frac{2\Delta^2\rho_0}{T} \sum_{m=1}^{\infty} (-1)^{m+1} K_2\left(\frac{m\Delta}{T}\right), \qquad (11)$$

$$\ln\left(\frac{\Delta_0}{\Delta}\right) = 2 \sum_{m=1}^{\infty} (-1)^{m+1} K_0\left(\frac{m\Delta}{T}\right). \qquad (12)$$

Using (10)—(12) we can readily obtain the excitation energy E* as a function of the temperature. Figure 1 shows (E^*/E^*_c) plotted as a function of (T/T_c). The value of E^*_c, which is the excitation energy at the phase transition point, is $3.1 (\rho_n\Delta_n^2/4 + \rho_p\Delta_p^2/4)$. The quantity $\rho_n\Delta_n^2/4 + \rho_p\Delta_p^2/4$ is the pair correlation energy of the neutrons and protons in the ground state and amounts to ~ 3 MeV for heavy nuclei. For the sake of comparison we show a plot of the excitation energy as a function of the temperature for a Fermi gas.

Figure 2 shows the dependence of $J_{eff}T/(J_{eff}T)_c$, calculated from formulas (5), (9), and (10), on the excitation energy. The deformation of the nucleus corresponded to the fission barrier. Here $\alpha_2 = (3/7)(1 - x)$ [where $x = (Z^2/A)/(Z^2/A)_{cr}$], $J_\perp^0 = 2.2 J_0^0$, and $J_\parallel^0 = 0.5 J_0^0$ (J_0^0 is the moment of inertia of the spherical nucleus). For comparison we show the analogous dependence for a Fermi gas and the experimental points obtained in [3]. In reducing the data we calculated two cases, $E^*_c \approx 10$ MeV and $E^*_c \approx 8$ MeV, with Δ = 0.7 MeV for both neutrons and protons.

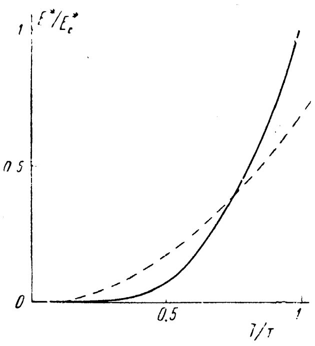

FIG. 1. Excitation energy of the nucleus E^*/E_c^* as a function of the temperature T/T_c. The continuous line corresponds to the theory developed here, while the dashed line is for a free Fermi gas.

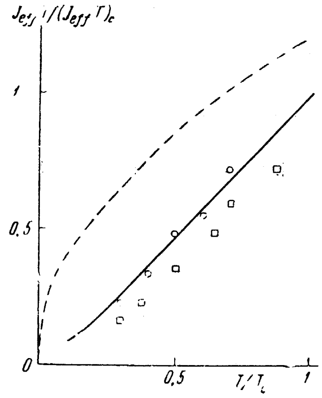

FIG. 2. Dependence of $J_{eff}T/(J_{eff}T)_c$ on the excitation energy. The continuous curve corresponds to the theory developed in the present paper; the dashed curve was obtained for a Fermi gas. The experimental points were taken for $E_c^* = 10$ MeV (o) and $E_c^* = 8$ MeV (□).

A similar curve for $J_{eff}T$ was obtained by Griffin [2]. This is connected with the fact that although Griffin used a cruder expression for $J_\perp$, the value of J_{eff} depends little on $J_\perp$ at the large deformations corresponding to the fission barrier. However, the experimental data in the cited paper [2] were inaccurate, since it was assumed, as in [1], that $E^*_c \approx 15$ MeV.

Thus, a consistent account of the pair correlation effects makes it possible to obtain good agreement between theory and experiment on the angular distribution of the fission fragments.

The author is grateful to V. T. Geĭlikman and V. M. Strutinskiĭ for a discussion of the results obtained.

[1] J. Halpern and V. Strutinskiĭ, Second Intern. Conf. Peaceful Uses Atom. Energy, Geneva, P/1513, **15**, 408 (1958).

[2] J. J. Griffin, Proc. Intern. Conference Nucl. Structure Kingston, Canada, (1960), p. 843.

[3] J. E. Simons and R. L. Henkel, Phys. Rev. **120**, 198 (1960).

[4] A. B. Migdal, JETP **37**, 249 (1959). Soviet Phys. JETP **10**, 176 (1960).

[5] S. T. Belyaev, Mat-Fys. Med. Dan Vid. Selsk. **31**, No. 11 (1959).

[6] Abrikosov, Gor'kov, and Khalatnikov, JETP **35**, 265 (1958), Soviet Phys. JETP **8**, 182 (1959).

[7] V. Strutinskiĭ, Compt. Rend. Congres Intern. Phys. Nucl. Paris (1959), p. 617.

Translated by J. G. Adashko
318