AMPLITUDES FOR THE $K\bar{K} \to \pi\pi$ PROCESS AND K_{e4} DECAYS

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It is shown that, if the waves with $l \geq 2$ are neglected, the K_{e4} decay mass spectra for the two π mesons are completely determined by the effective $K_{\mu 2}$ decay constant and the partial S and P wave amplitudes for the $K\bar{K} \to \pi\pi$ process.

1. INTRODUCTION

DISPERSION relations have been applied to the study of the electromagnetic structure of the K meson[1] and the scattering of K mesons on nucleons.[2,3] The integral equations derived from the dispersion relations and the unitarity condition contain the amplitude for the process

$$K + \bar{K} \to \pi + \pi. \tag{1}$$

This process can be investigated experimentally only in an indirect manner. The aim of the present paper is to consider the possibility of an experimental determination of the amplitude for the process (1) from the data on K_{e4} decays.

According to the $\Delta S = \Delta Q$ rule, there exist only the following K_{e4} decays:

$$K^0 \to e^+ + \nu + \pi^- + \pi^0, \tag{I}$$

$$K^+ \to e^+ + \nu + \pi^0 + \pi^0, \tag{II}$$

$$K^+ \to e^+ + \nu + \pi^+ + \pi^-, \tag{III}$$

together with the corresponding decays of the $\bar{K}$ meson. The $\Delta I = 1/2$ rule requires that the two-pion system can only be in states with I equal to 0 or 1.

Using the method employed in [4,5] for obtaining the Goldberger-Treiman relation[6] between the axial vector β decay constant, the πN coupling constant, and the effective pion decay constant, we seek a relation between the amplitudes for the process (1) and the decays (I) to (III). We shall show that, if the waves with $l \geq 2$ are neglected, the spectra of the decays (I) to (III) with respect to the effective mass of the two π mesons are completely determined by the amplitude for the process (1) and the known $K_{\mu 2}$ decay constant.

2. MATRIX ELEMENT FOR THE K_{e4} DECAYS

For simplicity, we shall first ignore the isospin structure of the amplitudes for the processes under consideration. Let p, q_1, q_2, k_1, and k_2 denote the four-momenta of the K meson, the π mesons, and the leptons and set

$$k = k_1 + k_2, \qquad s = -(q_1 + q_2)^2,$$
$$t_i = -(q_i + k)^2, \qquad \nu = t_2 - t_1. \tag{2}$$

The mass of the electron is set equal to zero.

In first order of weak interaction perturbation theory, the matrix element for the decays under consideration has the form

$$M(K_{e4}) = (2\pi)^4 \delta^4 (p - q_1 - q_2 - k)\bar{u}_\nu \gamma_\alpha$$
$$\times (1 + \gamma_5) v_e \langle \pi_1\pi_2 | J_\alpha^V + J_\alpha^A | K \rangle. \tag{3}$$

Here J_α^V and J_α^A are the vector and axial vector strangeness changing currents multiplied by the interaction constant. The matrix elements of these currents do not interfere. Moreover, the vector current gives a very small contribution, as was shown by Shabalin.[7] It can therefore be neglected.

It follows from invariance considerations that the matrix element of the axial vector current has the following general form:

$$\langle \pi_1\pi_2 | J_\alpha^A | K \rangle$$
$$= (8p^0 q_1^0 q_2^0)^{-1/2} [A(q_1 + q_2)_\alpha + B(q_1 - q_2)_\alpha + Ck_\alpha], \tag{4}$$

where A, B, and C are functions of s, k^2, and one of the t_i. The amplitude C has a pole at $k^2 = -M^2$ (M is the mass of the K meson), and A and B have no poles. The mass of the electron is set equal to zero. Therefore

$$k_\alpha \bar{u}_\nu \gamma_\alpha (1 + \gamma_5) v_e = 0$$

and the amplitude C gives no contribution.

We assume that A and B satisfy one-dimensional dispersion relations in k^2 for fixed s and t_i and double dispersion relations in s and t_i for fixed k^2. The regions where the spectral functions are nonvanishing are found by the standard method. [8,9] It follows from the spectral representations for these amplitudes that in the physical region for the decay, A and B depend very weakly on k^2 and t_i, and have an appreciable dependence only on s. Up to terms of the order [1)]

$$\frac{(M-2m)^2}{2(M+2m)^2}+\frac{2m^2}{(M+3m)^2-2m^2}<\frac{1}{10}$$

(m is the mass of the π meson), we can neglect their dependence on k^2 and t_i. The matrix element for the decays under consideration can therefore be written in this approximation

$$M(K_{e4}) = (2\pi)^4\,\delta^4(p-q_1-q_2-k)\,(8p^0q_1^0q_2^0)^{-1/2}\,\bar{u}_\nu\gamma_\alpha \times (1+\gamma_5)\,v_e\,[a(s)(q_1+q_2)_\alpha + b(s)(q_1-q_2)_\alpha], \quad (5)$$

where a(s) and b(s) are the values of A and B for $k^2 = 0$ and $t_i = \bar{t}_i$ (the average value of t_i). As will be shown below, our approximation corresponds to the neglect of the waves with $l \geq 2$, which give a very small contribution at low energies.

3. RELATION BETWEEN THE DECAY AMPLITUDE AND THE AMPLITUDES FOR PROCESS (1)

Applying dispersion relations to the study of the β decay and the pion decay, Goldberger and Treiman [6] have obtained a relation between the effective pion decay constant, the axial vector β decay constant, and the πN coupling constant which is in excellent agreement with experiment. Assuming that the axial current j_α^A without change of strangeness has the property

$$i\partial j_\alpha^A/\partial x_\alpha = \lambda\varphi_\pi,$$

where φ_π is the field operator of the π meson and λ a constant, Gell-Mann and Levy [4] derived the Goldberger-Treiman relation in a simple way. This result can also be obtained from a more general assumption, viz., that the matrix element of the divergence $\partial j_\alpha^A/\partial x_\alpha$ satisfies a dispersion relation without subtractions, where the pole term gives the most important contribution for small momentum transfers ($k^2 \approx 0$). [4,5] Let us suppose that this last assumption is correct also for the axial vector strangeness changing current J_α^A.

[1)]This estimate is obtained from the spectral representations with the help of the method of Chou Kuang-chao.[5]

Then we obtain the relation

$$A(q_1+q_2)k + B(q_1-q_2)k + Ck^2 = \frac{gM^2F(k\bar{k}\to\pi\pi)}{k^2+M^2} + \frac{1}{\pi}\int_{(M+2m)^2}^{\infty}\frac{\sigma(k'^2)\,dk'^2}{k'^2-k^2}, \quad (6)$$

where $F(K\bar{K}\to\pi\pi)$ is the amplitude for process (1) and the effective $K_{\mu 2}$ decay constant g is determined from the probability for this decay (μ is the mass of the μ meson)

$$W(K_{\mu 2}) = \frac{g^2}{4\pi}M^3\frac{\mu^2}{M^2}\left(1-\frac{\mu^2}{M^2}\right)^2,$$

the dispersion integral in (6) can be neglected for $k^2 \approx 0$. We then find the following approximate relation:

$$\tfrac{1}{2}[a(s)(s-M^2)+b(s)\nu] = gF(K\bar{K}\to\pi\pi) \quad \text{for } k^2=0. \quad (7)$$

For the computation of the amplitudes for the particular decays (I) to (III) we must still consider the isotopic spin structure of these amplitudes and the amplitude $F(K\bar{K}\to\pi\pi)$. Let us denote the projection operator on the isotopic spin state I by Π_I. We have then

$$F(K\bar{K}\to\pi\pi) = \Pi_0\sum_{l=0}^{\infty}F_{2l}(s)P_{2l}(\cos\theta) + \Pi_1\sum_{l=0}^{\infty}F_{2l+1}(s)P_{2l+1}(\cos\theta). \quad (8)$$

The two-pion system in the decay (I) is in the state I = 0. Therefore the relation (7) gives for this decay

$$\frac{1}{2}[a_I(s)(s-M^2)+b_I(s)\nu] = \frac{g}{\sqrt{6}}\sum_{l=0}^{\infty}F_{2l+1}(s)P_{2l+1}(\cos\theta). \quad (9)$$

The two-pion system in the decay (II) has isospin I = 0. For this decay we have therefore

$$\frac{1}{2}[a_{II}(s)(s-M^2)+b_{II}(s)\nu] = \frac{g}{\sqrt{2}}\sum_{l=0}^{\infty}F_{2l}(s)P_{2l}(\cos\theta). \quad (10)$$

Noting the following relation between ν and $\cos\theta$,

$$\nu = (M^2-s)\sqrt{(s-4m^2)s}\cos\theta, \quad (11)$$

we see that (9) and (10) are satisfied if $F_l(s) = 0$ for $l \geq 2$, i.e., our approximation corresponds to the neglect of the waves with $l \geq 2$. It follows from (9) and (10) that

$$a_I(s) = 0,\quad b_I(s) = \sqrt{2}g\,F_1(s)(M^2-s)^{-1}(1-4m^2/s)^{-1/2}, \quad (12)$$

$$a_{II}(s) = (2g/\sqrt{6})\,F_0(s)(s-M^2)^{-1},\quad b_{II}(s) = 0. \quad (13)$$

4. THE SPECTRA OF THE K_{e4} DECAYS

From the matrix element (5) and the relations (12) and (13) we obtain the following expressions for the spectra of the decays (I) and (II):

$$dW_1 = \frac{4g^2}{9(4\pi)^5} M^3 |F_1(s)|^2 \rho_I(s)\, d\frac{s}{M^2}, \quad (14)$$

$$dW_2 = \frac{4g^2}{9(4\pi)^5} M^3 |F_0(s)|^2 \rho_{II}(s)\, d\frac{s}{M^2}, \quad (15)$$

where

$$\rho_I(s) = \left[\frac{21}{2} - 20x + \frac{39}{4}x^2 - \frac{3(1-x)^2(7-4x)}{x^2}\left(\ln\frac{1}{1-x} - x\right)\right] \times \left[1 - \frac{4m^2}{(1-x)M^2}\right]^{1/2},$$

$$\rho_{II}(s) = \left[\frac{3(1-x)^2}{x^2}\left(\ln\frac{1}{1-x} - x\right) - \frac{3}{2} + 2x - \frac{x^2}{4}\right] \times \left[1 - \frac{4m^2}{(1-x)M^2}\right]^{1/2},$$

$$s = M^2(1-x).$$

It follows from the $\Delta I = 1/2$ rule that the spectrum for the decay (III) is related to the spectra of the decays (I) and (II) by the formula[10]

$$dW_{III} = \frac{1}{2} dW_I + 2dW_{II}. \quad (16)$$

The spectra of the decays (I) to (III) are therefore completely determined by the S and P wave amplitudes for the process $K\bar{K} \to \pi\pi$. We could hence use the experimental data on these decays to obtain some definite information on the $\pi\pi$ and $K\pi$ interactions. Using the results of the present paper, we have shown, together with Arbuzov and Faustov,[11] that the hypothesis of a $\pi\pi$ resonance in the I = 0 state at low energies leads to a value for the decay probability for the decay (III) which lies above the upper limit given by experiment.

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