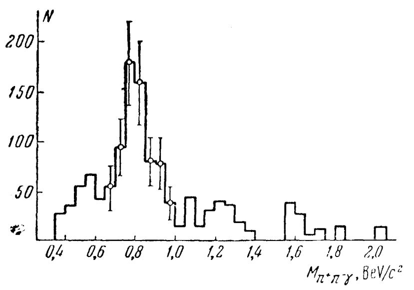

FIG. 3. Distribution of $M_{\pi\pi\gamma}$ for cases with E_γ = 0.5 – 0.8 BeV.

tistics do not enable us so far to consider this problem in greater detail.

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1)The corresponding kinematic considerations are developed, for example, in[2]. Analogous indications concerning the possible existence of gamma-quantum sources are contained also in[3,4].

2)The main results were reported by L. Strunov at the International Conference on High-Energy Particle Physics at CERN, in 1962.

3)In the region of the second maximum it amounts to about 10 per cent.

4)The average error in the determination of $M_{\pi\pi\gamma}$ did not exceed 5 per cent.

5)An analogous result was obtained also by another group in our laboratory[13].

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1 Lyubimov, Nikitin, and Trka, Preprint, Joint Inst. Nuc. Res. R-974, 1962.

2 G. I. Kopylov, JETP **33**, 430 (1957), Soviet Phys. JETP **6**, 335 (1958).

3 D. T. King, Bull. Am. Phys. Soc. 6, 39 (1961).

4 O. Czyzewski and J. Loskiewicz, Phys. Rev. Lett. **2**, 83 (1962).

5 Toohig, Kraemer, Madansky, Meer, Nussbaum, Pevsner, Richardson, Strand, and Block, Proc. 1962 Intern. Conf. on High Energy Phys. at CERN, p. 99.

6 Bastien, Berge, Dahl, Ferro-Luzzi, Miller, Murray, Rosenfeld, and Watson, Phys. Rev. Lett. **8**, 114 (1962).

7 Carmony, Rosenfeld, and Van de Walle, Phys. Rev. Lett. **8**, 117 (1962).

8 Pickup, Robinson, and Salant, Phys. Rev. Lett. **8**, 329 (1962).

9 R. Gatto, Proc. 1962 Intern. Conf. on High Energy Phys. at CERN, p. 36.

10 Mencuccini, Querzoli, Salvini, and Silverstrini, Proc. 1962, Intern. Conf. on High Energy Phys. at CERN, p. 33.

11 Chretien, Bulos, Crouch, et al., Phys. Rev. Lett. **9**, 127 (1962).

12 Meer, Kraemer, Madansky et al, Proc. 1962 Intern. Conf. on High Energy Phys. at CERN, p. 103.

13 Nguen Din Tu, Proc. 1962 Intern. Conf. on High Energy Phys. at CERN, p. 111.

14 M. I. Dyment and G. I. Kopylov, Preprint, Joint Inst. Nuc. Res. R-581, 1960.

Translated by J. G. Adashko
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ON THE BOHM DIFFUSION COEFFICIENT

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IN 1949 Bohm observed experimentally[1] that the coefficient for diffusion of plasma across a magnetic field is appreciably greater than that predicted by classical kinetic theory. He concluded that this anomalous behavior is due to an instability of unknown origin, which drives the plasma into a turbulent state, and proposed that the anomalous diffusion coefficient is given by

$$D_\perp = cT/16eH, \tag{1}$$

where H is the magnetic field, T is the plasma temperature, c is the velocity of light in vacuum and e is the charge of the electron. Since that time numerous attempts have been made to establish the nature of the instability and the resulting turbulent state, but there has been little progress toward an understanding of the anomaly (at best, through the use of additional hypotheses it has been possible to obtain numerical (!) values that approach the diffusion coefficient given above under certain conditions[2]). On the other hand, continuing experiments on plasma diffusion (see the review in [3]) frequently lead to contradictory results, some of which are in satisfactory agreement with classical theory.

We show below that a fully ionized low-pressure plasma in a strong magnetic field ($p \ll H^2/8\pi$) can exhibit an instability even when the lines of force of the magnetic field (along the z axis) are all of the same length. The sole reason for this instability is the existence of a density gradient dn/dx (along the x axis). An analysis of the turbulence arising as a consequence of this instability yields a diffusion coefficient that is approximately the same as that in (1). We show also that the relation in (1) is not universal; specifically, there are cases in which the diffusion coefficient can vary as H^{-2}.

In the present note, which is of preliminary nature, we do not give the details of the derivation. Only a brief outline of our approach is presented. It is well known that a plasma, which can frequently be regarded as a quasineutral mixture of two gases (ions and electrons), can support the propagation of so-called drift waves; these waves satisfy the dispersion relation

$$1 - c\frac{k_y T}{\omega e H}\frac{d\ln n(x)}{dx} = 0, \qquad k_y^2 \gg k_z^2 \tag{2}$$

(we assume a perturbation in the form $f(x)\exp(i\omega t + ik_y y + ik_z z)$.

It is found that the existence of a friction force between the electron and ion gases (in other words, the finite electrical conductivity of the plasma) can lead to the excitation of drift waves. We omit the derivation of the linear stability analysis, which differs from the usual drift-wave stability analysis only in the single term

$$\mathbf{F}_{\mathrm{fr}} = -m_e(\mathbf{v}_e - \mathbf{v}_i)\nu$$

(F_{fr} is the friction force, m_e is the electron mass, $v_{i,e}$ is the ion or electron velocity, ν is the effective frequency of electron-ion collisions) and give the final differential equation describing the perturbation of the electric potential φ:

$$\frac{d^2\varphi}{dx^2} - \left\{1 - i\frac{\omega_{He}\omega_{Hi}k_z^2}{\nu k_y^2\omega}\left[1 - \frac{\omega_e}{\omega}\right]\right\}k_y^2\varphi = 0,$$

$$\omega_{Hi,e} = \frac{eH}{m_{i,e}c}, \qquad \omega_e = k_y\frac{cT}{eH}\frac{d\ln n(x)}{dx}. \tag{3}$$

Here, m_i is the ion mass. Equation (3) is obtained under the assumption that the ions are cold (i.e., $T_e \gg T_i$). This equation is reminiscent of the Schrödinger equation, but with a complex potential:

$$U(x) = k_y^2\left\langle 1 + \frac{\omega_{He}\omega_{Hi}|\delta|[1 - 2\omega_e(x)\Omega/(\Omega^2+\delta^2)]k_z^2}{\nu(x)k_y^2(\Omega^2+\delta^2)} - i\frac{\omega_{He}\omega_{Hi}k_z^2}{\nu(x)k_y^2(\Omega^2+\delta^2)}\left(\Omega - \frac{\Omega^2-\delta^2}{\Omega^2+\delta^2}\omega_e(x)\right)\right\rangle,$$

$$\omega = \Omega + i\delta. \tag{4}$$

In using the WKB approximation to estimate the eigenvalues of the frequency ω corresponding to perturbations that vanish outside the plasma we make use of the condition[4]

$$U(\omega, k, x_0) = 0. \tag{5}$$

In the present case we find

$$\omega^2 - i\frac{\omega_{He}\omega_{Hi}k_z^2}{\nu k_y^2}\omega + i\frac{\omega_{He}\omega_{Hi}k_z^2}{\nu k_y^2}\omega_e = 0. \tag{6}$$

The maximum growth rate of the instability, which is of order $\delta = -\mathrm{Im}\,\omega \sim -\omega_e$, obtains for values of k_y and k_z that satisfy the condition

$$\omega_e \sim (\omega_{He}\omega_{Hi}/\nu)(k_z/k_y)^2. \tag{7}$$

However, the wavelength λ_x (in the direction of the gradient) of the perturbation is of order

$$\lambda_x = 2\pi/k_x \sim |U|^{-1/2}. \tag{8}$$

The quantity λ_x is automatically of order k_y^{-1} for values of k_y and k_z that satisfy Eq. (7).

Using a dimensional analysis we can write the turbulent diffusion coefficient in the form

$$D_\perp \sim \tau^{-1}\lambda_\perp^2, \tag{9}$$

where τ is the correlation time, which may reasonably be taken to be $\sim(\mathrm{Im}\,\omega)^{-1}$ and $\lambda_\perp$ is the characteristic dimension of the turbulent fluctuation in the direction perpendicular to H. It is reasonable to take $\lambda_\perp$ equal to the wavelength of the instability $\lambda_x \sim k_y^{-1}$. For the longest wavelengths $k_y \sim 2\pi/r$, where r is the transverse dimension of the system [the longitudinal dimension of the fluctuation k_z^{-1}, however, is obtained from the condition for the maximum growth rate (7)]. As a result (9) yields

$$D_\perp \sim cT/2\pi eH. \tag{10}$$

We now indicate possible deviations from Eq. (10). In very short tubes, where k_z is limited from below by the condition $k_z \geq 2\pi/L_{||}$ ($L_{||}$ is the longitudinal dimension of the system), it may turn out that the condition in (7), giving the maximum growth rate, cannot be satisfied with $k_y \sim 2\pi/r$ as H increases. It is evident from (7) that for a given tube length a point is reached at which the condition for maximum growth rate is violated. The critical field above which the nature of the diffusion changes is of order

$$H^* \sim L_{||}^{1/3}c(m_i m_e \nu T)^{1/3}/r^{4/3}e. \tag{11}$$

In this case the diffusion coefficient is given by

$$D_\perp \sim (cT/2\pi eH)H^*/H. \tag{12}$$

We have not carried out detailed comparisons with the numerous experimental data in the present note because there are frequently other factors in plasma diffusion, such as effects due to neutral gas and the longitudinal current, that are not taken into account. However, the results given here verify the possibility of observing an anomalous plasma diffusion proportional to 1/H.

[1] A. Guthrie and P. K. Wakerling, The Characteristics of Electrical Discharges in Magnetic Fields, N. Y., 1949.

[2] B. B. Kadomtsev, JETP **43**, 1688 (1962), Soviet Phys. JETP **16**, 1191 (1963).

[3] F. G. Hoh, Revs. Modern Phys. **34**, 267 (1962).

[4] A. A. Galeev, JETP (in press).

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POSSIBILITY OF DETERMINING $\pi\pi$-SCATTERING PHASES FROM ANGULAR CORRELATIONS IN K_{e4} DECAY

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EXPERIMENTS on K_{e4} decay are evidently to be initiated in the near future (one case of this decay has already been observed[1]). In this connection we wish to call the attention of experimental physicists to the fact that an investigation of angular correlations in K_{e4} decay can give information on the π-π interaction.

The effect of the interaction of the π mesons in the final state in K_{e4} decay has been discussed earlier by Chadan and Oneda[2] and by Chioccetti.[3] These authors have computed the "gain factor" for the decay probability due to the mutual attraction of the π mesons as compared with the decay probability in the absence of an interaction in the final state. It should be noted, however, that an estimate of the probability of K_{e4}-decay that neglects the π-π interaction is based on rather arbitrary assumptions as to the magnitude of the constant for this decay.[4-7] Hence, any experimental deviation of the probability from the value given by this estimate could be due to the interaction of the π mesons and/or to an incorrect estimate.

We consider here another effect related to the interaction of the π-mesons. Consider the asymmetry in positron emission in the decay

$$K^+ \to \pi^+ + \pi^- + e^+ + \nu \tag{1}$$

with respect to the plane formed by the tracks of the π-mesons.

Let us analyze the decay of a K meson at rest. The momentum of the π^+, $\mathbf{k}_1$, and the momentum of the π^-, $\mathbf{k}_2$, define a plane; the normal to this plane $\mathbf{n}$ is defined in such a way that for an observer looking from the end of the vector $\mathbf{n}$ the smallest rotation from the direction of the track of the π^+ to the track of the π^- is in the counterclockwise direction.

If we neglect the particle interaction in the final state the angular distribution of positrons must be symmetric with respect to the plane defined by $\mathbf{n}$. The absence of a term $(\mathbf{p}_e \cdot \mathbf{n})$ in the expression for the probability is a direct consequence of the conservation of time parity. The quantity $(\mathbf{p}_e \cdot \mathbf{n})$ changes sign under the T-transformation $\mathbf{k}_1 \to -\mathbf{k}_1$, $\mathbf{k}_2 \to -\mathbf{k}_2$, $\mathbf{p}_e \to -\mathbf{p}_e$. The situation is changed if we take account of the interaction in the final state. In this case $(\mathbf{p}_e \cdot \mathbf{n})$ can contain an odd function of the $\pi\pi$-scattering phase as a factor. Inasmuch as the signs of the phases are reversed under the T-transformation the term as a whole is T-invariant. However, it implies the violation of the symmetry indicated above.

We will assume the existence of the rule $|\Delta T| = 1/2$ for lepton decay of strange particles.[8-10] Then, using the S-matrix formalism for multichannel reactions[11] and assuming that decay leading to the formation of the system of π-mesons with orbital moment $l \geq 2$ is forbidden because of the high centrifugal barrier, we can write the decay amplitude (1) in the form

$$A = \frac{G}{\sqrt{2}} \bar{u}_\nu \{f_1 e^{i\varphi_0}(\hat{k}_1 + \hat{k}_2) + f_2 e^{i\varphi_1}(\hat{k}_1 - \hat{k}_2)\}(1 + \gamma_5) u_e \varphi_K \varphi_{\pi^+} \varphi_{\pi^-}. \tag{2}$$

In this expression we have neglected the axial part of the current of the strongly interacting particles since its contribution must be small.[7] The real quantities f_1 and f_2 are functions of the invariants (k_1k_2), (k_1q), (k_2q) (where q is the four-momentum of the K-meson); we assume that φ_0 and φ_1, the $\pi\pi$-scattering phases in the S and P states respectively, are constants.