

TRIPLE SCATTERING OF 660-MeV PROTONS. IV. ANGULAR DEPENDENCE OF THE PARAMETER A

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Further experiments on proton triple scattering are performed in the program of investigating p-p interactions at ~660 MeV. The experiments are described and the measurements of the parameter A for c.m.s. angles 54°, 72°, 90°, 108°, and 126° are reported. These and other results are employed for direct reconstruction of the p-p scattering matrix and for comparison with different variants of phase-shift analysis.

1. INTRODUCTION

In earlier articles [1-3] we described experiments in which a transversely polarized proton beam was triply scattered. [4] The results of these experiments together with measurements of differential p-p scattering cross sections, [5] polarization, [6] and the correlation of transverse polarization components, [7,8] had previously been used for the direct reconstruction of the p-p scattering matrix M [3,8,9] and for phase-shift analyses. [10-13] None of the attempted problems could be solved completely because the data then available did not determine the amplitudes of the matrix M in the general case (in [3] M was reconstructed only for $\theta = 90^\circ$) and because it was impossible to make a final choice among the sets of phase shifts given by different authors. [10-12]

As indicated in [3], for a unique reconstruction of M even at $\theta = 90^\circ$, i.e., when only three scattering amplitudes do not vanish, further experiments were necessary, especially in order to determine the triple scattering parameter A characterizing the transverse polarization component arising when a longitudinally polarized beam is scattered. [14] Moreover, the angular dependences of A calculated from different sets of phase shifts were far from identical; therefore an experimental determination of A over a broad range of angles was highly desirable.

The successful production of longitudinally polarized beams using the six-meter synchrotron of the Joint Institute for Nuclear Research [15] enabled more complex experimentation on the triple scattering of protons. The angular dependence of A was measured first.

2. EXPERIMENT AND RESULTS

The experimental geometry is represented in Fig. 1 where n_i are vectors normal to the planes π_i of i-th scattering, k_i and k'_i are unit vectors parallel to the proton momentum before and after i-th scattering, and $s_2 = n_2 \times k'_2$. The azimuthal angle φ of second scattering is defined by

$$\cos \varphi = n_1 n_2, \quad \sin \varphi = [n_1 n_2] k_2. \quad (1)^*$$

The protons were scattered first in the vertical plane π_1 ; therefore the polarization vector $P_1^{(0)}$ of the scattered beam along the normal $n_1^{(0)}$ lay in the horizontal plane π_2 . The transversely polarized scattered beam then entered a vertical magnetic field in which it was deflected through the angle δ . The normal vector n_1 , parallel to the transverse polarization component of the deflected beam, remained in the plane π_2 . Because of the

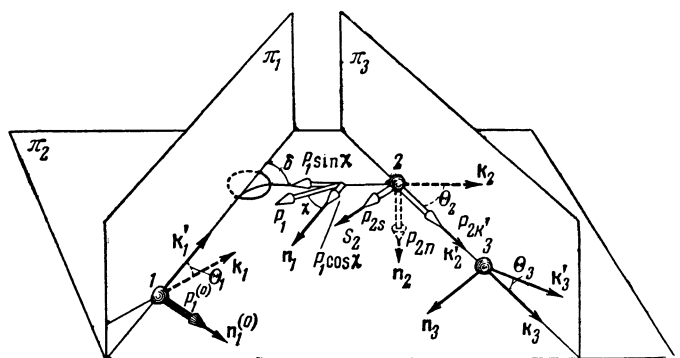


FIG. 1. Experimental geometry for measurement of the parameter A in triple scattering of protons.

$$[n_1 n_2] = n_1 \times n_2, \quad n_1 n_2 = n_1 \cdot n_2.$$

proton's anomalous magnetic moment its spin precessed in the magnetic field, the polarization vector P_1 of the deflected beam forming with the vector n_1 the angle

$$\chi = \delta (\mu_p - 1) / \sqrt{1 - \beta^2}, \quad (2)$$

where μ_p is the proton magnetic moment in nuclear magnetons, and β is the proton velocity in units of the velocity of light. This precession produced in the beam a longitudinal polarization component $P_1 \sin \chi$ directed opposite to the proton momentum, while leaving the transverse component $P_1 \cos \chi$ directed along the vector n_1 .

Under the given experimental conditions the second scattering occurred in the horizontal plane; Fig. 1 shows the case of $\varphi = 90^\circ$. In the measurement of A the polarization component P_{2S} along the vector s_2 was determined. For this purpose the third analyzing scattering was performed with either $n_3 = s_2$ (in Fig. 1 this is represented by third scattering upward in the vertical plane π_3), or $n_3 = -s_2$. From the respective counting rates $N(+)$ and $N(-)$ of triply scattered protons we calculated the asymmetry

$$\epsilon_{3s} = (N(+)-N(-))/(N(+)+N(-)), \quad (3)$$

which is related to the parameters A and R by^[14]

$$\epsilon_{3s} = P_1 P_3 (-A \sin \chi + R \cos \chi \sin \varphi) / (1 + P_1 \cos \chi P_2^{(0)} \cos \varphi). \quad (4)$$

The product $P_1 P_3$ was determined in a special double-scattering calibration experiment, where a transversely polarized beam with polarization P_1 was slowed down to the energy of doubly scattered protons, after which the beam was scattered by an analyzing target under conditions identical with those for measuring the asymmetry ϵ_{3s} . We here measured the asymmetry

$$\epsilon_3 = P_1 P_3, \quad (5)$$

which characterizes the analyzing power of the third target.

Second scattering occurred at $\varphi = \pm 90^\circ$; therefore A was determined from the relation

$$A \sin \chi = -\epsilon_{3s}/\epsilon_3 + R \cos \chi \sin \varphi. \quad (6)$$

Since the longitudinal polarization of the beam had been practically complete ($\chi = 89^\circ \pm 2.5^\circ$)^[15] the second term in (6) gave an extremely small correction. Appropriate values of R were taken from^[3].

A detailed description of the apparatus and procedure for measuring A has been given in^[16]. The disposition of the scatterers and detecting

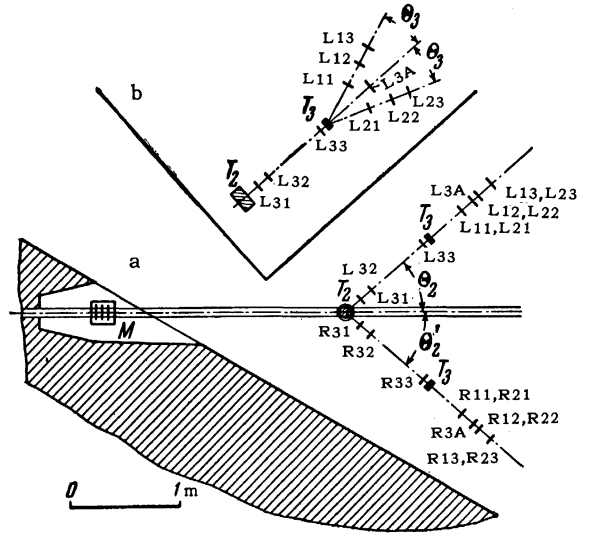


FIG. 2. Arrangement of scatterers and detecting apparatus. a—in horizontal plane, b—in vertical plane of third scattering. M—monitor (ionization chamber), T_2 —second scatterer (vessel containing liquid hydrogen), T_3 —third scatterers (70×60 -mm graphite blocks 50 mm thick); L11, L12, L13, L21, L22, L23, L31, L32, L33, L3A and R11–R3A—scintillators of counters, having dimensions (height \times width) 60×40 , 70×50 , 80×60 , 60×40 , 70×50 , 80×60 , 50×80 , 60×100 , 50×50 , 100×80 mm and thickness 6 mm. For measurement of the parameter A at 126° the third scatterer had cross section 70×60 mm and thickness 20 mm.

apparatus is shown in Fig. 2. The hydrogen target T_2 was a glass Dewar vessel filled with liquid hydrogen, at the center of which the mean proton energy was 608 MeV, with a current of 2×10^7 protons/sec in a beam of 4-cm diameter. In order to discriminate elastic p-p scattering the scattered protons and recoil protons were registered with coupled telescopes connected for coincidence. The parameter A was measured simultaneously at both coupled angles Θ_2 and Θ_2' . The third analyzing scatterings in the hydrogen targets T_3 were observed simultaneously at the angles $\Theta_3 = \pm 12^\circ$. To reduce the accidental coincidence background, protons not scattered in the targets T_3 were registered with the scintillation counters L3A and R3A connected in anticoincidence with the other counters. A block diagram of the scintillation counter and telescope circuit has been given in^[16].

At each observation angle for A five to ten independent measurements of the asymmetry ϵ_{3s} were obtained; the results were mutually consistent within the limits of statistical errors. Before starting each individual measurement we carefully determined the "profile" of the beam of doubly scattered protons in order to obviate

Table I. Asymmetries ϵ_{3S} and ϵ_3 , and the parameter A

θ , deg	$\epsilon_{3S} \pm \Delta \epsilon_{3S}$, %	$\epsilon_3 \pm \Delta \epsilon_3$, %	$A \pm \Delta A$
54	-4.6 ± 0.9	9.8 ± 0.3	0.48 ± 0.09
72	-6.0 ± 1.2	13.1 ± 0.4	0.46 ± 0.10
90	-3.2 ± 1.0	16.1 ± 0.4	0.20 ± 0.06
108	1.2 ± 1.4	16.2 ± 0.4	-0.08 ± 0.09
126	1.9 ± 1.4	9.7 ± 0.5	-0.20 ± 0.14

spurious asymmetry. The counting rates of triply scattered protons were corrected for a) the background observed in the absence of triple scatterings and induced almost entirely by proton scattering in the scintillators of counters L33 and R33, b) the effect observed when liquid hydrogen was absent from the Dewar vessel, and c) accidental coincidences.

The counting rates were negligibly small at all angles in the absence of second and third scatterers, and also when second and third scatterers were present but the angle $\Theta_2 + \Theta_2'$ was not in accord with the kinematics of elastic p-p collisions.

Table I gives the values obtained for the asymmetries ϵ_{3S} and ϵ_3 together with their statistical errors, and the values of A calculated from (6).

3. RECONSTRUCTION OF THE p-p SCATTERING MATRIX

The ultimate goal of the p-p scattering experiments is the determination of the scattering matrix M. In a phase-shift analysis this is attempted by seeking a set of phase shifts that represent fully all experimental data. Further study of the energy dependence of phase shifts, using dispersion relations, for example, [17] will evidently enable us to describe all p-p interaction data using a minimum number of phenomenological parameters. However, with increasing incident proton energy a phase-shift analysis encounters ever greater difficulties. The phase shifts become complex above the threshold of pion production. This doubling of the number of required parameters and the increased number of partial waves participating effectively in scattering at higher energies make a phase-shift analysis very difficult to achieve. The direct reconstruction of the scattering matrix is therefore a more realistic method at high energies. [18,19] In this method the matrix amplitudes are determined for certain selected scattering angles from the results of different independent experiments at a given energy. The method is applicable at all energies since it does not employ the unitarity condition. However, for this reason the overall phase of the matrix M be-

comes indeterminate [20] unless we consider the interference of nuclear and Coulomb scatterings, for example.

Previously obtained experimental results at ~ 660 MeV enabled a complete reconstruction of M only at $\theta = 90^\circ$; [3] however, in this case the solution was four-valued.

At the present time, the angular dependence of A having been measured, the experimental information is sufficient for a direct reconstruction of the moduli and relative phases of scattering matrix amplitudes in the range $54^\circ \leq \theta \leq 90^\circ$. However, since the expressions relating the measured quantities to the scattering amplitudes are nonlinear, the solutions are not unique. Further independent experimental data are required for a unique solution.

A. Scattering angle $\theta = 90^\circ$. We now have available at 90° for ~ 660 MeV: measurements of the differential scattering cross section of an unpolarized beam (σ), [5] the correlation coefficients C_{KP} [7] and C_{nn} , [8] the parameters D, [1] R, [3] and A (present work). These data can be related most simply to the matrix elements in a singlet-triplet representation: [21]

$$\sigma(90^\circ) = \frac{1}{2} |M_{01}|^2 + \frac{1}{2} |M_{10}|^2 + \frac{1}{4} |M_{ss}|^2, \quad (7a)$$

$$\sigma(90^\circ) C_{nn}(90^\circ) = \frac{1}{2} |M_{01}|^2 + \frac{1}{2} |M_{10}|^2 - \frac{1}{4} |M_{ss}|^2, \quad (7b)$$

$$\sigma(90^\circ) C_{KP}(90^\circ) = \frac{1}{2} |M_{01}|^2 - \frac{1}{2} |M_{10}|^2 + \frac{1}{4} |M_{ss}|^2 E / (4mc^2 + E), \quad (7c)$$

$$\sigma(90^\circ) D(90^\circ) = - |M_{01}| |M_{10}| \cos \varphi_{01,10}, \quad (7d)$$

$$\sigma(90^\circ) R(90^\circ) = |M_{01}| |M_{ss}| \cos \varphi_{01,ss} \left[\frac{1}{2} \frac{E + 2mc^2}{E + 4mc^2} \right]^{1/2}, \quad (7e)$$

$$\sigma(90^\circ) A(90^\circ) = - |M_{10}| |M_{ss}| \cos \varphi_{10,ss} \left[\frac{mc^2}{E + 4mc^2} \right]^{1/2}. \quad (7f)$$

Here m is the proton mass, c is the velocity of light, and E is the kinetic energy (lab.system) of an incident proton. Eqs. (7c), (7e), and (7f) include relativistic corrections. [22]

The Eqs. (7) form for five independent parameters (three moduli of matrix elements and two relative phases) an indeterminate system per-

mitting a double-valued solution. The best values of the parameters were determined by least squares. The minimum of the functional

$$\chi^2 = \frac{(\sigma_{\text{exp}} - \sigma_{\text{calc}})^2}{(\Delta\sigma)^2} + \frac{(C_{nn \text{ exp}} - C_{nn \text{ calc}})^2}{(\Delta C_{nn})^2} + \dots + \frac{(A_{\text{exp}} - A_{\text{calc}})^2}{(\Delta A)^2} \quad (8)$$

was obtained by linearization.^[23] Here σ_{exp} is the experimental differential cross section, σ_{calc} is the cross section calculated from (7a), $(\Delta\sigma)^2$ is the dispersion of the experimental values etc.

The calculations give the following moduli of the matrix elements in units of 10^{-13} cm:

$$\begin{aligned} |M_{01}| &= 0.48 \pm 0.04; & |M_{10}| &= 0.39 \pm 0.05; \\ |M_{ss}| &= 0.21 \pm 0.12 \end{aligned} \quad (9)$$

and the relative phases of these elements:

$$\begin{aligned} \varphi_{01,10} &= + (176^\circ \pm 34^\circ), & \varphi_{01,ss} &= - (6^\circ \pm 60^\circ) - \text{solution a} \\ \varphi_{01,10} &= - (176^\circ \pm 34^\circ), & \varphi_{01,ss} &= + (6^\circ \pm 60^\circ) - \text{solution b.} \end{aligned} \quad (10)$$

Errors of the parameters were calculated from $\chi^2 = \chi_{\text{min}}^2 + 1$.^[24] Since the Eqs. (7) are nonlinear these errors indicate the region of possible parameter values only provisionally.

Figure 3 shows in the complex plane two possible locations of values of $M_{01}/\sqrt{2}$, $M_{10}/\sqrt{2}$, and $M_{ss}/2$ determined from the minimum of χ^2 for the system (7). The circle of radius $\sqrt{\sigma}$ corresponds to the maximum possible moduli of these quantities; the absolute phase of the element M_{01} was taken to be 180° . The solutions are double-valued because Eqs. (7) contain only the cosines of the relative phases. At ~ 660 MeV both solutions are seen to practically coincide. However, for a unique reconstruction of $M(90^\circ)$ in the general case further independent experiments are

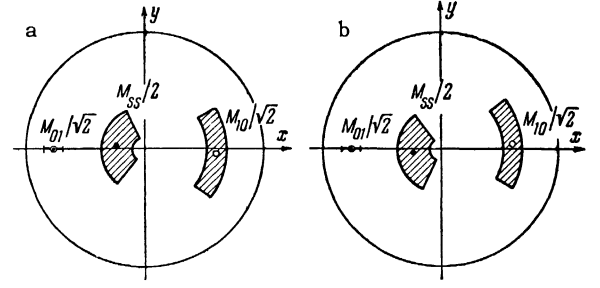


FIG. 3. Matrix elements $M_{PP}(90^\circ)$ at 660 MeV in the complex plane. The absolute phase of M_{01} is taken to be 180° . The solution remains double-valued because of insufficient experiments at $\theta = 90^\circ$.

required, such as measurement of the spin correlation when transversely and longitudinally polarized proton beams are used.

B. Reconstruction of scattering matrix at $\theta \neq 90^\circ$. For $0 < \theta < 90^\circ$ all five complex amplitudes of the p-p scattering matrix M are different from zero in the general case. These amplitudes can be determined from the results of nine independent experiments, with accuracy to within the overall phase and indeterminacy arising from the nonlinearity of the equations. At ~ 660 MeV for c.m. angles 54° and 72° measurements are available: of the differential scattering cross section $\sigma(\theta)$,^[5] polarization $P(\theta)$,^[6] the correlation coefficient $C_{nn}(\theta)$,^[8] the triple scattering parameters $D(\theta)$ and $D(\pi - \theta)$,^[2] $R(\theta)$ and $R(\pi - \theta)$,^[3] $A(\theta)$ and $A(\pi - \theta)$ (present work). Table II gives the values of five moduli and four relative phases of amplitudes of the matrix M in the form given by Oehme:^[25]

$$\begin{aligned} M &= \frac{1}{2} \{ (a + b) + (a - b) \sigma_{1n} \sigma_{2n} + e(\sigma_{1n} + \sigma_{2n}) \\ &\quad + (c + d) \sigma_{1K} \sigma_{2K} + (c - d) \sigma_{1P} \sigma_{2P} \}. \end{aligned} \quad (11)$$

These values were obtained from the indicated ex-

Table II. Moduli (in units of 10^{-13} cm) and relative phases of amplitudes of the p-p scattering matrix at ~ 660 MeV

θ	54°	72°	90°	
			Solution a	Solution b
χ_{min}^2	2.45	0.25	0.07	0.07
$ a $	0.19 ± 0.13	0.10 ± 0.05		
$ b $	0.35 ± 0.07	0.28 ± 0.06	0.11 ± 0.06	0.11 ± 0.06
$ c $	0.22 ± 0.13	0.10 ± 0.07		
$ d $	0.22 ± 0.12	0.30 ± 0.07	0.07 ± 0.05	0.07 ± 0.05
$ e $	0.73 ± 0.06	0.58 ± 0.04	0.62 ± 0.04	0.62 ± 0.04
φ_{ea}	-4°	-3°		
φ_{eb}	79°	94°	$85 \pm 30^\circ$	$95 \pm 30^\circ$
φ_{ec}	-56°	-86°		
φ_{ed}	-90°	-49°	$-72 \pm 40^\circ$	$-108 \pm 40^\circ$

perimental data by least squares. Table II also gives values of the parameters at 90° which were calculated from (9) and (10).

Table II gives for 54° and 72° only the errors of the amplitude moduli, calculated from $\chi^2 = \chi_{\min}^2 + 1$. The relative phases of the amplitudes at 54° and 72° are highly correlated with the moduli so that the errors cannot be calculated from the available experimental information. Additional experiments are required to refine the solution, especially measurement of the correlation coefficient C_{KP} .

Table II shows that for $54^\circ \leq \theta < 90^\circ$ all five amplitudes $a, b, c, d,$ and e differ from zero. The principal contribution to the p-p scattering cross section in this range comes from the spin-orbit amplitude $e(\theta)$; at 90° this term makes an especially large contribution (of the order of 90%, as already noted in [1,8,9]). The moduli and relative phases of the amplitudes of $M(90^\circ)$ in [3] and the squares of the moduli calculated in [8] agree with Table II within the error limits.

4. COMPARISON OF EXPERIMENTAL DATA AND PHASE SHIFT ANALYSIS OF p-p SCATTERING

Figure 4 shows the angular dependences of triple scattering parameters $D, R,$ and A at 660

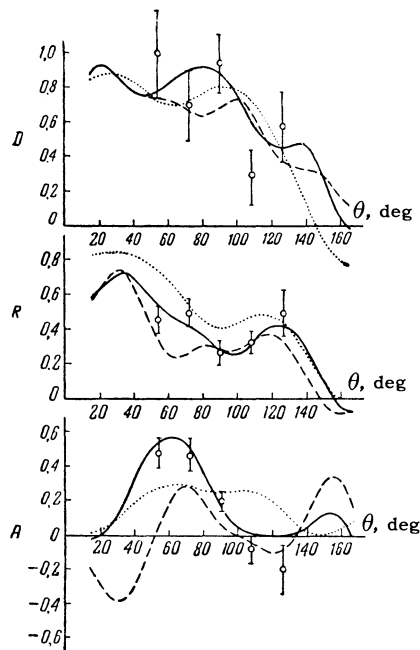


FIG. 4. Angular dependences of the triple scattering parameters D , [1,2] R , [3] and A (present work). The curves were calculated from the phase shifts in different variants of phase-shift analysis: dotted curves, [10] dashed curves, [11] and continuous curves. [12]

MeV, compared with the results obtained by different variants of phase shift analysis.

The first phase shift analysis of elastic p-p scattering at ~ 660 MeV was performed by Hoshizaki and Machida, [10] who used data obtained at Dubna regarding differential cross sections, polarization, and depolarization. They evaluated the imaginary parts of the phase shifts using the resonance model of pion production in N-N collisions. [26] The dotted curves in Fig. 4 represent the angular dependences of $D, R,$ and A calculated in [10] from the phase shifts.

Zul'karneev and Silin [11] used, in addition, information regarding the correlation coefficients C_{nn} and C_{KP} , the parameter R , and the total cross section σ_{tot} . They assumed that meson production occurs only in 3P and 1D states of the p-p system. The dashed curves in Fig. 4 represent the angular dependences of $D, R,$ and A calculated from the phase shifts in their most probable solution ($\chi^2/\bar{\chi}^2 = 1.48$).

Azhgirei et al [12] assumed in their phase-shift analysis of p-p scattering at ~ 660 MeV that at this energy we must not neglect meson production in $^3F_{2,3}$ states, which on the resonance model of pion production can contribute to inelastic processes. All imaginary parts of the phase shifts were characterized by three independent parameters, as in [10]. The analysis included experimental data on $A(90^\circ)$ and on the total cross sections for inelastic processes. The solution was obtained with considerably higher probability ($\chi^2/\bar{\chi}^2 = 1.11$) than the solutions in [10,11]. The continuous curves in Fig. 4 represent the angular dependences of $D, R,$ and A calculated from the phase shifts in this solution.

Bystritskiĭ and Zul'karneev in their recent [13] phase-shift analysis of p-p scattering at 660 MeV varied separately the imaginary parts of all phase shifts of $^3P, ^1D,$ and 3F states. The two most probable of their four solutions resemble the solutions in [11,12].

Table III. Contributions to the sums χ^2 for experimental triple scattering parameters in different variants of phase-shift analysis

	[10]	[11]	[12]
D	9.7	9.5	6.1
R	23.4	11.6	2.2
A	32.2	33.9	4.8
Total	65.3	55.0	13.1

Table III gives the contributions to the sums χ^2 from the different solutions, corresponding to experimental information regarding the triple scattering parameters. It is evident from this table and Fig. 4 that only the set of phase shifts obtained in [12] agrees satisfactorily with all experimental triple scattering parameters.¹⁾ The refinement of this set by including in the analysis the values of A for 54°, 72°, 108°, and 126° will be described in a separate article. Nevertheless, a more exact phase analysis of p-p scattering at 660 MeV will require additional measurements of the triple scattering parameters D, R, and A. New experimental data will also be required regarding inelastic processes in this energy region and also a more consistent inclusion of such data in the phase shift analysis.

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