OPTICAL MODEL FOR ANTINUCLEON-NUCLEON COLLISIONS

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The interaction between nonrelativistic antinucleons and nucleons is considered. This interaction can be effectively described by a complex potential which depends on the spin and isotopic spin and contains a tensor force. The results obtained for the total cross sections, the angular distribution of the elastic scattering, and the charge-transfer cross section are in satisfactory agreement with experiment.

A treatment has previously been given[1] of the simplest model for the interaction between antinucleons and nucleons. This model contained the following simplifications: 1) the interaction was assumed independent of the spin and the isotopic spin; 2) the tensor force and the spin-orbit interaction were not taken into account. In spite of these simplifications there was satisfactory agreement between the theoretical total cross sections and experiment, although not over the entire energy range.

In the present paper we consider the influences of the tensor force, the spin-orbit interaction, and the isotopic dependence of the potential on the total cross sections, and we also calculate the angular distribution of the elastically scattered nucleons and the cross section for charge transfer in the reaction $\bar{p}p \to \bar{n}n$.

1. FUNDAMENTAL EQUATIONS OF THE PROBLEM

The system antinucleon-nucleon can have the isotopic spins T = 1 and T = 0. The states with $T_Z = 1$ and $T_Z = -1$ correspond to the systems proton-antineutron ($p\bar{n}$) and neutron-antiproton ($n\bar{p}$). In the states with $T_Z = 0$ the isotopic spin can be 0 or 1, and therefore these states are mixed.

Thus for the systems $n\bar{p}$ and $p\bar{n}$ we have the wave functions[2]:

$$\psi_{n\bar{p}} = \psi_{\bar{n}p} = \psi^3, \quad (1)$$

where the index 3 denotes the multiplicity (2T + 1) with respect to isotopic spin. The respective functions for the systems $\bar{p}p$ and $\bar{n}n$ are

$$\psi_{\bar{p}p} = (\psi^1 - \psi^3)/\sqrt{2}, \qquad \psi_{\bar{n}n} = (\psi^1 + \psi^3)/\sqrt{2}. \quad (2)$$

When antinucleons are scattered by nucleons the state of the system will be described by the following wave functions, depending on what antinucleons and nucleons are involved in the system:

$$(\psi_{\bar{n}n} + \psi_{\bar{p}p})/\sqrt{2} = \psi^1,$$
$$(\psi_{\bar{n}n} - \psi_{\bar{p}p})/\sqrt{2} = \psi^3 \qquad (\text{for } T_z = 0),$$
$$\psi_{n\bar{p}} = \psi^3 \qquad (\text{for } T_z = -1),$$
$$\psi_{\bar{n}p} = \psi^3 \qquad (\text{for } T_z = 1). \quad (3)$$

The problem will be treated in the nonrelativistic approximation. Then we are to solve the Schrödinger equation with a complex potential

$$H_{\text{kin}}\psi^{1,3} + (V_c^{1,3} + V_T^{1,3} + V_{s.o} + iW)\,\psi^{1,3} = E\psi^{1,3}. \quad (4)$$

Here E is the energy in the center-of-mass system; V_C is the central potential, which is a function of the spin S and the isotopic spin T of the system; V_T is the tensor force; $V_{S.O}$ is the spin-orbit interaction; and W is the imaginary part of the potential, which describes the absorption. It is assumed that W does not depend on S and T.[1] In the energy range in which there is no meson production this assumption can be supposed to be justified, since here W is due to annihilation, i.e., to a multiple process which should not depend much on the parameters of the input channel.

In the general case V_C is of the form

$$V_c = V_1 + V_2 + V_3.$$

The potential V_1 is defined as the scalar part of the one-meson interaction. Then[1)]

$$V_1 = \begin{cases} -f^2 \dfrac{e^{-x}}{x} \dfrac{(\tau_1\tau_2)(\sigma_1\sigma_2)}{3} & (\text{for } (x \geqslant x_0), \\ -f^2 \dfrac{e^{-x_0}}{x_0} \dfrac{(\tau_1\tau_2)(\sigma_1\sigma_2)}{3} & (\text{for } x < x_0). \end{cases} \quad (5)$$

The cut-off parameter $x_0 = 0.1$ is chosen sufficiently small and has no significant effect on the behavior of the cross sections. The sign of V_1 for $\bar{p}p$ is opposite to the sign of the np interaction.

As is well known, the potential (5) is inadequate for the description of np scattering in the $1S_0$

1)In what follows it is assumed that $\hbar = c = 1$, and the unit of length is $1/\mu$, where μ is the mass of the π meson.

state, and it is necessary to assume that there are shorter-range attractive forces. They can be ascribed to the exchange of two or more π mesons, and also to the exchange of heavier particles. We have assumed that there is an attractive potential

$$V_2 = -V_{20}x^{-2}e^{-2x}[1 - e^{-(x/\alpha)^4}], \qquad (6)$$

which is independent of S and T and does not change sign on charge conjugation (as is well known, the exchange of an even number of mesons leads to an interaction which keeps the same sign under charge conjugation).

The potential

$$V_3 = -V_{3,0}e^{-5.4x}[1 + (\sigma_1\sigma_2)/6] \qquad (7)$$

was chosen on the assumption that it is caused by a vector meson with T = 0 and changes sign under charge conjugation. Then the large repulsion for the np system (repulsive core) corresponds to the attractive potential (7). The earlier calculations [1] showed, however, that there is not much dependence of the cross sections on V_3.

The tensor potential V_T corresponds to the tensor part of the one-meson interaction. To obtain agreement with experiment the cut-off of this potential must be made at comparatively large x:

$$V_T = -\tfrac{1}{3}(\tau_1\tau_2)\left(\frac{3}{x^2} + \frac{3}{x} + 1\right) f^2 \frac{e^{-x}}{x}[1 - e^{-(x/\beta)^4}]. \qquad (8)$$

The form of the spin-orbit interaction is chosen as in the paper of Signell and Marshak.[3] Also $V_{s.o}$ does not depend on T:

$$V_{s.o} = \begin{cases} -V_{s.o}^0 \dfrac{9e^{-4x/3}}{16x^2}\left(1 + \dfrac{3}{4x}\right) & (\text{for } x \geqslant x_0), \\ -V_{s.o}^0 \dfrac{9e^{-4x_0/3}}{16x_0^2}\left(1 + \dfrac{3}{4x_0}\right) & (\text{for } x < x_0). \end{cases} \qquad (9)$$

The calculation is made for both signs of $V_{s.o}^0$.

The problem formulated in Eq. (4) allows us to find the connection between the five total cross sections of the following processes: a) annihilation, elastic scattering, and charge transfer in the system $\bar{p}p$; b) annihilation and elastic scattering in the system $n\bar{p}$. For elastic scattering and charge transfer we can find the angular distributions and the polarization. Thus the total number of experiments that can be explained by the optical model for a given energy of the particles is not smaller than eleven.

In a paper by Fulco[4] it is shown that the elastic-scattering cross section in a system with T = 0 is determined by the amplitude

$$f_s = \tfrac{1}{2}(f^1 + f^3), \qquad (10)$$

and the charge-transfer cross section by the amplitude

$$f_{c.e} = \tfrac{1}{2}(f^1 - f^3). \qquad (11)$$

The amplitudes for scattering of singlet and triplet types with respect to the isotopic spin can be written in the form

$$f^{1,3} = \frac{\lambda\!\!\!^{-}}{2}\sum_{lJm}\sqrt{g_{lJ}^{1,3}}\,(1 - \eta_{lJ}^{1,3})\, X_{lJ}^m(\vartheta, \varphi), \qquad (12)$$

where $X_{lJ}^m(\vartheta,\varphi)$ are linear combinations of spherical harmonics, and $\eta_{lJ}^{1,3}$ are scattering coefficients for waves with prescribed quantum numbers.

The cross section for annihilation is given by the well known formula

$$\sigma_a = \pi\lambda\!\!\!^{-2}\sum_{lTJ} g_{lJ}^T\{1 - |\eta_{lJ}^T|^2\}. \qquad (13)$$

Equation (4) is formulated for a theory in which isotopic spin is conserved. If the energy is small, two factors begin to be important: the Coulomb interaction and the difference between the neutron and proton masses. In this case the equations for the functions ψ^1 and ψ^3 are coupled:

$$(H_{\text{kin}} + V_{\text{nuc}}^1 + iW)\psi^1 + \tfrac{1}{2}V_Q(\psi^1 - \psi^3)$$
$$= E\psi^1 - (M_n - M_p)(\psi^1 + \psi^3),$$
$$(H_{\text{kin}} + V_{\text{nuc}}^3 + iW)\psi^3 + \tfrac{1}{2}V_Q(\psi^3 - \psi^1)$$
$$= E\psi^3 - (M_n - M_p)(\psi^1 + \psi^3). \qquad (14)$$

Here V_{nuc} is the real part of the nuclear interaction, and V_Q is the Coulomb interaction.

These corrections, however, are already small at energy 20 MeV. In our calculations we shall neglect the deviations from isotopic invariance.

2. SOLUTION OF THE EQUATIONS AND RESULTS OF THE CALCULATIONS

In [1] the problem of antinucleon-nucleon collisions was solved quasi-classically. In doing so one separates the radial equations, which are coupled by the tensor force, and this gives rise to considerable inaccuracy. In the present work Eq. (4) has been solved exactly with an M-20 computer.

We introduce the notation ${}^3S_1^3$ for a partial wave, where the multiplicity with respect to the spin is at the upper left, that with respect to the isotopic spin at the upper right, and the total angular momentum J at the lower right.

It is obvious that the determination of the coefficients η_{lJ}^m for waves that are singlet with respect to the ordinary spin (and for which there is no tensor force) reduces to the solution of one radial equation. States that are triplets with re-

	U_I		U_{II}		U_{III}		U_{IV}	
E, MeV	20	120	20	120	20	120	20	120
σ_a, mb	175	83	173	86	178	73	141	76
σ_s^t, mb	124	72	123	69	135	78	101	56
$\sigma_{c.e}$, mb	18	11	19	12	21	18	23	11
σ^t, mb	299	155	296	155	313	151	242	132

spect to the ordinary spin and have $l = J$ are also described by one equation, since for given J and parity only the one value of l is possible. The state with $J = l + 1$ is connected with the state with $J = l' - 1$, where $l' = l + 2$. For example, the state ${}^3S_1^1$ is connected with the state ${}^3D_1^1$. For such states we get a system of coupled radial equations. The solution of this system is found in the form of a linear combination of two particular solutions. We choose for these the solutions that satisfy the following initial conditions for $x \ll x_0$: 1) $\psi_1 = \psi_{10} \neq 0$, $\psi_2 = 0$; 2) $\psi_1 = 0$, $\psi_2 = \psi_{20} \neq 0$. For convenience the singularity of the potential has been removed by joining-on to the nonsingular function at very small distances.

For the determination of the cross sections for annihilation, elastic scattering, and charge transfer of antinucleons with energies 10 and 20 MeV it was sufficient to solve the equations with $l = 0$, 1, 2; for the larger energies (70 and 120 MeV) it was necessary to solve also the equations with $l = 3$, 4.

In [1] certain real central potentials and imaginary parts of the potential in the form $W = -W_0 \times e^{-(r/c_n)^n}$ were studied. It was shown that the most satisfactory results are given by the exponential law for W and a potential V_2 that is attractive at moderate distances, and also that the total cross sections depend only weakly on V_3.

In the present work the imaginary part of the potential has been chosen in the form $W = -W_0 \times e^{-x/C}$. The constant V_{20} was found from the np scattering in the $1S_0$ state at $E = 0$. The cut-off parameter α of the potential V_2 was taken equal to 0.5. This quantity has a much weaker influence on the results than does V_{20}. We have taken $f^2 = \mu^2 g^2/4M^2 = 0.09$. The cut-off parameter of the tensor force was found from the binding energy of the deuteron.

The table gives the results of calculations of the annihilation cross sections σ_a, the elastic-scattering cross sections $\sigma_s + \sigma_{c.e} = \sigma_s^t$, and the charge-transfer cross section $\sigma_{c.e}$, for various potentials.

For the potential U_I, $W_0 = -4$ BeV, $V_{s.o}^0 = -30$ MeV; for U_{II}, $W_0 = -4$ BeV, $V_{s.o}^0 = 30$ MeV; for U_{III}, $W_0 = -2$ BeV, $V_{s.o}^0 = -30$ MeV. For these three potentials $V_{20} = 121.9$ MeV, $\beta = 0.71$, $V_{30} = 1750$ MeV. The parameters for the potential U_{IV} are: $V_{s.o}^0 = 0$, $V_{30} = 0$, $V_{20} = 65$ MeV, $\beta = 1, 1$, $W_0 = -4$ BeV. For all four potentials $\alpha = 0.5$, $C = 0.2$.

The dependence on the sign of the spin-orbit interaction is weak. For small energies the dependence on W_0 is also insignificant, but for larger energies the annihilation cross section is considerably larger for $W_0 = -4$ BeV than for $W_0 = -2$ BeV, and the charge-transfer cross section, on the contrary, is much smaller. It must be pointed out that for the potentials U_I and U_{IV} the ratio $(\sigma_s + \sigma_{c.e})/\sigma_a$ varies over the range from 0.7 to 1; i.e., it is much larger than without the tensor force.[1] In fact, the tensor force leads to the appearance of a large repulsion in certain states, for example in ${}^3P_0^3$ and ${}^3P_1^1$. In these states there is not much penetration of the particles into the region of annihilation, and although the absorption cross section is not zero, as in [1], it is much smaller than the scattering cross section. This also leads to an increase of the total cross section for scattering. Nevertheless for a broad class of potentials containing a tensor force and a spin-orbit interaction the scattering cross section is smaller than the absorption cross section; i.e., the results obtained in [1] can be extended to the case of a broad class of nuclear interactions.

3. COMPARISON WITH EXPERIMENT

The potential U_{IV} gives better agreement with experiment than the interactions with other parameters. Besides the constants which fix the one-meson exchange and are well known from the data on πN interactions, this potential contains six parameters. Two of them are determined from the binding energy of the deuteron (the cut-off parameter of the tensor force) and from NN scattering in the $1S_0$ state (the constant V_{20}), and the remaining four are determined from the experiments with antinucleons.

The theoretical curves found for the annihilation

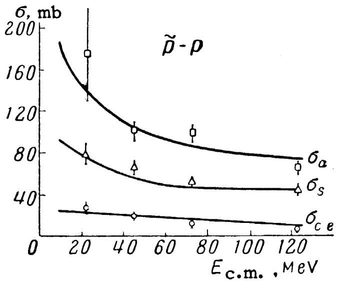

FIG. 1

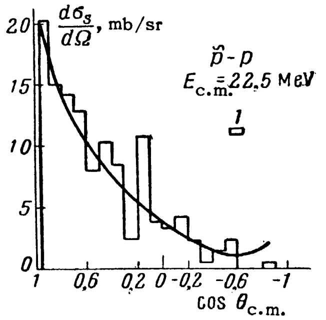

FIG. 2

FIG. 1. Energy dependences of the annihilation cross section σ_a, the true-elastic-scattering cross section σ_s, and the charge-transfer cross section $\sigma_{c.e}$ for the reaction p̄p.

FIG. 2. Angular dependence $d\sigma_s/d\Omega$ of the true elastic scattering (in the c.m.s.) for the energy E = 22.5 MeV.

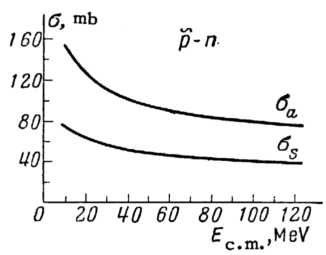

FIG. 5

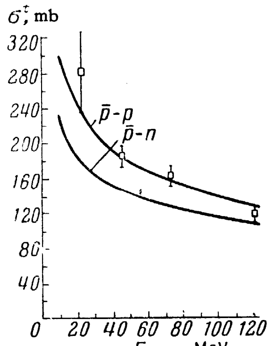

FIG. 6

FIG. 5. Energy dependences of the annihilation cross section σ_a and the elastic-scattering cross section σ_s for the reaction p̄n.

FIG. 6. Energy dependences of the total interaction cross section σ^t for the reactions p̄p and p̄n.

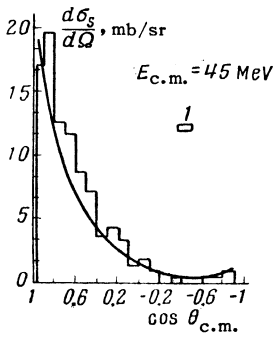

FIG. 3

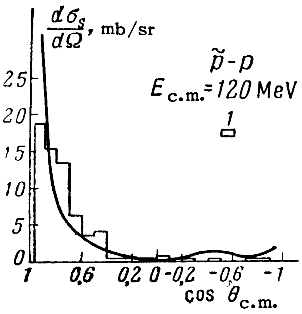

FIG. 4

FIG. 3. Angular dependence $d\sigma_s/d\Omega$ of the true elastic scattering (in the c.m.s.) for the energy E = 45 MeV.

FIG. 4. Angular dependence $d\sigma_s/d\Omega$ of the true elastic scattering (in the c.m.s.) for the energy E = 120 MeV.

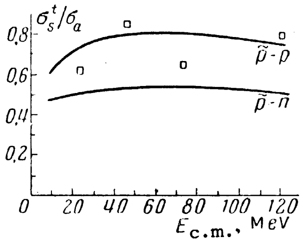

FIG. 7

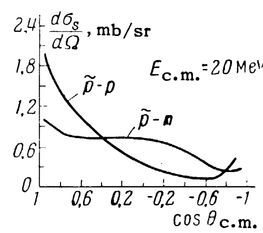

FIG. 8

FIG. 7. Energy dependences of the ratio $(\sigma_s + \sigma_{c.e.})/\sigma_a$ for the reactions p̄p and p̄n.

FIG. 8. Angular dependences $d\sigma_s/d\Omega$ (in the c.m.s.) of the true elastic scattering in the reactions p̄p and p̄n, for energy E = 20 MeV.

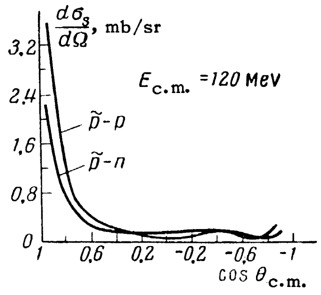

FIG. 9

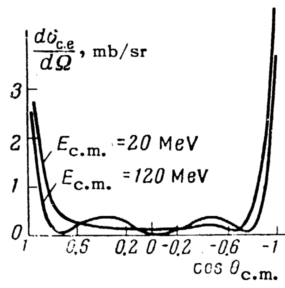

FIG. 10

FIG. 9. Angular distributions $d\sigma_s/d\Omega$ of the true elastic scattering (in the c.m.s.) for energy E = 120 MeV, for the reactions p̄p and p̄n.

FIG. 10. Charge-transfer angular distributions $d\sigma_{c.e.}/d\Omega$ in reaction p̄p for energies E = 20 MeV and E = 120 MeV.

cross section σ_a, the true-elastic-scattering cross section σ_s, and the charge-transfer cross section $\sigma_{c.e}$ for the reaction p̄p are compared in Fig. 1 with the available experimental data. [5]

In Figs. 2—4 the angular dependences $d\sigma_s/d\Omega$ are compared with experiment. [5] It can be seen that in the energy range 20 MeV ≤ E ≤ 120 MeV there is good agreement with experiment.

Figure 5 shows the shapes of the variation with energy of the cross sections for annihilation and elastic scattering in the reaction p̄n. It can be seen that whereas the σ_a for the reaction p̄n is practically the same as the σ_a for the reaction p̄p, the σ_s^t for p̄n is 30—40 percent smaller than σ_s^t for p̄p. There is also considerable difference between the total cross sections σ^t, which are shown in Fig. 6.

Figure 7 shows the ratio $(\sigma_s + \sigma_{c.e})/\sigma_a$ for p̄p and p̄n. This ratio is considerably smaller for p̄n. Figures 8 and 9 show the angular distribution $d\sigma_s/d\Omega$ for the reaction p̄n, and also for comparison the $d\sigma_s/d\Omega$ for p̄p. Each of the curves is normalized to the corresponding cross section, and the ordinates are in arbitrary units. It can be seen that for small energies the angular distribution for p̄n is more isotropic than that for p̄p.

Figure 10 gives the angular distributions for charge transfer of antinucleons at different energies; there are as yet no experimental data for this distribution. Also there are no experiments on $\bar{p}n$ scattering. Such experiments could give additional evidence about the applicability of the model proposed here.

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