ELECTROMAGNETIC WAVES IN A METAL UNDER CONDITIONS OF FERROMAGNETIC RESONANCE

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The high-frequency properties of a ferromagnetic conductor located in a stationary magnetic field are investigated. Under these conditions a ferromagnetic conductor is characterized by a strong coupling between weakly decaying electromagnetic waves and the magnetic-moment oscillations. The case of coupled helicons (a conductor with one type of carrier) and the case of coupled magnetohydrodynamic waves (a conductor with equal concentrations of two groups of carriers) are considered. The dependence of the spectrum and coupled-wave attenuation, as well as the impedance tensor, on frequency, magnitude of the magnetic field, and its orientation relative to the symmetry axes is studied.

WEAKLY decaying electromagnetic waves in conductors located in a strong magnetic field have been observed by a number of authors.[1-10] At the present time helicons have been discovered in many metals[1-3] and in InSb.[4] Magnetohydrodynamic waves have been observed in bismuth having equal concentrations of electrons and holes. The theory of electromagnetic waves in metals in a magnetic field has been considered by various authors.[10-13] The existence of weakly decaying waves leads to positive values of the effective dielectric constant in the region of frequencies less than cyclotron frequencies and causes a number of resonant effects.[1-10]

The weak attenuation of these excitations in the absence of spatial dispersion is caused by scatterings of the carriers, and in the case of strong dispersion, as Kaner and Skobov[12] showed, it is determined by Landau decay and depends in an essential fashion on the orientation of the magnetic field relative to the symmetry axes of the crystal. The shape of the equal-energy surface of the conduction electrons also influences the spectrum and decay of the electromagnetic waves.[14]

The electromagnetic properties of conductors that are connected with the existence of helicons and magnetohydrodynamic waves should manifest themselves during observation of ferromagnetic resonance. It is shown below that a strong interaction between the electromagnetic waves and the oscillations of the magnetic moment produces a significant change in the spectrum of the helicons and magnetohydrodynamic waves. Thereby, in the frequency region in which the effective magnetic permeability is positive, the conductor has a selective transparency and displays resonant properties similar to those observed in ordinary metals. In other words, in a characteristic range of frequencies a ferromagnetic conductor behaves like a dielectric.

Stern and Callen[15] have made a close investigation of the propagation of a helicon in a uniaxial ferromagnetic along the applied magnetic field with a quadratic dependence of the electron energy on momentum and without taking dissipation into account.[1)]

In treating a ferromagnetic metal it is necessary to consider spatial dispersion and anisotropy of the energy spectrum of the carriers.

DISPERSION EQUATION

We consider a homogeneous, unbounded ferromagnetic metal. The propagation of a plane monochromatic electromagnetic wave of frequency ω in the ferromagnetic is determined by the Maxwell equations:

$$i[\mathbf{kh}] = 4\pi c^{-1}\mathbf{j}, \qquad [\mathbf{ke}] = \omega c^{-1}\mathbf{b}, \tag{1*}$$

$$j_i = \sigma_{ik}e_k, \qquad b_i = \mu_{ik}h_k, \tag{2}$$

where **e** and **h** are the alternating electric and magnetic fields, **j** and **b** are respectively the current density and magnetic induction, σ_{ik} and μ_{ik} are the conductivity and magnetic permeability tensors, and **k** is the wave vector.

[1)]F. G. Bass had expressed the idea of coupled waves in ferromagnetic conductors independently of Stern and Callen.[15]

*$[\mathbf{kh}] = \mathbf{k} \times \mathbf{h}$

We choose axis 3 (Z) along the constant magnetic field **H**, and axis 1 (X) directed perpendicular to the vectors **k** and **H**. The angle between **k** and **H** is Φ. We also need a coordinate system X, η, ζ, where $\zeta \parallel$ **k**.

Setting the determinant of the system of Eqs. (1) and (2) equal to zero leads to the dispersion equation of the natural electromagnetic oscillations of the unbounded metal, which can be written in invariant form:

$$\begin{aligned} &{}^1/_2 k_i k_j e_{ikl} e_{jmn} \eta_{km} \eta_{ln} \sigma_{pq} k_p k_q \\ &- 4\pi i\omega c^{-2}[(\sigma_{ij}k_ik_j)\eta_{kl}\sigma_{kl} - k_i\sigma_{ij}\eta_{lj}\sigma_{lm}k_m] \\ &+ (4\pi\omega c^{-2})^2|\sigma_{ij}| = 0. \end{aligned} \tag{3}$$

Here e_{ikl} is a completely antisymmetric unit tensor, $\eta_{ij} = \mu_{ij}^{-1}$ is the reciprocal magnetic permeability tensor.

The applied magnetic field will be assumed strong, which means

$$k_z v \ll \omega_c, \tag{4}$$

where $\omega_c = |e|B/m^*c$ is the cyclotron frequency, the magnetic induction $\mathbf{B} = \mathbf{H} + 4\pi \mathbf{M}_0$, $\mathbf{M}_0$ is the saturation magnetic moment, $k_z = k|\cos\Phi|$, v is the carrier velocity, of the order of the Fermi velocity v_F, and m^* is the effective mass of the carriers.

In the considered case of a strong magnetic field it is possible to neglect the magnetic anisotropy energy and to assume that the total magnetic moment is parallel to the field **H**. The condition (4) also allows us to neglect in the tensor μ_{ik} of the spatial dispersion, which is important only at wavelengths small compared with interatomic spacings.

The elements of the tensor μ_{ik} in the system 123 have the usual form (see, for example, [16]):

$$\mu_{ik} = \begin{pmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_1 & \mu_1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\begin{aligned} \mu_1 &= 1 + 4\pi\gamma M_0\Omega(\Omega^2 - \omega^2 + 2i\lambda\omega) / (\Omega^2 - \omega^2)^2, \\ \mu_2 &= 4\pi\gamma M_0\omega(\Omega^2 - \omega^2 + 2i\lambda\omega) / (\Omega^2 - \omega^2)^2, \end{aligned} \tag{5}$$

where $\gamma = g'|e|/2mc$ (g' is the spectroscopic factor, m is the mass of a free electron), $\Omega = \gamma H$ is the ferromagnetic resonance frequency, λ is the relaxation constant.

HELICONS ($n_1 \neq n_2$)

1. Helicons are propagated in conductors with one group of carriers in the region of frequencies less than the cyclotron frequency.[11-14] Following Kaner and Skobov,[12] we consider the case of strong spatial dispersion

$$|\nu - i\omega| \ll k_z v, \tag{6}$$

where ν is the effective electron collision frequency.

If the attenuation of the wave is to be small it is also necessary that the frequency ω substantially exceed the inverse relaxation time of the magnetic moment

$$\omega \gg \lambda. \tag{7}$$

Under the conditions (4) and (6), for an isotropic electronic dispersion law in the case of a single group of carriers, the asymptote of the tensor σ_{ik} in the 123 system calculated in[12] has the form

$$\sigma_{ik} = \frac{n|e|c}{B}\begin{pmatrix} (\nu - i\omega)/\omega_c + {}^3/_8\pi k_z R \operatorname{tg}^2\Phi & 1 & -\operatorname{tg}\Phi \\ -1 & (\nu - i\omega)/\omega_c & 0 \\ \operatorname{tg}\Phi & 0 & 3(\nu - i\omega)\omega_c^{-1}(k_z R)^{-2} \end{pmatrix}, \tag{8*}$$

where $R = cp_F/|e|B$; p_F is the Fermi limiting momentum.

As is seen from Eq. (8), the Hermitian part of the tensor σ_{ik}, which pertains to the dissipation of energy, contains besides a collision term a component associated with the calculation of spatial dispersion. Under the conditions (6) it will be the most important term in calculating the attenuation of the wave.

In the 123 system, Eq. (3) takes the form

$$\begin{aligned} &(\mu_1 s_2^2 + s_3^2)(\sigma_{22}s_2^2 + \sigma_{33}s_3^2)k^4 - 4\pi i\omega c^{-2}\{s_2^2[\mu_1(\sigma_{11}\sigma_{22} + \sigma_{12}^2) \\ &+ |\mu_{ik}|\sigma_{22}\sigma_{33}] + s_3^2[\mu_1(\sigma_{11}\sigma_{33} + \sigma_{13}^2 + \sigma_{22}\sigma_{33}) \\ &+ 2\mu_2\sigma_{12}\sigma_{33}] + 2s_2s_3[\mu_1\sigma_{12} - \mu_2\sigma_{22}]\sigma_{13}\}k^2 \\ &- (4\pi\omega / c^2)^2|\sigma_{ik}||\mu_{ik}| = 0, \quad \mathbf{s} = \mathbf{k}/k. \end{aligned} \tag{9}$$

Substituting Eqs. (5) and (8) into (9) and omitting the dissipative terms, it is not difficult to obtain the expression for the spectrum of the wave and its dependence on the frequency, the magnitude of the applied magnetic field, and the angle Φ:

$$k_\pm^2 = \frac{4\pi\omega\sigma_{12}}{c^2}\,\frac{4\pi\gamma M_0\omega \pm (\Omega\Omega_1 - \omega^2)^{1/2}[\Omega_1(\Omega\cos^2\Phi + \Omega_1\sin^2\Phi) - \omega^2]^{1/2}|\cos\Phi|^{-1}}{\omega_r^2 - \omega^2}, \tag{10}$$

*tg = tan

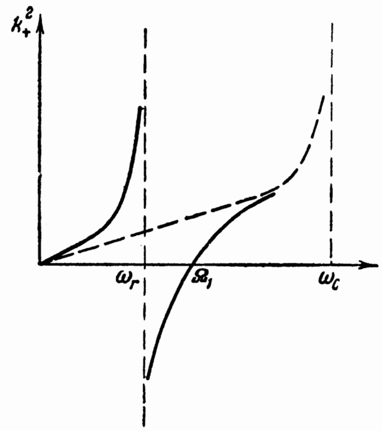

FIG. 1

where

$$\Omega_1 = \Omega + 4\pi\gamma M_0, \qquad \omega_r^2 = \Omega(\Omega\cos^2\Phi + \Omega_1\sin^2\Phi).$$

Direct calculations show that $k_-^2 < 0$ for all values of ω.

The dependence of k_+^2 on ω is represented schematically in Fig. 1. The dashed lines indicate the dependence of k^2 on ω for helicons and spin waves, neglecting the interaction between them. A particularly strong interaction is caused by the fact that the coupling parameter $4\pi\gamma M_0/\Omega$ is, generally speaking, not small.

When $\Phi = 0$ the wave spectrum has a more simple form:

$$k_+^2 = \frac{4\pi\omega\sigma_{12}}{c^2}\frac{\Omega_1 - \omega}{\Omega - \omega}. \qquad (11)$$

At small and large frequencies the spectrum approaches the spectrum of a helicon.

At small ω

$$k_+^2 \approx \frac{4\pi\omega\sigma_{12}}{c^2}\frac{\Omega_1|\cos\Phi|^{-1}}{\omega_r}; \qquad (12)$$

for $\omega \gg \Omega_1$

$$k_+^2 \approx \frac{4\pi\omega\sigma_{12}}{c^2}\left(|\cos\Phi|^{-1} - \frac{4\pi\gamma M_0}{\omega}\right). \qquad (13)$$

Substituting (10) into (4), we obtain the condition of nearness to the point of ferromagnetic resonance:

$$\frac{\omega_r - \omega}{\omega_r} \gg \frac{v_F^2}{c^2}\frac{n|e|c}{B}\frac{4\pi\gamma M_0}{\omega}. \qquad (14)$$

Near the point $\omega = \Omega_1$ the inequality (6) ceases to be valid. The condition of nearness to this point has the form

$$\frac{\omega - \Omega_1}{\Omega_1} \gg \frac{c^2}{v_F^2}\frac{|\nu - i\omega|^2}{\Omega_1^2}\frac{\gamma M_0}{\sigma_{12}}. \qquad (15)$$

In the general case $\Phi \neq 0$, the part of the field of the coupled wave that is transverse relative to **k** is elliptically polarized. The coefficient of ellipticity is determined from the expression

$$\alpha = \frac{e_x}{e_\eta} = i\left[\frac{\Omega\Omega_1 - \omega^2}{\Omega_1(\Omega\cos^2\Phi + \Omega_1\sin^2\Phi) - \omega^2}\right]^{1/2}. \qquad (16)$$

When $\Phi = 0$ the wave becomes circularly polarized.

The attenuation of the coupled wave for $\Phi \neq 0$ is due principally to spatial dispersion and has the form ($k = k_0 + ik''$)

$$\frac{k''}{k_0} = \frac{3}{32}\pi\frac{kR\sin^2\Phi}{(\cos^2\Phi + (\Omega_1^2 - \omega^2)(\Omega\Omega_1 - \omega^2)^{-1}\sin^2\Phi)^{1/2}}. \qquad (17)$$

For $\Phi = 0$ the spatial dispersion in (8) is absent and the attenuation of the coupled wave is determined by the relaxation times of the conduction electrons and of the magnetic moment

$$\frac{k''}{k_0} = \frac{1}{2}\frac{\nu}{\omega_c} + \frac{4\pi\gamma M_0\omega}{(\Omega - \omega)(\Omega_1 - \omega)}\lambda. \qquad (18)$$

Consider the case of a positive Hall constant. If the current carriers are "holes," the spectrum of the coupled wave has an entirely different form. For simplicity we shall give the results for $\Phi = 0$. In this case the "plus" wave has the spectrum

$$k_+^2 = \frac{4\pi\omega|\sigma_{12}|}{c^2}\frac{\Omega_1 - \omega}{\omega - \Omega}, \qquad (19)$$

which is illustrated in Fig. 2.

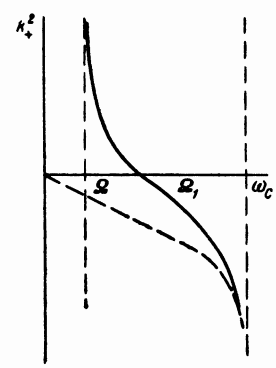

FIG. 2

The values of the wave vector of the "minus" wave also become positive:

$$k_-^2 = \frac{4\pi\omega|\sigma_{12}|}{c^2}\frac{\omega + \Omega_1}{\omega + \Omega}. \qquad (20)$$

This spectrum is shown in Fig. 3.

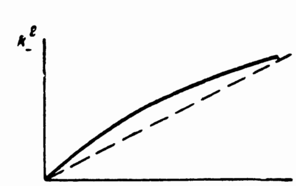

FIG. 3

2. Account of the anisotropy of the Fermi surface leads to a number of peculiarities in the propagation of the helicons, in particular to a change in the dependence of the attenuation on magnetic field.[14] Similar results obtain also in the case of a ferromagnetic metal that has a singly-connected convex Fermi surface.

Use of the asymptotic tensor σ_{ik} calculated by Kaner and Skobov[14], with arbitrary dependence of the electron energy on momentum, for a different orientation of the magnetic field relative to the symmetry axes, yields the following results.

Let **H** be parallel to the symmetry axis. Then the corrections to the wave spectrum (10) due to spatial dispersion of the conduction electrons are found to be less than the attenuation determined by the expression

$$\frac{k''}{k_0} = \frac{1}{4}\frac{B}{n|e|c} A w_1^2 \left(\frac{\Omega\Omega_1 - \omega^2}{\Omega_1(\Omega\cos^2\Phi + \Omega_1\sin^2\Phi) - \omega^2}\right)^{1/2} \sim kR;$$

$$A = \frac{(2\pi e)^2}{n^3}\left|\frac{m^*}{k_z \overline{v_z'}}\right|_{\bar{v}_z=0}, \qquad v_z' = \frac{\partial v_z}{\partial p_z},$$

$$w_1 = (c/eB)\,\overline{kv\,(p_2 - \bar{p}_2)}. \tag{21}$$

The bar indicates the average over the trajectory in momentum space.

Note that this case of orientation of the field along the symmetry axis is analogous to the case of an isotropic spectrum.

As already mentioned, when $\Phi = 0$ the Landau damping vanishes and the attenuation of the coupled wave is determined by the values of ν and λ.

If the direction of **H** is not along the symmetry axis, it can be shown that the attenuation of the coupled wave undergoes a resonance $(k''/k_0)_{max} \sim |(\nu - i\omega)/k_z v|$ similar to that which takes place in the case of the helicon and due to the interaction with the decaying longitudinal wave.[14] The role of the magnetic permeability then reduces to an insignificant change in the factor preceding the resonance term.

3. The existence of a weakly damped coupled wave leads to transparency of the metal in a particular frequency region. This fact is manifest in the reality of the elements of the impedance tensor, which is defined by

$$Z_{\alpha\beta} = \partial e_\alpha(0)/\partial J_\beta, \quad \alpha, \beta = x, \eta, \tag{22}$$

where J_β is the total current in the volume of the metal.

Let the ferromagnetic metal fill the half-space $\zeta > 0$ and let **H** lie in the plane $X = 0$.

Going over to the elliptically polarized wave, we write the system of Maxwell's equations after eliminating the variable magnetic field in the form

$$\partial^2 e_\pm/\partial\zeta^2 + 4\pi\omega c^{-2}\left(\tilde{\mu}''_{x\eta} \pm \sqrt{\tilde{\mu}_{xx}\tilde{\mu}_{\eta\eta}}\right)\tilde{\sigma}_{x\eta} e_\pm = 0. \tag{23}$$

Here

$$e_\pm = e_x \pm \alpha e_\eta, \qquad \tilde{\sigma}_{x\eta} = n|e|cB^{-1}|\cos\Phi|^{-1},$$

$$\alpha = i(\tilde{\mu}_{\eta\eta}/\tilde{\mu}_{xx})^{1/2}$$

is the coefficient of ellipticity determined from Eq. (16). The tensor $\mu_{\alpha\beta}$ has the form

$$\tilde{\mu}_{\alpha\beta} = \frac{1}{\omega_r^2 - \omega^2} \times \begin{pmatrix} \Omega_1(\Omega\cos^2\Phi + \Omega_1\sin^2\Phi) - \omega^2 & i4\pi\gamma M_0\omega\cos\Phi \\ -4i\pi\gamma M_0\omega\cos\Phi & \Omega\Omega_1 - \omega^2 \end{pmatrix}. \tag{24}$$

In order to obtain Eq. (23) the dissipative terms were omitted.

We introduce the surface impedance tensor for elliptically polarized waves by means of the formulas

$$e_\pm(0) = Z_\pm J_\pm + Z' J_\mp, \qquad J_\pm = J_x \pm \alpha J_\eta. \tag{25}$$

The elements of this tensor are related to the elements of $Z_{\alpha\beta}$ by the following relations:

$$Z_\pm = \frac{1}{2}(Z_{xx} + Z_{\eta\eta}) \pm \frac{1-\alpha^2}{2\alpha} Z_{x\eta},$$

$$Z' = \frac{1}{2}(Z_{xx} - Z_{\eta\eta}) - \frac{1+\alpha^2}{2\alpha} Z_{x\eta}. \tag{26}$$

Expressing by means of Maxwell's equations the electric field on the boundary via the total current in the volume of the metal and using the definition (25), we obtain after uncomplicated calculations

$$Z_- = \frac{1}{c}\sqrt{\frac{\pi\omega}{\tilde{\sigma}_{x\eta}}}\,(\tilde{\mu}_{xx} + \tilde{\mu}_{\eta\eta})\,\frac{[(\tilde{\mu}_{xx}\tilde{\mu}_{\eta\eta})^{1/2} + \tilde{\mu}''_{x\eta}]^{1/2}}{(\tilde{\mu}_{xx}\tilde{\mu}_{\eta\eta})^{1/2}}, \tag{27}$$

$$Z_+ = -\frac{i}{c}\sqrt{\frac{\pi\omega}{\tilde{\sigma}_{x\eta}}}\,(\tilde{\mu}_{xx} + \tilde{\mu}_{\eta\eta})\,\frac{[(\tilde{\mu}_{xx}\tilde{\mu}_{\eta\eta})^{1/2} - \tilde{\mu}''_{x\eta}]^{1/2}}{(\tilde{\mu}_{xx}\tilde{\mu}_{\eta\eta})^{1/2}}, \tag{28}$$

$$Z' = \frac{i}{c}\sqrt{\frac{\pi\omega}{\tilde{\sigma}_{x\eta}}}\,(\tilde{\mu}_{xx} - \tilde{\mu}_{\eta\eta})\,\frac{[(\tilde{\mu}_{xx}\tilde{\mu}_{\eta\eta})^{1/2} - \tilde{\mu}''_{x\eta}]^{1/2}}{(\tilde{\mu}_{xx}\tilde{\mu}_{\eta\eta})^{1/2}}. \tag{29}$$

In Eqs. (27)—(29), we have omitted for simplicity small quantities of the order kR associated with spatial dispersion.

Thus, the impedance element Z_- is real in the approximation used, owing to the penetration of the wave with "minus" polarization into the metal. The element Z_+ is imaginary, corresponding to reflection of the wave with "plus" polarization from the surface of the ferromagnet.

With the aid of (27)—(29) it is not difficult to obtain the explicit dependence of the impedance tensor on the frequency, the magnetic field strength, and the angle Φ. In the general case this dependence is rather complicated. Limiting ourselves for simplicity to the case $\Phi = 0$, we find

$$Z_- = \frac{4\pi}{c}\omega_0^{-1}\sqrt{\omega\omega_c|\cos\Phi|}\sqrt{\frac{\Omega_1-\omega}{\Omega-\omega}}, \quad (30)$$

$$Z_+ = -i\frac{4\pi}{c}\omega_0^{-1}\sqrt{\omega\omega_c|\cos\Phi|}\sqrt{\frac{\Omega_1+\omega}{\Omega+\omega}}, \quad (31)$$

where $\omega_0 = (4\pi e^2 n/m)^{1/2}$ is the plasma frequency.

The small complex corrections omitted in (30) and (31) are proportional to the inverse relaxation times of the conduction electrons and of the magnetic moment. The formulas reflect the frequency dependence of the spectrum shown in Fig. 1.

Note that according to (30) $\mathrm{Re}Z_-$ goes to infinity when ω is close to Ω and to zero when ω is close to Ω_1. This makes the observation of ferromagnetic resonance very convenient.

MAGNETOHYDRODYNAMIC WAVES ($n_1 = n_2$)

Kaner and Skobov have shown[12] that magnetohydrodynamic waves can propagate in metals with equal concentrations of electrons and ''holes'' ($n_1 = n_2$) at frequencies $\nu_s \ll \omega \ll \omega_{cs}$ (s = 1, 2).

Now the role of spatial dispersion turns out to be important. In the case of weak spatial dispersion there is propagation of Alfven and fast magnetic-sound waves, and for strong dispersion, Alfven and slow magnetic sound waves. In the latter case, however, Landau damping becomes significant, and the existence of magnetohydrodynamic waves is possible only with a magnetic field parallel to a symmetry axis of high order.

Equality of the concentrations of carriers of different sign is characteristic of a number of ferromagnetic metals, in particular iron. In this section we consider the coupling of magnetohydrodynamic waves with the oscillations of the magnetic moment.

1. We consider the case of weak spatial dispersion, when the frequency of the wave is limited by the following inequality:

$$k_z v_s \ll \omega \ll \omega_{cs}. \quad (32)$$

Neglecting collisions of the carriers, we write the total tensor σ_{ik} in the form[12]

$$\sigma_{ik} = \frac{n|e|c}{B}\begin{pmatrix} -i\omega a_1/\omega_c & -i\omega a_{12}/\omega_c & a_{13} \\ -i\omega a_{12}/\omega_c & -i\omega a_2/\omega_c & -a_{32} \\ -a_{13} & a_{32} & i\omega_c\omega^{-1}a_3 \end{pmatrix}. \quad (33)$$

Here $\omega_c = |e|B/(m_1 + |m_2|)c$, and the dimensionless elements of the matrix a_{ik} depend only on the shape of the equal-energy surface and the orientation of the magnetic field.

The solution of the dispersion equation (3) in this case has the form

$$k^2 = \frac{\omega^2}{v_a^2}\{A_{xx}\tilde{\mu}_{\eta\eta} + A_{\eta\eta}\tilde{\mu}_{xx} \pm [(A_{xx}\tilde{\mu}_{\eta\eta} + A_{\eta\eta}\tilde{\mu}_{xx})^2 - 4\det A_{\alpha\beta}\det\tilde{\mu}_{\alpha\beta}]^{1/2}\}, \quad (34)$$

where $v_a = H/[4\pi m(m_1 + |m_2|)]^{1/2}$, the tensor $\tilde{\mu}_{\alpha\beta}$ is determined by the expression (24), and the matrix $A_{\alpha\beta}$ is related to $a_{\alpha\beta}$ in the following way[12]:

$$A_{\alpha\beta} = \begin{pmatrix} a_1 + a_{13}^2/a_3 & (a_{12} + a_{13}a_{32}/a_3)|\cos\Phi|^{-1} \\ (a_{12} + a_{13}a_{32}/a_3)|\cos\Phi|^{-1} & (a_2 + a_{23}^2/a_3)\cos^2\Phi \end{pmatrix}; \quad (35)$$

in calculating $A_{\alpha\beta}$ it is supposed that $|\cos\Phi| \gg \omega/\omega_c$.

For an isotropic dependence of the energy of the carriers on the momentum or for a magnetic field parallel to the symmetry axis $A_{xx} = A_{\eta\eta} \times \cos^2\Phi = 1$, $A_{x\eta} = 0$, and the spectrum of the coupled waves takes the form

$$k_\pm^2 = \frac{\omega^2}{v_a^2}\frac{\cos^{-2}\Phi}{\omega_r^2 - \omega^2}\{(\Omega_1^2 - \omega^2)\sin^2\Phi + 2(\Omega\Omega_1 - \omega^2)\cos^2\Phi \pm [(\Omega_1^2 - \omega^2)^2\sin^4\Phi + 4(4\pi\gamma M_0\omega)^2\cos^4\Phi]^{1/2}\}. \quad (36)$$

Neglecting interaction, the wave bearing the ''minus'' sign in (36) becomes the Alfven wave, and the one with the ''plus'' sign the fast magnetic-sound wave. Calculations show that the spectrum of the coupled fast wave is analogous to the spectrum of the coupled helicon pictured in Fig. 1. With the Alfven wave the dependence of the wave vector on frequency has a non-resonant character, similar to that shown in Fig. 3 for the case when the carriers are ''holes.''

When $\Phi = 0$, Eq. (36) for the spectrum simplifies to

$$k_\pm = \frac{\omega}{v_a}\sqrt{\frac{\Omega_1 \mp \omega}{\Omega \mp \omega}}. \quad (37)$$

The coupled magnetohydrodynamic waves have in the case (36), generally speaking, elliptical polarization. For $\Phi = 0$ the part of the field transverse to k is circularly polarized, and the electric field vectors in the ±-waves rotate in opposite directions.

In the particular case $\Phi = \pi/2$ the asymptotic expression for the tensor has the form (in the X, η, ζ system)

$$\sigma_{ik} = \frac{n|e|c}{B}\begin{pmatrix} -i\omega\omega_c^{-1}a_1 & a_{13} & i\omega\omega_c^{-1}a_{12} \\ -a_{13} & i\omega_c\omega^{-1}a_3 & -a_{32} \\ i\omega\omega_c^{-1}a_{12} & a_{32} & -i\omega\omega_c^{-1}a_2 \end{pmatrix}. \quad (38)$$

Here the fast magnetic-sound wave propagates as in an ordinary metal,[12] without interacting with the magnetic-moment oscillations. The wave corresponding to the choice of a "minus" sign in the solution of the dispersion equation is not the analog of an Alfven wave in this case and has the following spectrum:

$$k_-^2 = \frac{\omega_0^2}{c^2}\left(a_3 + \frac{a_{23}^2}{a_2}\right)\frac{\Omega_1^2 - \omega^2}{\omega^2 - \Omega\Omega_1}. \quad (39)$$

The wave vector is real in a relatively narrow interval of frequency $\Omega < \omega < (\Omega\Omega_1)^{1/2}$. The transverse part of the field in this wave is parallel to **H**.

In ordinary metals, as Kaner and Skobov[14] have pointed out, the condition (32) $k_z v_s \ll \omega$ can be fulfilled only in fields of the order of 10^6 Oe, whereas in semiconductors and metals with not too high a concentration it is perfectly feasible.

2. The possibility of propagation of magnetohydrodynamic waves in metals with $n_1 = n_2$ in the case of strong spatial dispersion

$$v_s \ll \omega \ll kv_s \ll \omega_{cs} \quad (40)$$

exists in fields that are not too high and satisfy the condition[12]

$$v_a \ll v_s. \quad (41)$$

As already mentioned, for the existence of magnetohydrodynamic waves it is necessary here that the magnetic field be parallel to the symmetry axis of the crystal.

The electronic part of the conductivity tensor, by virtue of the assumptions made above, has the form (8); the "hole" part can be obtained by replacing the electronic characteristics by "hole" characteristics. As a result the total tensor σ_{ik} is found to be diagonal:

$$\sigma_{xx} = \frac{n|e|c}{B}\left(\frac{\nu - i\omega}{\omega_c} + \frac{3}{8}\pi k_z R\,\mathrm{tg}^2\,\Phi\right),$$

$$\sigma_{yy} = \frac{n|e|c}{B}\frac{\nu - i\omega}{\omega_c},$$

$$\sigma_{zz} = 3\frac{ne^2}{\mu}(k_z v_F)^{-2}\left(\frac{\nu_1|m_2| + \nu_2 m_1}{m_1 + |m_2|} - i\omega\right). \quad (42)$$

Here $\nu = (\nu_1 m_1 + \nu_2 m_2)/(m_1 + |m_2|)$ and μ is the reduced mass. Since in the general case $\Phi \neq 0$, as seen from Eq. (42),

$$|\sigma_{yy}| \ll |\sigma_{xx}| \ll |\sigma_{zz}|,$$

the x and z components of the electric field in the coupled wave are negligibly small. Setting e_x and e_z equal to zero, we obtain from the system (1) and (2) a dispersion equation for the coupled Alfven wave:

$$k_-^2 = \frac{\omega^2}{v_a^2}\frac{\omega^2 - \Omega_1^2}{\omega^2 - \Omega\Omega_1}\cos^{-2}\Phi. \quad (43)$$

The slow magnetic-sound wave in this case has an imaginary wave vector.

For $\Phi = 0$, the spatial dispersion does not play any role, and both waves become weakly damped. Their spectrum coincides with the spectrum given by Eq. (34), in which it is necessary to set $A_{ik} = \delta_{ik}$. Their attenuation, as in the case of the wave (43), is due to the finite relaxation of the carriers and of the magnetic moment.

We consider now the case of arbitrary direction of the magnetic field with respect to the symmetry axes, for which the most important components in the conductivity tensor are those due to Landau damping (see [14]). In these conditions Eq. (3) does not have real solutions, with the exception of an electromagnetic wave linearly polarized along **H**. Its dispersion equation can be obtained from the system (1), (2) by setting the field e_α and the elements $\sigma_{\alpha z}$ (α = x, y) equal to zero. The spectrum of this wave takes the form

$$\frac{\hbar k^2}{2M} = \omega\sqrt{\frac{\omega^2 - \Omega_1^2}{\omega^2 - \Omega\Omega_1}}\,|\sin 2\Phi|^{-1}, \quad M = \frac{\hbar}{c}\left|4\pi e^2 \sum \frac{dn}{d\varepsilon}\right|^{1/2}, \quad (44)$$

where ϵ is the Fermi energy.

As the estimates presented in [14] show, the wave can exist in fields limited by the inequality

$$(\omega / \omega_{cs})(v_s / v_a)^3 \gg 1. \quad (45)$$

For metals ($n \sim 10^{22}$) this means $H \ll 10^3\,\omega^{1/4}$. The possibility of propagation of this wave is essentially connected with the anisotropy of the Fermi surface. For $\Phi = 0$ the wave is also absent. As seen from (44) the wave vector is real over the entire allowable frequency range, except the interval $\Omega < \omega < (\Omega\Omega_1)^{1/2}$. The impedance of the wave with linear polarization, determined by means of Eq. (22), has the form

$$Z_{\eta\eta} = \frac{4\pi}{c^2}\sqrt{\frac{\hbar\omega}{2M}|\sin 2\Phi|}\sqrt{\frac{\omega^2 - \Omega_1^2}{\omega^2 - \Omega\Omega_1}}. \quad (46)$$

Thus, weakly damped electromagnetic waves of the coupled helicon or magnetohydrodynamic type can propagate in a wide class of ferromagnetic conductors. The existence of these waves should be manifest in the transparency of the ferromagnet in a particular frequency region and in the resonant excitation of standing waves in a plate, similar to that which occurs in a number of experiments on the observation of undamped waves in ordinary metals and semiconductors.[1-10] A characteristic feature is the asymmetric shape of the ferromagnetic resonance line.

Since the spectrum and attenuation of the coupled waves are extremely sensitive to the shape of the equal-energy surface, their experimental detection could give specific information about the energy spectrum of the carriers in ferromagnetic conductors.

It is difficult to make numerical estimates because of the absence of experimental data on ferromagnets. The values of the pertinent quantities for iron presented by Stern and Callen[15] allow one to expect that magnetohydrodynamic waves can propagate in iron under the conditions of (40).

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