ACCELERATION OF CHARGED PARTICLES BY A RADIATION BEAM

L. É. GUREVICH and A. A. RUMYANTSEV

A. F. Ioffe Physico-technical Institute, Academy of Sciences, U.S.S.R.

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We consider acceleration of charged particles by a beam of radiation as a result of Compton scattering of the beam photons on electrons. We show that for the radiation fluxes encountered in nature (such as during supernova flares) the nuclei may acquire energies of the order of their rest energy.

TSYTOVICH [1] noted the possibility of plasma particle acceleration by Cerenkov absorption of protons in the presence of a magnetic field or if the dielectric constant is greater than 1. We consider here a different mechanism of acceleration, with Compton scattering of photons, which does not depend on these conditions.

1. Let the radiation flux pass through a layer of plasma of concentration n (for example, the outer layer of a star) and of optical scattering thickness < 1. The photons are multiply scattered by the electrons, which are in turn "dragged" by the photons. Since the electrons transfer momentum to the nuclei by collision or via the electric field, the entire layer is accelerated as a whole.

An electron situated in the beam of the radiation that it scatters (in which $\hbar\omega \ll mc^2$) experiences an effective force

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = 2\int \mathbf{\Delta}\frac{d^3k}{(2\pi)^3} N_\mathbf{k}(1+N_{\mathbf{k}'})\, dW_{\mathbf{kk}'}^{\mathbf{pp}'}, \qquad (1)$$

where

$$\mathbf{\Delta} = \hbar(\mathbf{k}-\mathbf{k}'); \quad dW_{\mathbf{kk}'}^{\mathbf{pp}'} = [c - v\cos(\mathbf{k},\mathbf{v})]\, d\Sigma_{\mathbf{kk}'}^{\mathbf{pp}'};$$

$d\Sigma_{\mathbf{kk}'}^{\mathbf{pp}'}$—differential scattering cross section.

We express the quantities $\mathbf{k}$ and Δ in terms of their values $\mathbf{k}_0$ and $\mathbf{\Delta}_0$ in the electron rest system, using the invariance of $d\Sigma$:

$$\frac{d\mathbf{p}}{dt} = 2\int \frac{cd^3k_0}{\gamma(2\pi)^3}\left\{\gamma\mathbf{\Delta}_0 + \frac{\gamma-1}{v^2}[[\mathbf{\Delta}_0\mathbf{v}]\,\mathbf{v}]\right\} N_{\mathbf{k}_0}(1+N_{\mathbf{k}_0'})\, d\Sigma_0$$
$$= 2\int \frac{cd^3k_0}{\gamma(2\pi)^3}\left\{\gamma\mathbf{\Delta}_0 + \frac{\gamma-1}{v^2}[[\mathbf{\Delta}_0\mathbf{v}]\,\mathbf{v}]\right\} N_{\mathbf{k}_0}\, d\Sigma_0;$$
$$\gamma = (1 - v^2/c^2)^{-1/2}. \qquad (2)^*$$

The integral of the second term in the curly brackets vanishes, since

$$d\Sigma_0 = \frac{r_0^2}{2}[1+\cos^2(\mathbf{k}_0\mathbf{k}_0')]\, d\Omega_0'.$$

The remaining expression takes the form

$$2\int \mathbf{\Delta}_0 N_{\mathbf{k}_0}\, d\Sigma_0 \frac{cd^3k_0}{(2\pi)^3} = \frac{8\pi r_0^2 \mathbf{S}_0}{3c},$$

where $\mathbf{S}_0$—density of radiation flux in the electron rest system, and therefore

$$\mathbf{F} = \mathbf{F}_0 + \left(1 - \frac{1}{\gamma}\right)\left[[\mathbf{F}_0\mathbf{v}]\frac{\mathbf{v}}{v^2}\right],$$

where $\mathbf{F}_0 = 8\pi r_0^2 \mathbf{S}_0/3c$—effective force in the electron rest system.

After the electrons have become noticeably accelerated, their velocity in the direction of the radiation flux exceeds greatly the transverse components, so that we can put $\mathbf{v} \parallel \mathbf{S}_0$; then

$$F = F_0; \qquad S_0 = \frac{1+\beta^2}{1-\beta^2}S + \frac{\beta c}{1-\beta^2}(T_{11} - T_{00}),$$

where T_{11}—component of the stress tensor in the flux direction, T_{00}—energy density; $\beta = v/c$.

2. Assume that a spherically symmetrical radiation flux enters from the outside into a spherical layer of plasma with inside radius R. Then the point P at a distance r > R receives not all the radiation, but only photons having wave vectors k in the solid angle subtended by the star at the point P. Their maximum angle with the radius vector P is determined by the equation

$$x_m = \cos\vartheta_m = (1 - R^2/r^2)^{1/2}.$$

For a spherically symmetrical distribution N_k, the calculations yield

$$S = J(1 - x_m^2); \qquad T_{00} = \frac{2J}{c}(1 - x_m);$$

$$T_{11} = -\frac{2J}{3c}(1 - x_m^3),$$

where J—radiation flux density at r = R. Therefore

*$[\Delta_0\nu] = \Delta_0 \times \nu$.

$$\frac{dp}{dt}=\frac{16\pi r_0^2 J}{3c(1-\beta^2)}\left[\frac{1-x_m^2}{2}(1+\beta^2)-\beta\left(\frac{4}{3}-x_m-\frac{x_m^3}{3}\right)\right]. \quad (3)$$

An electron displacement δ gives rise to an electric field $E \approx 4\pi ne\delta$, which increases the force F even when $\delta \ll R$ and $\beta < 1$. Indeed, in this case $F = 8\pi r_0^2 J/3c$, meaning that $eE = F$ for $\delta/R \approx r_0^2 J/ne^2cR \ll 1$, at all realizable values of J, n, and R. This field accelerates the nuclei and it can be assumed that the force ZF (Z = atomic number) is applied to the assembly with total mass $M + Zm \approx M$.

Equation (3) is easy to solve in the limiting cases of small and large particle displacements Δr:

a) $\Delta r = r - R \ll R$; we can assume here that $\beta \ll 1$ and, in accordance with (3)

$$M\frac{dv}{dt}=\frac{8\pi r_0^2 ZJ}{3c}\left(1-\frac{8}{3}\beta\right),$$

hence

$${}^8/_3\beta = 1-\exp(-t/t_0).$$

The characteristic time is $t_0 = 3Mc^2/8\pi r_0^2 J$; for $J \sim 10^{21}$ erg/cm^2 sec we have $t_0 \approx 1$ sec; for $J \sim 10^{16}$ erg/cm^2 sec [2] we get $t_0 \sim 10^5$ sec.

b) $R^2/r^2 \ll 1$; in this case, multiplying (3) by v, we get

$$\frac{d\varepsilon}{dt}=\frac{8\pi Zr_0^2 JR^2\beta(1-\beta)}{3r^2(1+\beta)}=\frac{8\pi Zr_0^2 JR^2}{3c(1+\beta)^2}\left(\frac{Mc^2}{\varepsilon}\right)^2\frac{d}{dt}\frac{1}{r(t)}$$

In the extremely relativistic case ($\epsilon \gg Mc^2$) we can put $1 + \beta = 2$ and for $r(t) \gg r(0) \sim R$ we obtain the largest attainable particle energy:

$$\varepsilon = Mc^2\left(\frac{2\pi Zr_0^2 RJ}{Mc^3}\right)^{1/3}. \quad (4)$$

In the opposite limiting case ($\epsilon - \epsilon_0 \ll Mc^2$) we have

$$\varepsilon-\varepsilon_0 = 8\pi ZRJr_0^2/3c.$$

3. The foregoing acceleration processes can be realized, for example, during explosions of supernovas of type II on the surface of an expanding shell. Imshennik and Nadezhin [3] have shown that at the instant when the shock wave emerges to the surface of the star, the temperature of the surface is 500,000°K, and consequently the radiation flux density is $J \sim 10^{19}$ erg/cm^2 sec. Within approximately one hour, a star expanding with a velocity $\sim 10^4$ km/sec increases in radius from an initial $R_0 \sim 10$—$20\ R_\odot$ to $R \sim 4\times 10^{12}$ cm, i.e., by a factor of several times. For $J \sim 10^{19}$ erg/cm^2 sec the characteristic time is $t_0 \sim 10^2$ sec so that at the instant of emergence of the wave we can use the formula (4), which yields $\epsilon_{kin} \equiv \epsilon - Mc^2 \sim Mc^2$. The condition that the optical thickness of the accelerated layer be small yields for the number of particles in the layer $N \approx 4\pi R^2/\Sigma \sim 10^{50}$ for $R \sim 10^{12}$ cm and for $\Sigma \sim 10^{-24}$ cm^2.

The fluctuations of the radiation flux lead to an energy distribution of the particles and increase the upper limit of attainable energies. In the presence of a magnetic field, the accelerated particles remain in the region surrounding the star. This process can be the mechanism for the injection of fast electrons and nuclei.

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[3] V. S. Imshennik and D. K. Nadezhin, Astr. zh. 41, 829 (1964), Soviet Astronomy 8, in press.

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