ELECTROMAGNETIC RADIATION FROM A NEUTRON IN AN EXTERNAL MAGNETIC FIELD

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Submitted to JETP editor September 12, 1964

J. Exptl. Theoret. Phys. (U.S.S.R.) 48, 921-927 (March, 1965)

It is shown that a neutron moving in a magnetic field can radiate electromagnetic waves. The properties of the radiation are investigated. The problem of orientation of the neutron spin as a result of the radiation is investigated.

WE consider in this paper the electromagnetic radiation emitted by a neutron moving in a constant and homogeneous magnetic field. The possibility of this radiation is connected with the fact that the neutron has an anomalous magnetic moment

$$\mu_n = -\mu = -1.93\ \mu_{\text{nuc}}$$

where $\mu_{\text{nuc}} = e_0\hbar/2Mc$—Bohr nuclear magneton.[1]

For a neutron situated in a magnetic field, the presence of a magnetic moment is manifest, in particular, in the fact that the neutron energy becomes dependent on the orientation of the magnetic moment relative to the field direction. The two possible orientations of the neutron magnetic moment have different characters of stability with respect to the additional interaction between the neutron and the electromagnetic vacuum. Consequently, spontaneous transitions of the neutron between states of different magnetic-moment orientation become possible. Such transitions are accompanied by photon emission and by a simultaneous change in the orientation of the neutron magnetic moment, which becomes oriented in the direction possessing the greatest stability.

Let us assume that the motion of the neutron in an external electromagnetic field (H, E) can be described by a Dirac equation

$$i\hbar\partial\psi / \partial t = \hat{\mathcal{H}}\psi \qquad (1)$$

with a Hamiltonian containing the additional interaction due to the anomalous magnetic and electric moments:

$$\hat{\mathcal{H}} = c(\boldsymbol{\alpha}\mathbf{p}) + \rho_3 mc^2 + \mu\{\rho_3(\boldsymbol{\sigma}\mathbf{H}) + \rho_2(\boldsymbol{\sigma}\mathbf{E})\}. \qquad (2)$$

Here m is the neutron mass and **E** and H the external electric and magnetic fields, with **E** ≡ 0 in our problem. The magnetic field **H** is assumed to be constant in time, homogeneous, and directed along the z axis of a rectangular coordinate system. Inasmuch as the momentum operator $\mathbf{p} = -i\hbar\nabla$ commutes with the Hamiltonian (2), the solutions of (1) can be chosen in the form of wave functions that are eigenfunctions for the momentum operator p.

Further, as can be readily verified, the operator

$$\Pi_{12} = mc^2\sigma_3 + c\rho_2[\boldsymbol{\sigma}\mathbf{p}]_3 + \mu H \qquad (3)^*$$

(the polarization tensor[2]) also commutes with the Hamiltonian, and can therefore be used to determine the spin states of the neutron. As is well known, this operator describes the projection of the neutron spin on the direction of the magnetic field, and it is important to note that the operator of "longitudinal polarization" $\boldsymbol{\sigma}\cdot\mathbf{p}$ in our problem does not commute with the Hamiltonian and is not an integral of the motion.

Thus, subjecting the wave function to the two equations

$$\mathbf{p}\psi = \hbar\mathbf{k}\psi, \qquad (4)$$

$$\Pi_{12}\psi = c\hbar\lambda\zeta\psi, \qquad (5)$$

we obtain a stationary solution in the form of a plane wave

$$\psi = L^{-3/2}\hat{b}\exp\{-icKt + i\mathbf{kr}\}, \qquad (6)$$

where the spinor $\hat{b}$ has the following form

$$\hat{b} = \begin{pmatrix} C_1 \\ C_2 e^{i\varphi} \\ C_3 \\ C_4 e^{i\varphi} \end{pmatrix}, \qquad (7)$$

*$[\boldsymbol{\sigma}\mathbf{p}] = \boldsymbol{\sigma}\times\mathbf{p}$.

$$C_1=\frac{1}{2}\left(\frac{1}{2}\left(1+\zeta\frac{k_0}{S}\right)\right)^{1/2}\left(\left(1+\frac{k_3}{K}\right)^{1/2}+\zeta\left(1-\frac{k_3}{K}\right)^{1/2}\right),$$

$$C_2=-\frac{\zeta}{2}\left(\frac{1}{2}\left(1-\zeta\frac{k_0}{S}\right)\right)^{1/2}\left(\left(1+\frac{k_3}{K}\right)^{1/2}-\zeta\left(1-\frac{k_3}{K}\right)^{1/2}\right),$$

$$C_3=\frac{1}{2}\left(\frac{1}{2}\left(1+\zeta\frac{k_0}{S}\right)\right)^{1/2}\left(\left(1+\frac{k_3}{K}\right)^{1/2}-\zeta\left(1-\frac{k_3}{K}\right)^{1/2}\right),$$

$$C_4=\frac{\zeta}{2}\left(\frac{1}{2}\left(1-\zeta\frac{k_0}{S}\right)\right)^{1/2}\left(\left(1+\frac{k_3}{K}\right)^{1/2}+\zeta\left(1-\frac{k_3}{K}\right)^{1/2}\right). \quad (7')$$

Here

$$\lambda=(K^2-k_3^2)^{1/2},\quad k_\varphi=(k_1^2+k_2^2)^{1/2};$$

The neutron energy is

$$E=c\hbar K=c\hbar\left(k_3^2+\left(S+\zeta\frac{\mu H}{c\hbar}\right)^2\right)^{1/2},\qquad k_0=\frac{mc}{\hbar},$$

$$S=(k_0^2+k_\varphi^2)^{1/2},\quad \tan\varphi=\frac{k_2}{k_1},$$

and $\zeta=\pm 1$ characterizes the direction of the neutron spin relative to the direction of the magnetic field: $\zeta = 1$ along the field and $\zeta=-1$ against the field.

The interaction of the neutron with the quantized radiation field $\mathbf{A}^\Phi$ will obviously have, in accordance with (2), the form

$$U_{\text{int}}=\mu\left\{\rho_3(\sigma\,\text{rot}\,\mathbf{A}^\Phi)-\frac{1}{c}\rho_2\left(\sigma\frac{\partial\mathbf{A}^\Phi}{\partial t}\right)\right\}, \qquad (8)^*$$

since the transverse electric and magnetic field of the radiation are connected with $\mathbf{A}^\Phi$ by the relations

$$\mathbf{H}^\Phi=\text{rot}\,\mathbf{A}^\Phi,\quad \mathbf{E}^\Phi=-\frac{1}{c}\frac{\partial\mathbf{A}^\Phi}{\partial t}.$$

By expanding $\mathbf{A}^\Phi$ in a Fourier series[3] we readily obtain the energy of interaction between the neutron and the radiation field in the form

$$U_{\text{int}}=U+U^+, \qquad (9)$$

where

$$U=\frac{\mu}{L^{3/2}}\sum_{\varkappa}\varkappa\left(\frac{2\pi c\hbar}{\varkappa}\right)^{1/2}\exp(-ic\varkappa t+i\boldsymbol{\varkappa}\mathbf{r})\,(\mathbf{Ba}). \qquad (10)$$

with

$$\mathbf{B}=i\{\rho_3[\sigma\boldsymbol{\varkappa}_0]+\rho_2\sigma\}, \qquad (11)$$

$\boldsymbol{\kappa}_0=\boldsymbol{\kappa}/\kappa$—unit vector in the photon propagation direction.

The radiation probability

$$W=\frac{\mu^2}{2\pi\hbar}\int\delta(K-K'-\varkappa)\,|\langle\mathbf{B}\rangle|^2\varkappa d^3\varkappa \qquad (12)$$

*rot = curl.

is connected with the matrix elements $\langle\mathbf{B}\rangle$ defined by the expression

$$\langle\mathbf{B}\rangle=\int\psi'^+e^{-i\boldsymbol{\varkappa}\mathbf{r}}\,\mathbf{B}\psi d^3x. \qquad (13)$$

This integral must be calculated only accurate to terms that are linear in μ, since the initial equation (2) takes into account the magnetic anomalous moment of the neutron only in the approximation linear in μ. In this approximation, the matrix elements $\langle\mathbf{B}\rangle$ have a simple form

$$\langle B_x\rangle=\left[\cos\varphi-i\sin\varphi\left(\frac{1-\beta^2}{1-\beta^2\cos^2\theta}\right)^{1/2}\right]\times(\beta\cos\theta-\cos\theta')\,\delta(\mathbf{k}-\mathbf{k}'-\boldsymbol{\varkappa}),$$

$$\langle B_y\rangle=\left[\sin\varphi+i\cos\varphi\left(\frac{1-\beta^2}{1-\beta^2\cos^2\theta}\right)^{1/2}\right]\times(\beta\cos\theta-\cos\theta')\,\delta(\mathbf{k}-\mathbf{k}'-\boldsymbol{\varkappa}),$$

$$\langle B_z\rangle=\left\{\left[\cos(\varphi-\varphi')-i\sin(\varphi-\varphi')\left(\frac{1-\beta^2}{1-\beta^2\cos^2\theta}\right)^{1/2}\right]\times\sin\theta'-\beta\sin\theta\right\}\delta(\mathbf{k}-\mathbf{k}'-\boldsymbol{\varkappa}). \qquad (14)$$

We have introduced here for the vectors of neutron momentum $\mathbf{k}$ and photon momentum $\boldsymbol{\kappa}$ the spherical system of coordinates

$$\mathbf{k}=\{k\sin\theta\cos\varphi,\quad k\sin\theta\sin\varphi,\quad k\cos\theta\},$$

$$\boldsymbol{\varkappa}=\{\varkappa\sin\theta'\cos\varphi',\quad \varkappa\sin\theta'\sin\varphi',\quad \varkappa\cos\theta'\};$$

$$1-\beta^2=(mc^2/E)^2=(k_0/K)^2. \qquad (15)$$

From the energy conservation law

$$K-K'-\varkappa=0 \qquad (16)$$

and from the momentum conservation law

$$\mathbf{k}-\mathbf{k}'-\boldsymbol{\varkappa}=0 \qquad (17)$$

we find that in the approximation linear in μ the frequency of the radiated phonons is

$$\varkappa=\begin{cases}\dfrac{2\mu H}{c\hbar}\dfrac{(1-\beta^2\cos^2\theta)^{1/2}}{1-\beta\cos\Omega}, & \text{if } \zeta=-\zeta'=1\\ 0 \text{ otherwise.}\end{cases} \qquad (18)$$

Here Ω—angle between the vectors $\boldsymbol{\kappa}$ and $\mathbf{k}$,

$$\cos\Omega=\sin\theta\sin\theta'\cos(\varphi-\varphi')+\cos\theta\cos\theta'.$$

It follows from this formula that the radiation is monochromatic for each fixed direction relative to the neutron velocity (Ω = const), but for different directions (different angles Ω) the radiation frequency will be different. We note further that formula (18) establishes the following selection rule: only a neutron whose spin is directed along the direction of the magnetic field radiates. The radiation is always accompanied by a change in spin orientation.

Thus, spontaneous polarization of the neutron beam is possible when the initially unpolarized neutrons radiate electromagnetic waves. After some time interval, the neutron spin acquires a preferred orientation opposite that of the magnetic field.

To investigate the polarization properties of the neutron radiation we make use of a method described earlier [4]. We then obtain for the intensity of radiation of components with specified linear polarization

$$W_i = \frac{8}{\pi}\frac{\mu^6 H^4}{c^3\hbar^4}\int \frac{(1-\beta^2\cos^2\theta)^2}{(1-\beta\cos\Omega)^5} S_i \sin\theta' \, d\theta' \, d\varphi', \quad (19)$$

where S_i takes for two components of linear polarization of radiation the form

$$S_2 = \frac{[1-\beta^2+\beta^2\sin^2\theta\sin^2(\varphi-\varphi')](\beta\cos\theta-\cos\theta')^2}{1-\beta^2\cos^2\theta}, \quad (20)$$

$$S_3 = (1-\beta\cos\Omega)^2 - \frac{\beta^2\sin^2\theta(\beta\cos\theta-\cos\theta')^2\sin^2(\varphi-\varphi')}{1-\beta^2\cos^2\theta}. \quad (21)$$

For the right-hand ($l = 1$) and left-hand ($l = -1$) circular polarizations we have, respectively,

$$S_l = \frac{1}{2}(S_2+S_3) - lN, \quad (22)$$

$$N = \left(\frac{1-\beta^2}{1-\beta^2\cos^2\theta}\right)^{1/2}(\beta\cos\theta-\cos\theta')(1-\beta\cos\Omega). \quad (23)$$

We note that the dependence on the angle φ is contained in all the formulas given above for the angular distribution of the radiation intensity only in the form of the difference $\varphi - \varphi'$, thus evidencing the symmetry of radiation relative to the z axis—the selected direction of the magnetic field. It is interesting to note further that the radiation of a rapidly moving neutron is concentrated in a narrow cone around the instantaneous direction of its momentum.

After integrating over the angles we obtain the following expressions for the total radiation intensity:

$$W_2 = \frac{W_0}{16}\left(\frac{H}{H_0}\right)^4\left\{5\left(\frac{1-\beta^2\cos^2\theta}{1-\beta^2}\right)^2 - 1\right\}, \quad (24)$$

$$W_3 = \frac{W_0}{16}\left(\frac{H}{H_0}\right)^4\left\{11\left(\frac{1-\beta^2\cos^2\theta}{1-\beta^2}\right)^2 + 1\right\}, \quad (25)$$

$$W_l = \frac{W_0}{2}\left(\frac{H}{H_0}\right)^4\left(\frac{1-\beta^2\cos^2\theta}{1-\beta^2}\right)^2 \times\left\{1+\frac{1}{2}l\beta\cos\theta\left(\frac{1-\beta^2}{1-\beta^2\cos^2\theta}\right)^{1/2}\right\}, \quad (26)$$

with

$$W = W_2 + W_3 = W_0\left(\frac{H}{H_0}\right)^4\left(\frac{1-\beta^2\cos^2\theta}{1-\beta^2}\right)^2. \quad (27)$$

We have put here

$$W_0 = \frac{128}{9}c^3\hbar^2/\mu^2 = 8.58\cdot 10^{30}\,\text{MeV/sec},$$
$$H_0 = (\hbar c)^{3/2}/\mu^2 = 1.91\cdot 10^{21}\ \text{Oe}. \quad (28)$$

Defining the degree of polarization with the aid of the relation

$$p = (W_\alpha - W_\beta)/W, \quad W = W_\alpha + W_\beta, \quad (29)$$

we can easily find that the degree of linear polarization

$$p_1 = \frac{3}{8}\left[1+\frac{1}{3}\left(\frac{1-\beta^2}{1-\beta^2\cos^2\theta}\right)^2\right] \quad (30)$$

lies in the region of values $3/8 \le p \le 1/2$, that is, the radiation of the neutron is noticeably polarized. In exactly the same way, we have for circular polarization

$$p_{\text{circ}} = \frac{1}{2}l\beta\cos\theta\left(\frac{1-\beta^2}{1-\beta^2\cos^2\theta}\right)^{1/2}, \quad (31)$$

that is,

$$|p_{\text{circ}}| \leqslant \frac{1}{2}.$$

We emphasize that the radiation of a neutron at rest differs from zero ($\beta = 0$)

$$W = W_0\left(\frac{H}{H_0}\right)^4 \quad (32)$$

and coincides with the radiation of a neutron moving along the field ($\theta = 0$). On the other hand, the radiation of a neutron moving perpendicular to the field H increases with energy like E^4:

$$W_\perp = W_0\left(\frac{H}{H_0}\right)^4\left(\frac{E}{mc^2}\right)^4. \quad (33)$$

To calculate the radiation probability we can use formula (19) with allowance for the fact that this expression must be divided by the energy of the photon $c\hbar\kappa$. We then obtain

$$W = \frac{4}{\pi}\frac{\mu^5 H^3}{c^3\hbar^4}\int\frac{(1-\beta^2\cos^2\theta)^{3/2}}{(1-\beta\cos\Omega)^4}(S_2+S_3)\sin\theta'\,d\theta'\,d\varphi' = \frac{1}{T_0}\left(\frac{H}{H_0}\right)^3\frac{(1-\beta^2\cos^2\theta)^{3/2}}{1-\beta^2}, \quad (34)$$

where

$$T_0 = 3\mu/64\sqrt{c^3\hbar} = 2{,}68\cdot 10^{-27}\ \text{sec}.$$

We see therefore that the probability of radiation from a neutron moving along the field ($\theta = 0$) decreases in inverse proportion to the first power of the energy:

$$W_{\parallel} = \frac{1}{T_0}\left(\frac{H}{H_0}\right)^3 \frac{mc^2}{E};$$

if the neutron moves perpendicular to the field H, the radiation probability increases in proportion to the square of the energy:

$$W_{\perp} = \frac{1}{T_0}\left(\frac{H}{H_0}\right)^3 \left(\frac{E}{mc^2}\right)^2.$$

For a numerical estimate of the results it is necessary to compare the probability of emission of a neutron with the corresponding expression for the probability of spontaneous β decay. As is well known[5], in the case of β decay of a neutron at rest the lifetime is $T_\beta \approx 12$ min. It is obvious that the probability of decay of the moving neutron is

$$W_\beta = mc^2 / T_\beta E. \quad (35)$$

Taking the most favorable case of motion perpendicular to the field, we obtain

$$\frac{W_\perp}{W_\beta} = \frac{T_\beta}{T_0}\left(\frac{H}{H_0}\right)^3 \left(\frac{E}{mc^2}\right)^3. \quad (36)$$

It follows therefore that this ratio becomes of the order of unity if the energy of the neutron and the magnitude of the field are connected by the relation

$$\frac{E}{mc^2} \sim 1.6 \cdot 10^{-10} \frac{H_0}{H}. \quad (37)$$

Inasmuch as $H_0 \sim 10^{21}$ Oe, we see that the processes connected with the radiation of electromagnetic waves by neutrons can be of practical interest only in the case of large particle energies or at high magnetic field intensities.

We note that the shortest wavelength radiated by the neutron is, in accordance with (18)

$$\lambda_{min} = \frac{64\pi}{3} \frac{cT_0}{1+\beta} \frac{H_0}{H}\left(\frac{mc^2}{E}\right)^2; \quad (38)$$

in the case of relativistic motion of the neutron ($\beta \sim 1$)

$$\lambda_{min} \approx 3.39 \cdot 10^{-15} \frac{H_0}{H}\left(\frac{mc^2}{E}\right)^2 \text{cm} \quad (39)$$

and can be small at high energies.

In conclusion let us discuss the behavior of the spin of a longitudinally-polarized neutron in a magnetic field. It is easy to verify that the operator of longitudinal polarization $(\boldsymbol{\sigma} \cdot \mathbf{p})$ does not commute with the Hamiltonian (2) and is not an integral of the motion. We construct the function $\Psi(t)$ such that at the initial instant it is an eigenfunction for the operator $(\boldsymbol{\sigma} \cdot \mathbf{p})$:

$$(\boldsymbol{\sigma}\mathbf{p})\Psi(0) = \hbar\lambda_0\Psi(0), \quad (40)$$

$$\Psi(t) = A\psi_1 + B\psi_{-1}, \quad (41)$$

where the subscripts ±1 denote the corresponding values of $\zeta = \pm 1$. From the condition (40) we can easily obtain AB in the approximation linear in μ, and then determine from the function (41) the mean value $\langle(\boldsymbol{\sigma} \cdot \mathbf{p})\rangle = \hbar\lambda_0 f(t)$. Here

$$\lambda_0 = \zeta' k, \quad \zeta' = \pm 1, \quad (42)$$

$$f(t) = \frac{(1-\beta^2)\cos^2\theta + \sin^2\theta\cos\omega t}{1 - \beta^2\cos^2\theta}, \quad (43)$$

where we put

$$\omega = \omega_0(1 - \beta^2\cos^2\theta)^{1/2}, \quad \omega_0 = \frac{3}{64}\frac{1}{T_0}\frac{H}{H_0}. \quad (44)$$

Thus, when a neutron moves perpendicular to the field, the longitudinal polarization precesses with a period

$$T = \frac{2\pi}{\omega} = \frac{128\pi}{3} T_0 \frac{H_0}{H}, \quad (45)$$

which coincides with the behavior of a longitudinally-polarized electron in a magnetic field[6]. When the neutron moves along the field ($\theta = 0$), the longitudinal polarization is conserved, coinciding with the transverse polarization. For fields $H \sim 10^4$ Oe, the precession period is $T \sim 7 \times 10^{-8}$ sec. Apparently, by passing a beam of longitudinally-polarized slow neutrons through a magnetic field, is is possible to observe experimentally the change in the spin orientation.

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Translated by J. G. Adashko
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