# NON-NEWTONIAN GRAVITATIONAL FIELDS

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The solutions of Einstein's field equations are of two types: those which in nonrelativistic approximation (in the limit $c \to \infty$) satisfy the field equations of Newton's theory of gravitation (Newtonian gravitational fields), and those for which this condition is not fulfilled (non-Newtonian gravitational fields). In the nonrelativistic approximation of Einstein's theory the two types can be designated appropriately as irrotational and rotational gravitational fields, respectively. The theory of irrotational gravitational fields is identical with Newton's gravitational theory. A rotational gravitational field is characterized by an ordinary Newtonian potential satisfying Poisson's equation and by a vector potential satisfying Laplace's equation. In a rotational gravitational field inertial systems of reference in the sense of Newtonian mechanics do not exist. The law of motion of a freely falling particle in a rotational gravitational field is Eq. (5.5), where $\Phi$ and $\mathbf{a}$ are the Newtonian and vector potentials and $\psi$ is given by (3.15).

## 1. INTRODUCTION

IT is well known that when $c \to \infty$ (by comparison with the velocities of material bodies) the mechanics of special relativity theory becomes identical with Newtonian mechanics, and relativistic inertial systems are transformed into Newtonian inertial systems. We can also consider more general reference systems characterized by a metric[1)]

$$ds^2 = c^2 dt^2 - \gamma_{rs} d\xi^r d\xi^s, \tag{1.1}$$

where the coefficients $\gamma_{ik}$ have nonvanishing finite limits when $c \to \infty$. Each point $\xi^i = \text{const}$ moves uniformly in a straight line with respect to an inertial system (this is the inertial motion of a free material particle); in the general case different points move in different directions at different velocities. Such reference systems can be defined with the aid of freely moving mass points. As $c \to \infty$ they become the corresponding nonstatic reference systems of Newtonian mechanics, defined similarly with the aid of freely moving mass points.

We have applied the same concept to the general theory of relativity, where there are no inertial systems in the sense of the special theory but there exist reference systems represented by (1.1). The world lines of points $\xi^i = \text{const}$ are geodesics, i.e., these points move like freely falling particles in a given gravitational field. Therefore the reference systems can be defined by means of freely falling mass points.

In the Newtonian gravitational theory reference systems can be constructed completely analogously with the aid of freely falling mass points. These systems clearly correspond to the aforementioned relativistic reference systems, the correspondence being of the same character as in special relativity. In the limit $c \to \infty$, on exactly the same physical basis as for special relativity, the relativistic $(\xi, t)$ systems become nonrelativistic $(\xi, t)$ systems. Moreover, Einstein's equations written for a $(\xi, t)$ system become the Newtonian equations of a gravitational field written for the corresponding nonrelativistic $(\xi, t)$ system,[1] subject to the condition that the components $R_{0ijk}$ of the curvature tensor approach zero as $c \to \infty$ in the $(\xi, t)$ system. If this additional condition is not fulfilled, then in the limit $c \to \infty$ Einstein's equations are not transformed into Newton's equations. In other words, if the condition

$$\lim_{c\to\infty} R_{0ijk} = 0 \tag{1.2}$$

is fulfilled in the $(\xi, t)$ systems, then in the limit $c \to \infty$ the gravitational field satisfies Newton's equations, and will be called a Newtonian gravitational field. If (1.2) is not fulfilled, then in the limit $c \to \infty$ the gravitational field does not satisfy Newton's equations and will be called a non-Newtonian gravitational field.

1)Latin indices have the values 1, 2, 3; Greek indices have the values 0, 1, 2, 3.

It is general knowledge that in both special and general relativity [2] the limiting transition $c \to \infty$ denotes a transition from the more exact relativistic theory to its nonrelativistic (Newtonian) approximation. The theory that is obtained in the limit includes the Newtonian gravitational theory as a special case, but is broader as a whole than the Newtonian theory.

## 2. FIELD EQUATIONS

In a coordinate system represented by (1.1) for $c \to \infty$ Einstein's equations assume the limiting form [1]

$$R_{00} = -4\pi G\rho, \quad R_{i0} = 0, \quad P_{ik} = 0. \tag{2.1}$$

Here [2)]

$$R_{00} = \partial^2 \ln \sqrt{\gamma} / \partial t^2 + \Gamma_r{}^s \Gamma_s{}^r, \tag{2.2}$$

$$R_{i0} = \Gamma_s{}^s{}_{,i} - \Gamma_i{}^s{}_{;s}, \tag{2.3}$$

$P_{ik}$ is the Ricci tensor of three-dimensional space, consisting of the limiting values

$$\underset{0}{\gamma}_{ik} = \lim_{c\to\infty} \gamma_{ik}; \tag{2.4}$$

G is the gravitational constant; $\rho$ is the density limit of matter giving rise to the gravitational field;

$$\Gamma_k{}^i = \underset{0}{\gamma}^{is}\Gamma_{ks}, \quad \Gamma_{ik} = {}^1/_2 \underset{0}{\gamma}_{ik,0},$$
$$\gamma = \det(\gamma_{ik}).$$

The covariant differentiation in (2.3) is performed with the aid of the coefficients $\gamma_{ik}$. Equations (2.1) comprise the basis of the nonrelativistic approximation of Einstein's gravitational theory which we shall consider here.

## 3. SOLUTION OF FIELD EQUATIONS

The general solution of the equation $P_{ik} = 0$ is

$$\underset{0}{\gamma}_{ik} = x^s{}_{,i} x^s{}_{,k}, \tag{3.1}$$

where $x^i(\xi^k, t)$ are three independent arbitrary functions. For any choice of the functions $x^i$, (3.1) satisfies $P_{ik} = 0$, as can be verified directly. On the other hand, if $\underset{0}{\gamma}_{ik}(\xi^j, t)$ is a solution of $P_{ik} = 0$, the three-dimensional space $S_\xi$ of the points $\xi^i = \text{const}$ with the metric tensor $\underset{0}{\gamma}_{ik}$ is flat. Consequently, there exists a system of Cartesian rectangular coordinates $x^i$ related to the coordinates $\xi^i$ by a transformation

$$x^i = x^i(\xi^k; t). \tag{3.2}$$

Since in the coordinate system $x^i$ the metric tensor components are $\delta_{ik}$, while in the coordinate system $\xi^i$ they are $\underset{0}{\gamma}_{ik}$, by a tensor transformation the coefficients $\gamma_{ik}$ are expressed in terms of (3.2) by means of (3.1).

It is evident from (3.1) that a specific choice of the functions $x^i(\xi^k, t)$ determines $\underset{0}{\gamma}_{ik}$ uniquely. However, two triplets $x^i(\xi^k, t)$ and $\bar{x}^i(\xi^k, t)$ determine the same solution for $\underset{0}{\gamma}_{ik}$ if, and only if, $x^i$ and $\bar{x}^i$ are related by the orthogonal transformation

$$\bar{x}^i = \alpha_s{}^i x^s + \alpha^i, \qquad \alpha_i{}^s \alpha_k{}^s = \delta_{ik} \tag{3.3}$$

having the time-dependent coefficients $\alpha_k^i(t)$ and $\alpha^i(t)$. Consequently, in the case of each solution for $\underset{0}{\gamma}_{ik}$ there exists an infinite set of Cartesian rectangular coordinate systems that are mutually transformable by means of (3.3); we shall make use of these systems.

Let $S_x$ denote the three-dimensional space of the points $x^i = \text{const}$ defined by one of the rectangular coordinate systems pertaining to a given solution for $\underset{0}{\gamma}_{ik}$. Equation (3.2) describes the motion of the space $S_\xi$ relative to the space $S_x$. The velocity of this motion is calculated from

$$v^i(x^k, t) = x^i{}_{,0}. \tag{3.4}$$

Taking $x^i{}_{,0k} = v^i{}_{,s} x^s{}_{,k}$ from (3.4), we obtain from (3.1):

$$\Gamma_{ik} = A_{rs} x^r{}_{,i} x^s{}_{,k}, \tag{3.5}$$

$$A_{ik} = {}^1/_2 (v^i{}_{,k} + v^k{}_{,i}). \tag{3.6}$$

Equation (3.5) shows that the three-dimensional tensor $\Gamma_{ik}$ has the components $A_{ik}$ in the $x^i, t$ coordinate system. According to (2.3), therefore, in the $x^i, t$ coordinate system the tensor equation $R_{i0} = 0$ assumes the form $A_{ss,i} - A_{is,s} = 0$, or

$$\omega_{is,s} = 0, \tag{3.7}$$

$$\omega_{ik} = {}^1/_2 (v^i{}_{,k} - v^k{}_{,i}). \tag{3.8}$$

Finally, remembering that

$$\partial \ln \sqrt{\gamma} / \partial t = \Gamma_s{}^s = A_{ss},$$

the equation $R_{00} = -4\pi G\rho$, which is invariant under the transformation (3.2), assumes the following form in the $x^i, t$ system:

$$A_{ss,0} + A_{ss,r} v^r + A_{rs} A_{rs} = -4\pi G\rho, \tag{3.9}$$

since

$$\partial \Gamma_s{}^s / \partial t = \partial A_{ss} / \partial t + (\partial A_{ss} / \partial x^r) x^r{}_{,0}.$$

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2) A comma preceding an index denotes ordinary differentiation; a semicolon denotes covariant differentiation.

Equation (3.7) can be written in the form $\omega_{i,k} - \omega_{k,i} = 0$, or

$$[\nabla \omega] = 0,$$

using the notation $\omega_{23} = \omega_1$, $\omega_{31} = \omega_2$, $\omega_{12} = \omega_3$, and $\omega = (\omega_1, \omega_2, \omega_3)$. It follows that

$$\omega = \nabla \psi, \tag{3.10}$$

where $\Psi(x^i, t)$ satisfies Laplace's equation

$$\Delta \psi = 0, \tag{3.11}$$

since, from (3.10),

$$\psi_{,ss} \equiv \omega_{1,1} + \omega_{2,2} + \omega_{3,3} \equiv \omega_{23,1} + \omega_{31,2} + \omega_{12,3},$$

which vanishes identically according to (3.8). For an arbitrary function $\Psi$, (3.10) and (3.11) comprise the general solution of (3.7).

Considering that, according to (3.8), $2\omega = -\nabla \times v$, we obtain an equation for $v$ from (3.10):

$$[\nabla v] = -2\nabla \psi. \tag{3.12}$$

This equation has the general solution

$$v = \nabla \varphi + [\nabla a], \tag{3.13}$$

where $\varphi(x^i, t)$ is a second arbitrary function, and the vector $a$ is determined from

$$\nabla(\nabla a) - \Delta a = -2\nabla \psi, \tag{3.14}$$

which is obtained by substituting (3.13) into (3.12). Equation (3.14) can be satisfied by assuming

$$\Delta a = 0, \qquad \nabla a = -2\psi. \tag{3.15}$$

Equation (3.11) is then satisfied automatically.

To solve (3.9) we take into consideration:

$$\Delta(\tfrac{1}{2}v^s v^s) - \omega_{rs}\omega_{rs} \equiv v^s v^s{}_{,rr} + A_{rs}A_{rs},$$

$$v^s{}_{,rr} = v^r{}_{,rs} = A_{rr,s},$$

$$A_{ss} = \Delta \varphi,$$

$$\omega_{rs}\omega_{rs} = 2\psi_{,s}\psi_{,s} = \Delta(\psi^2).$$

Then (3.9) can be written in the form of Poisson's equation:

$$\Delta \Phi = -4\pi G \rho, \tag{3.16}$$

$$\Phi = \varphi_{,0} + \tfrac{1}{2}v^s v^s - \psi^2. \tag{3.17}$$

The solution of (3.9) thus reduces to the determination of the ordinary Newtonian potential $\Phi$.

The solution for the complete system (2.1) now proceeds as follows. We begin by determining $a$ from the first equation of (3.15), and we then determine $\Psi$ from the second equation of (3.15), $\Phi$ from (3.16), and finally $\varphi$ from (3.17). We then have a specific form of $v^i = v^i(x^k, t)$ from (3.13), and can determine the functions $x^i(\xi^k, t)$ from (3.4); for example, with the initial condition $t = 0$ we have $x^i = \xi^i$. Substituting these functions into (3.1), we obtain the functions $\underset{0}{\gamma}_{ik}(\xi^j, t)$ satisfying all equations of (2.1).

## 4. ROTATIONAL AND IRROTATIONAL GRAVITATIONAL FIELDS

We have seen that one of the field equations, $P_{ik} = 0$, ensures the existence of Cartesian rectangular coordinate systems interrelated by the orthogonal transformations (3.3). Each of these coordinate systems is the basis of a three-dimensional space $S_x$ of points $x^i = \text{const}$, and the coordinate systems that are mutually transformable by means of orthogonal transformations with constant coefficients define a single identical space $S_x$. All spaces $S_x$ have the Euclidean metric and move arbitrarily relative to each other. Therefore, to treat the nonrelativistic approximation of Einstein's theory we can use reference systems associated with Euclidean space and absolute time $t$, as is the case for the Newtonian gravitational theory.

The gravitational field is characterized by the functions $\underset{0}{\gamma}_{ik}(\xi^j, t)$, which depend in turn on the functions $x^i(\xi^k, t)$ describing the motion of a space $S_\xi$ relative to one of the spaces $S_x$. We can therefore state that a gravitational field is characterized by a field of velocities $v^i(x^k, t)$ associated in a space $S_x$ with a given system of freely falling particles, since the points $\xi^i = \text{const}$ of the space $S_\xi$ move like freely falling particles. It follows that the properties of the field of velocities $v^i(x^k, t)$, which are independent of the coordinate system $x^i$ (or of the space $S_x$), are at the same time properties of the gravitational field.

If in some other space $\bar{S}_x$ the points $\xi^i = \text{const}$ induce a velocity field $\bar{v}^i(\bar{x}^k, t)$, then by virtue of (3.3) and (3.4) we have

$$\bar{v}^i = \alpha^i_s v^s + \dot{\alpha}^i_s x^s + \dot{\alpha}^i. \tag{4.1}$$

Differentiating with respect to $x^k$, using $\bar{x}^s{}_{,k} = \alpha^s_k$ in accordance with (3.3), and multiplying by $\alpha^j_k$, we obtain

$$\bar{v}^i{}_{,j} = \alpha_s{}^i \alpha_k{}^j v^s{}_{,k} + \dot{\alpha}_k{}^i \alpha_k{}^j,$$

whence it follows that

$$\bar{\omega}_{ik} = \alpha_s{}^i \alpha_r{}^k \omega_{sr} + \underset{0}{\bar{\omega}}_{ik}, \tag{4.2}$$

where

$$\underset{0}{\bar{\omega}}_{ik} = \tfrac{1}{2}(\dot{\alpha}^i_s \alpha^k_s - \dot{\alpha}^k_s \alpha^i_s) \tag{4.3}$$

is the angular velocity of space $S_x$ relative to $\bar{S}_x$.

It is seen from (4.2) and (4.3) that if $\omega_{ik}$

$= \omega_{ik}(t)$, i.e., $\psi_{,i} = \psi_{,i}(t)$ on the basis of (3.10), or

$$\psi_{,ik} = 0, \tag{4.4}$$

then $\bar{\omega}_{ik} = \bar{\omega}_{ik}(t)$, so that (4.4) is also fulfilled for the space $\bar{S}_x$. Therefore in all spaces $S_x$ we have simultaneously either $\psi_{,ik} = 0$ or $\psi_{,ik} \neq 0$. Equation (4.4), or the inequality $\psi_{,ik} \neq 0$, is therefore independent of the choice of space $S_x$ and can, in accordance with the foregoing discussion, be considered to characterize the gravitational field. We shall divide all gravitational fields on this basis into two classes.

If $\psi_{,ik} = 0$ we can choose a space $\bar{S}_x$ such that

$$\alpha_s{}^i \alpha_r{}^k \omega_{sr} + \underset{0}{\bar{\omega}}_{ik} = 0,$$

and at the same time $\bar{\omega}_{ik} = 0$. In some spaces $S_x$ the velocity field $v^i(x^k, t)$ is irrotational; vortices can be eliminated by the proper choice of $S_x$. Therefore the gravitational field is not characterized by the presence of vortices, and in the case $\psi_{,ik} = 0$ a gravitational field will be designated as irrotational.

If $\psi_{,ik} \neq 0$ there are no spaces $S_x$ for which $\omega_{ik} = 0$. The inequality $\omega_{ik} \neq 0$ now does not depend on the space $S_x$ and characterizes the gravitational field. Therefore in the case $\psi_{,ik} \neq 0$ the gravitational field will be designated as rotational.

As already mentioned in the Introduction, the Newtonian gravitational fields satisfy the limiting condition

$$\lim_{c\to\infty} R_{0ijk} \equiv \Gamma_{ik;\,j} - \Gamma_{ij;\,k} = 0. \tag{4.5}$$

In the nonrelativistic approximation the theory of these gravitational fields is identical with the Newtonian theory of gravitation. However, in a space $S_x$ this equation (4.5) has the form $\omega_{jk,i} = 0$, which is equivalent to (4.4). We thus see that Newtonian gravitational fields coincide in nonrelativistic approximation with the irrotational gravitational fields considered in our approximate theory, and that Newton's theory of gravitation is the theory of irrotational gravitational fields. The rotational gravitational fields are nonrelativistic approximations of non-Newtonian gravitational fields.

## 5. LAW OF FREE FALL AND INERTIAL REFERENCE SYSTEMS

The equations of geodesics written for a coordinate system described by (1.1),

$$c^2 \frac{d^2\xi^\nu}{ds^2} + c^2 \left\{ {\nu \atop \alpha\ \beta} \right\} \frac{d\xi^\alpha}{ds}\frac{d\xi^\beta}{ds} = 0 \qquad (\xi^0 = t),$$

become in the limit $c \to \infty$:

$$\ddot{\xi}^i + 2\Gamma_s{}^i \dot{\xi}^s + \Gamma_{rs}{}^i \dot{\xi}^r \dot{\xi}^s = 0. \tag{5.1}$$

We have here taken into account that, in accordance with (1.1), $ds/c \to dt$, and the limiting values of the Christoffel symbols were calculated from formulas given in [1]. In view of the equations

$$\Gamma_{ik,\,j} = x^s{}_{,ik} x^s{}_{,j}, \qquad \underset{0}{\gamma}^{rs} x^i{}_{,r} x^k{}_{,s} = \delta^{ik}$$

derived from (3.1), together with (3.5), we can write (5.1) as

$$x^i{}_{,s}\ddot{\xi}^s = -2A_{ir}x^r{}_{,s}\dot{\xi}^s - x^i{}_{,rs}\dot{\xi}^r\dot{\xi}^s. \tag{5.2}$$

Furthermore, using (3.4) we arrive at

$$x^i{}_{,s}\dot{\xi}^s = \dot{x}^i - v^i, \tag{5.3}$$

whence by differentiating with respect to t and using the equations $x^i{}_{,k0} = v^i{}_{,s} x^x{}_{,k}$, as well as (5.2), and (5.3), we obtain

$$\begin{aligned} \ddot{x}^i &= v^i{}_{,0} + v^i{}_{,s} v^s + 2\omega_{is}(\dot{x}^s - v^s) \\ &= v^i{}_{,0} + {}^1\!/_2 (v^s v^s)_{,i} + 2\omega_{is}\dot{x}^s \end{aligned} \tag{5.4}$$

or, by virtue of (3.17) and (3.13),

$$\ddot{\mathbf{r}} = \nabla(\Phi + \psi^2) + 2[\dot{\mathbf{r}}\nabla\psi] + \left[\nabla \frac{\partial \mathbf{a}}{\partial t}\right]. \tag{5.5}$$

This is the law of free fall in nonrelativistic approximation as written for a reference system with a Euclidean space $S_x$. It follows from (5.5) in conjunction with (3.15) that the gravitational force depends on a scalar potential $\Phi$ and a vector potential **a**, with the former satisfying Poisson's equation and the latter satisfying Laplace's equation.

Let us consider, for example, $\mathbf{a} = (0, 0, -2/r)$ $(r^2 = x^2 + y^2 + z^2)$. Then $\psi = -z/r^3$ is the potential of a dipole located at the origin. Since $\psi_{,ik} \neq 0$, the gravitational field here is rotational. We assume, furthermore, $\Phi = 0$ [in which case (3.17) has a solution for $\varphi$], so that the ordinary Newtonian attraction is not present. In this gravitational field the law of free fall (5.5) has the form

$$\ddot{\mathbf{r}} = \nabla(\psi^2) + 2[\dot{\mathbf{r}}\nabla\psi].$$

Here $\nabla\psi \sim r^{-3}$ and $\nabla(\psi^2) \sim r^{-5}$. Therefore in the vicinity of the origin the dominant force is $\nabla(\psi^2)$, while weak gyroscopic effects dominate at great distances from the center.

We have seen in Sec. 4 that in an irrotational gravitational field a set of spaces $S_x$ includes a more limited group of spaces (to be denoted by $S'_x$) for which $\omega_{ik} = 0$. Any space $S'_x$ is transformed into another by means of the orthogonal transformations (3.3); as a consequence of the condition $\omega_{ik} = \bar{\omega}_{ik} = 0$ we have, from (4.2), $\bar{\omega}_{ik} = 0$, or, from (4.3), $\alpha^i_s\alpha^k_s - \alpha^k_s\alpha^i_s = 0$, which from orthogonality can be written as $\alpha_s^i \alpha_s^k = 0$ or $\alpha_k^i = 0$, so that the $\alpha_k^i$ are constants. The spaces $S'_X$ do not rotate relative to each other. By a suitable orientation of the rectangular coordinate axes the considered transformations can be put into the simple form

$$\bar{x}^i = x^i + \alpha^i(t). \tag{5.6}$$

The equation $\omega_{ik} = 0$ is a required condition for deriving the velocities $v^i$ from a scalar potential. Therefore, for each space $S'_X$ we can assume $\mathbf{a} = 0$ in (3.13). The law of free fall (5.5) then becomes

$$\ddot{x}^i = \Phi_{,i}. \tag{5.7}$$

However, the function $\Phi$ cannot be identical for all spaces $S'_X$. It is easily seen that by virtue of (5.6) it follows from the equations

$$\ddot{\bar{x}}^i = \bar{\Phi}_{,i}, \qquad \ddot{x}^i = \Phi_{,i}$$

that

$$\bar{\Phi}_{,i} = \Phi_{,i} + \ddot{\alpha}^i. \tag{5.8}$$

From the set of possible potentials $\Phi$ a single potential can be determined using the limiting conditions at infinity; for example, in the case of an isolated material system $\Phi = 0$ is required at infinity. Then among the spaces $S'_X$ we distinguish a more limited class of spaces (denoted by $S''_X$) associated with this uniquely determined $\Phi$. Each space $S''_X$ is transformed into another by means of (5.6); now by virtue of $\bar{\Phi} = \Phi$ it follows from (5.8) that $\ddot{\alpha}^i = 0$ or $\alpha^i = \beta^i t + \beta$, where $\beta^i$ and $\beta$ are constants. By choosing a suitable origin of the rectangular coordinate system we obtain $\beta = 0$; the transformation then becomes simply

$$\bar{x}^i = x^i + \beta t. \tag{5.9}$$

This shows that the spaces $S''_X$ move uniformly in straight lines with respect to each other. In these spaces the law of free fall has the form (5.7) with a potential $\Phi$ that is invariant under the transformations (5.9).

We have thus presented a theory of irrotational gravitational fields in the customary form of the Newtonian gravitational theory. The spaces $S''_X$ are the inertial reference systems of Newtonian mechanics. The role of the conditions at infinity in determining these spaces must be emphasized.

In a rotational gravitational field, if the conditions at infinity are not taken into account a preferred subset of spaces $S'_X$ cannot be distinguished in the set of spaces $S_X$. On the basis of (3.10) we can write (4.2) as

$$\bar{\psi}_{,i} = \alpha_s{}^i \psi_{,s} + \underset{0}{\bar{\omega}}_i \quad \text{or} \quad \nabla\bar{\psi} = \nabla\psi + \underset{0}{\bar{\boldsymbol{\omega}}},$$

where $\underset{0}{\bar{\omega}}$ is the angular velocity of $S_X$ relative to $\bar{S}_X$. It therefore follows from $\nabla\bar{\psi} = \nabla\psi$ that $\underset{0}{\bar{\omega}} = 0$, i.e., $\nabla\psi$ has an identical value only in the spaces $S_X$ that do not rotate with respect to each other.

If we now determine $\nabla\psi$ uniquely by means of the limiting conditions at infinity, such as the requirement $\nabla\psi = 0$ at infinity (giving an irrotational field at infinity), then among the spaces $S_X$ we distinguish a class of spaces (denoted by $S'^*_X$) for which $\nabla\psi$ has this uniquely determined value. The spaces $S'^*_X$ are interrelated by the transformations (5.6). It is impossible to separate out from the set of $S'^*_X$ a still more limited class of spaces that would correspond to Newtonian inertial systems.

The form of the law of free fall (5.5) cannot be simplified for the spaces $S'^*_X$. The vector potential $\mathbf{a}$ is an essential characteristic of a rotational gravitational field. Coriolis force terms cannot be eliminated from (5.5) by any choice of the reference system.

## 6. CONCLUSION

We have seen that some solutions of Einstein's equations in nonrelativistic approximation (irrotational gravitational fields) satisfy the field equations of the Newtonian theory, while others (rotational gravitational fields) do not. It follows that, either Newton's theory of gravitation does not account for all gravitational phenomena in nature and therefore requires nonrelativistic corrections, or a large class of solutions of Einstein's equations cannot be used to describe real gravitational fields.

[1] H. Keres, JETP **46**, 1741 (1964), Soviet Phys. JETP **19**, 1174 (1964).

[2] G. C. McVittie, The General Theory of Relativity and Cosmology, Wiley, New York, 1956 (Russian transl., IIL, 1961).

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196