NUCLEAR FORM FACTORS IN MUON CAPTURE BY He^3

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From the experimental form factors obtained from the capture of pions by He^3 and the scattering of electrons by He^3 and H^3, we have calculated the probability of the reaction $\mu^- + He^3 \rightarrow H^3 + \nu$. We obtained the value $\Lambda_{He^3,\text{theor}} = 1515 \pm 55\ \text{sec}^{-1}$ for $g_A^\beta/g_V^\beta = -1.16$ and $g_P^\mu/g_A^\mu = 7$. From comparison of the calculated probability with the experimental result we have estimated the value of the pseudoscalar constant to be $g_P^\mu = (8 \pm 3)g_A^\mu$.

As has been remarked more than once,[1-4] the reaction

$$\mu^- + He^3 \rightarrow H^3 + \nu \tag{1}$$

is one of the most easily interpreted μ-capture processes. However, even in this case, when the simplest nuclei are involved in the process, there is some uncertainty in the calculations of the nuclear matrix element. The nuclear matrix element for reaction (1) can be expressed to a good approximation in terms of a single parameter—the mean-square nuclear radius corresponding to the distribution of nucleon centers. Attempts to estimate the mean-square radius from the binding energy of the mirror nuclei He^3 and H^3 have demonstrated the sensitivity of this type of evaluation to choice of the phenomenological nucleon-nucleon potential[1,4,5], as the result of which an appreciable uncertainty arises in the nuclear matrix element.

In the present paper we obtain improved values of the nuclear matrix element for reaction (1) from the experimental data on form factors obtained in experiments on capture of π^- mesons[6] by He^3 and on scattering of electrons[7] by He^3 and H^3.

The nuclear matrix element for reaction (1) can be expressed in terms of the form factors F_1 and F_2 introduced by Schiff.[8] We will write the wave function of the S state of the nucleus in the form

$$\psi = \Phi_0 U + \Phi_1 V_2 - \Phi_2 V_1,$$

where Φ_0, Φ_1, and Φ_2 are the spin-isospin functions and U, V_1, and V_2 are the spatial functions. Here it is assumed that the wave functions of the He^3 and H^3 nuclei are dominated by the $\Phi_0 U$ state, which is symmetrical in the spatial coordinates of the nucleons. The wave functions V_1 and V_2, as well as the functions Φ_1 and Φ_2, are antisymmetric with respect to interchange of one pair of nucleons and symmetric with respect to interchange of the other pair. The form factors F_1 and F_2 depend on the momentum transfer q and are expressed in terms of the wave functions in the following way:

$$F_1(q) = \langle U | e^{i\mathbf{qr}} | U \rangle, \qquad F_2(q) = -3 \langle U | e^{i\mathbf{qr}} | V_1 \rangle,$$

where **r** designates the radius vector of the nucleon.

Using the expression for the matrix element of reaction (1) taken from the work of Fujii and Primakoff,[2] we obtain as the result of calculations

$$|M^\mu(He^3 \rightarrow H^3)|^2 = (G_V{}^\mu)^2(F_1{}^2 - {}^8/_3 F_1 F_2) + [3(G_A{}^\mu)^2 + (G_P{}^\mu)^2 - 2G_P{}^\mu G_A{}^\mu] F_1{}^2,$$

where

$$G_V{}^\mu = g_V{}^\mu(1 + \nu/2m_p),$$
$$G_A{}^\mu = g_A{}^\mu - g_V{}^\mu(1 + \mu_p - \mu_n)\nu/2m_p;$$
$$G_P{}^\mu = [g_P{}^\mu - g_A{}^\mu - g_V{}^\mu(1 + \mu_p - \mu_n)]\nu/2m_p;$$
$$g_V{}^\mu = 0{,}97 g_V{}^\beta, \quad g_A{}^\mu = g_A{}^\beta, \quad g_P{}^\mu \approx 7 g_A{}^\mu;$$

ν is the neutrino momentum in process (1), m_p is the proton mass, μ_p and μ_n are the anomalous magnetic moments of the proton and neutron.

The same form factors F_1 and F_2 can also be used to describe the radiative capture of pions by He^3 and the scattering of electrons by He^3 and H^3 nuclei, for which experimental data have been obtained. Thus we have the possibility of determining the form factors F_1 and F_2 and then using them in a calculation of the matrix element for reaction (1). However, in the experiments in question there are no data on the form factors for the required value of momentum transfer. The measurements on electron scattering by He^3 and H^3 were made in the region $1\ F^{-2} \le q^2 \le 5\ F^{-2}$ and the momentum transfer in the radiative capture of pions by He^3 amounts to $q^2 = 0.47\ F^{-2}$, while in reaction (1) the value of q^2 is $0.27\ F^{-2}$. Therefore the experimental results must be extrapolated into the momentum-transfer region of interest to us.

In order to carry out the extrapolation it is necessary to know the explicit form of the form factors F_1 and F_2, which depends on the choice of wave functions. According to the analysis of the electron scattering experiments with He^3 and H^3, the experimental data are well described by two different assumptions as to the form of the single-particle wave function: the Gaussian and Irving functions. Here the form factors F_1 and F_2 have the form:

for the Gaussian function

$$F_1 = e^{-q^2r^2/6}, \qquad F_2 = {}^1/_2 (P/6)^{1/2} q^2 r^2 e^{-q^2r^2/6}$$

and for the Irving function

$$F_1 = (1 + q^2r^2/21)^{-7/2},$$

$$F_2 = (P/21)^{1/2} q^2 r^2 (1 + q^2r^2/21)^{-9/2}.$$

The values of mean-square radius of the nucleus are as follows:

$$r = 1.5^{+0.2}_{-0.1}\ \mathrm{F} \quad \text{for the Gaussian function,}$$

and

$$r = 1.7 \pm 0.1\ \mathrm{F} \quad \text{for the Irving function.}$$

The parameter P characterizes the weight of a state of mixed symmetry and, according to an estimate made by Schiff[8] and by one of us[9], is equal to 0.03.

Thus, in extrapolation of the experimental electron-scattering results for He^3 and H^3 to the point $q^2 = 0.27 F^{-2}$, we find that the form factor F_1^2, with allowance for the uncertainty associated with the choice of wave function, is given by

$$F_1{}^2(0.27) = 0.80^{+0.03}_{-0.05}, \qquad \text{(I)}$$

and the form factor F_2 is given by

$$F_2(0.27) = 0.023 \pm 0.005.$$

The Panofsky ratio in He^3 is expressed in terms of the Panofsky ratio in hydrogen P_H and the matrix element for radiative capture of a π^- meson by He^3, which is equal to the form factor F_1:

$$P_{He^3} = P_H K / F_1{}^2,$$

where K is a kinematic factor. Using the experimental value of the Panofsky ratio for He^3 obtained by Zaĭmidoroga et al.,[6] we obtain

$$F_1{}^2(0.47) = 0.75 \pm 0.06.$$

For the momentum transfer $q^2 = 0.47 F^{-2}$ corresponding to the radiative capture of a pion by He^3, the two different types of single-particle wave functions give the same value of mean-square radius within 2%, namely

$$r = 1.4 \pm 0.2\ \mathrm{F}.$$

The extrapolated value of the form factor F_1^2 for $q^2 = 0.27 F^{-2}$ in this case is

$$F_1{}^2(0.27) = 0.84 \pm 0.04. \qquad \text{(II)}$$

For the weighted mean value of the extrapolated results $F_1^2(I)$ and $F_1^2(II)$ we obtain the final value

$$F_1{}^2(0.27) = 0.82 \pm 0.03.$$

The partial probability for capture of muons by He^3 (reaction (1)), calculated on the basis of universal weak interaction theory with the values of the form factors F_1 and F_2 obtained, turns out to be

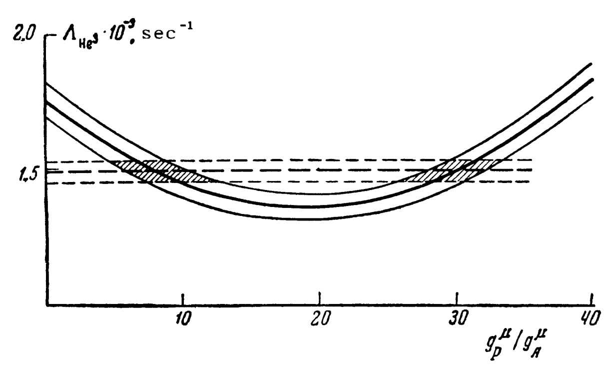
$$\Lambda_{He^3,\mathrm{theor}} = 1515 \pm 55\ \mathrm{sec}^{-1}.$$

(The error quoted reflects only the uncertainty in the nuclear form factors.) In calculating this probability we used the new value of the ratio $g_A^\beta / g_V^\beta = -1.16$[10] and $g_P^\mu / g_A^\mu = 7$.

The calculated value for the probability of reaction (1) is in good agreement with the weighted mean value of the results of three well known experiments[11-13] on muon capture by He^3:

$$\Lambda_{He^3,\mathrm{exp}} = 1490 \pm 40\ \mathrm{sec}^{-1}.$$

Within the framework of universal weak interaction theory the improved value of the nuclear matrix element for reaction (1), together with the experimental value of the probability for this reaction, permits us to estimate the poorly calculated pseudoscalar constant g_P^μ. The dependence of the probability for reaction (1) on the ratio of the constants g_P^μ / g_A^μ is shown in the figure. Also



shown are the existing uncertainties in the nuclear matrix element and in the experimental probability value. The smaller of the two possible values of the pseudoscalar constant is

$$g_P{}^{\mu} = (8 \pm 3) g_A{}^{\mu}.$$

This value of the pseudoscalar constant is in agreement with that calculated by Goldberger and Treiman[14]: $g_P^{\mu} \sim 7 g_A^{\mu}$.

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229