LANDAU DAMPING AND RESONANCE DAMPING OF MAGNETOPLASMA WAVES IN BISMUTH

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Measurements were made of the magnetic fields which define the region of strong damping of magnetoplasma waves due to the cyclotron resonance of carriers shifted by the Doppler effect. The Landau damping of magnetoplasma waves was observed and measurements were made of the values of the magnetic field and wave velocity at which this damping appeared. The experiments were carried out on single-crystal bismuth slabs, whose binary axis was parallel to the normal to the surface, at a frequency of 9.5 Gc and a temperature of 1.5°K. The following quantities were determined: the Fermi velocity of holes in the basal plane, $v_{Fh} = (2.35 \pm 0.1) \times 10^7$ cm/sec; the Fermi velocity of electrons in the basal plane, which was $v_{Fe} = (11.3 \pm 0.5) \times 10^7$ cm/sec along a binary axis; the effective mass of holes at the limiting point in the Fermi surface in the basal plane, $m_n^* = (0.220 \pm 0.002)m_0$. The results obtained are compared with some parameters of the Fermi surface.

RECENTLY, there have been several investigations of the cyclotron resonance at a frequency $\omega - \mathbf{k} \cdot \mathbf{v}$, shifted by the Doppler effect. The resonance shift occurs in the case of carriers moving at a velocity v in a metal in which an electromagnetic (or an acoustic) wave with a wave vector $\mathbf{k}$ is propagated. Miller and Hearing[1] considered theoretically the case of a metal with one type of carrier and an isotropic dispersion law for $\mathbf{H} \parallel \mathbf{k}$; they found that the value of the magnetic field H, defined by the relationship

$$\Omega = eH / m^*c = \omega - kv_F \qquad (1)$$

(v_F is the maximum Fermi velocity of carriers along the direction of the field **H**), represented the lower limit of the range of fields in which weakly damped waves—helicons—could be propagated in a metal. This approach is applicable to alkali metals, which were investigated experimentally by Taylor.[2] For an arbitrary angle $\measuredangle\, \mathbf{H}, \mathbf{k}$ in a metal with equal electron and hole densities and an anisotropic dispersion law (bismuth), the phenomenon is much more complex due to the presence of several groups of carriers in the metal and two modes of magnetoplasma waves.[3]

Kirsch[4] considered theoretically the possibility of an experimental investigation of the cyclotron resonance in bismuth, shifted by the Doppler effect, but the results of his actual experiments were limited basically to the observation of the resonance which was shifted most; thorough measurements were not carried out by Kirsch.

In the present study, we investigated: 1) the Doppler shift of the cyclotron resonance of electrons and holes in a magnetic field; 2) the position of the limit of the range of fields in which magnetoplasma waves experienced Landau damping. In both cases, the experiments involved the observation of the damping of two magnetoplasma wave modes, propagated in a bismuth single crystal in a magnetic field lying in the basal plane of the crystal. The Doppler-shifted cyclotron resonance was observed in the form of a strong resonance damping of the magnetoplasma waves due to the interaction of the waves with carriers whose Larmor frequency was equal to the Doppler frequency of the wave. Apart from this effect, we also observed the Landau damping caused by the interaction of a wave with carriers moving at a velocity equal to the wave velocity in a constant-phase field.

It follows formally from the relationship (3) given in [3] that the regions of existence of these two effects are the regions where the denominator of this expression vanishes:

$$n\Omega - \omega + k_H v_H = 0, \qquad (2)$$

where k_H is the projection of the wave vector, and v_H is the trajectory-average projection of the Fermi velocity onto the direction of the field **H**. For $n = 1$, we obtain the condition for the resonance damping of the waves in the first-order cyclotron resonance:

$$\Omega = \omega - k_H v_H. \quad (3)$$

For n = 0, we obtain the Landau damping condition:

$$\omega = k_H v_H. \quad (4)$$

A metal with a closed Fermi surface contains carriers having any value of v_H between the limits from 0 to $\pm v_F$, and this gives the limits of the regions of damping of the magnetoplasma waves.

The measurements of the cyclotron resonance shift made it possible to determine the Fermi velocities of electrons along directions close to a binary axis and the corresponding velocities of holes in the basal plane, and to obtain a more accurate value of the effective mass of holes at a limiting point in the Fermi surface in the basal plane. The Fermi velocity of electrons for a certain range of directions in the basal plane was obtained from measurements of the Landau damping limit.

EXPERIMENT

The measurements were carried out at a frequency of ≈ 9.4 Gc using two single-crystal disks of bismuth whose thicknesses were h = 1.00 mm and 0.47 mm. In both samples, the binary axis C_2 was parallel to within ≈ 1°, to the normal **N** to the flat surface of the disk (Fig. 1a). The same samples were used previously in the investigation of the cyclotron resonance.[5]

The excitation of standing magnetoplasma waves in a sample was observed by measuring the flow of power through a transmission strip resonator enclosing the sample, using a dc amplifier circuit. A microwave signal from a klystron passed through the resonator, was amplified by a traveling-wave tube and applied to a detector. The use of field modulation and a narrow-band amplifier with a synchronous detector gave an output voltage proportional to the quantity $R^{-3}\partial R/\partial H$, which was recorded by a two-coordinate automatic recorder as a function of H^{-1} (R was the active surface impedance of the sample). The strip resonator structure is shown schematically in Fig. 1b. During the measurements, the sample (Fig. 1a) was placed at a distance of ≈ 0.5 mm from a strip 4 covering the end of the cylindrical cavity of the resonator; the normal **N** was directed horizontally (during the experiments, the sample could be rotated about the normal). The polarization of the microwave current **J** flowing along the strip 4 into the sample could be measured by rotating the strip. In these

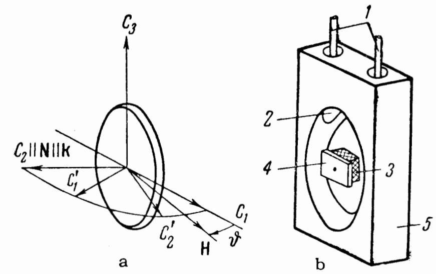

FIG. 1. a) Orientation of the crystallographic axes of bismuth in a disk sample: C_3 – trigonal axis; C_2, C_2' – binary axes; C_1, C_1' – bisectrix axes; N – normal to the sample surface; k – wave vector of a magnetoplasma wave; H – magnetic field. b) Schematic representation of the strip resonator structure: 1) – coaxial transmission lines; 2) – coupling aperture (the second aperture is not shown); 3) – foamed polystyrene; 4) – copper resonance strip; 5) – casing.

experiments, we used two polarizations: 1) **J** ∥ C_3, 2) **J** ∥ C_1.

A magnetic field of up to 10 kOe intensity was established by an electromagnet.[6] By rotating the magnet, the field could be directed at any angle ϑ to the sample surface (Fig. 1a) in the horizontal plane.

The experiments were carried out at a sample temperature of ≈ 1.5°K.

MEASUREMENTS OF THE MAGNETOPLASMA WAVE VELOCITY

The velocity of the S and P magnetoplasma waves in bismuth was measured earlier[3] as a function of the magnetic field along various crystallographic directions. The velocity was found from the serial number of the resonance of a standing wave in a sample, which was equal to the number of half-waves which could be fitted to the sample thickness. In these earlier experiments,[3] the odd-numbered surface impedance oscillations (counting from $H^{-1} = 0$) had lower amplitude and, under certain conditions, were shifted considerably with respect to the even-numbered oscillations. For this reason, it was assumed that only the even-numbered oscillations represented the standing wave resonances and the order of the resonance was determined by counting every other oscillation. This was incorrect: we should have counted every oscillation since the differences between the even and odd oscillations were obviously due to the influence of microwave currents leaking through the contact between the sample and the resonator to the opposite side of the sample, in the same way as observed in the experiments of Williams and Smith.[7] This assumption was checked by special control measurements, involving the observation of the passage of waves through a sample separating two wide-band lines. A circuit with an attenuator and a phase shifter was connected in parallel with the sample and an auxiliary compensation microwave signal was applied through this circuit. Such a circuit made it possible to produce interference of the auxiliary signal with the signal leaking through the contacts past the sample.

Thus the values of ΔH^{-1} (the oscillation period) and vH^{-1} (v is the wave velocity) given in [3] should be divided by 2. The resultant wave velocities then agree well with the theoretically calculated values[3] and with the results of other measurements.[4,7] A review of the data presented in [3] indicated the need to alter the estimate of the wave damping $\omega\tau_W \approx 10$, obtained by Fal'kovskiĭ:[8] since $\omega\tau_W \propto n^2$, other conditions being equal, the correct estimate should be $\omega\tau_W \approx 40$.

RESULTS OF EXPERIMENTS

The purpose of the experiments was to measure the magnetic field corresponding to the limit of the damping region of the magnetoplasma waves, due to the shifted cyclotron resonance or to the Landau damping. These two damping mechanisms begin to be effective at certain limiting values of the field [Eqs. (3) and (4)], causing a sudden change in the amplitude of the observed oscillations, which makes it possible to distinguish them from the usual relaxation damping that gives rise to a smooth reduction in the amplitude when the field intensity is decreased. The boundary field for each crystallographic direction is governed by that damping mechanism which is more effective under given experimental conditions. The two causes of damping can be distinguished by comparing the results of the experiments with the expected values of the limiting fields in accordance with Fig. 2. The values of the limits of the fields shown by continuous lines were calculated using Eqs. (3) and (4) and the Fermi velocities of carriers were determined using the published data.[5,9,10] The velocity of magnetoplasma waves was measured in the present investigation.

1. Cyclotron Resonance of Holes, Shifted by the Doppler Effect

A. $\mathbf{J} \parallel C_3$. In this case, we excite the S-wave having the spectrum $\omega \propto kH$ and existing at any angle between **k** and **H**.[3] According to Smith et al.,[11] when $\vartheta = 0$ the S-wave may be propagated even in a field $H < H_h$ (H_h is the hole cyclotron resonance field; $\omega = eH_h/m_h^*c$; the subscript "h" designates the quantities which refer to holes), but in $H = H_h$ the cyclotron resonance is not excited and there is no associated damping of the waves (curve $\vartheta = 0.5°$ in Fig. 3). When $\vartheta \neq 0$, the resonance is Doppler-broadened and its limits are given by Eq. (3). It is found that in this case the

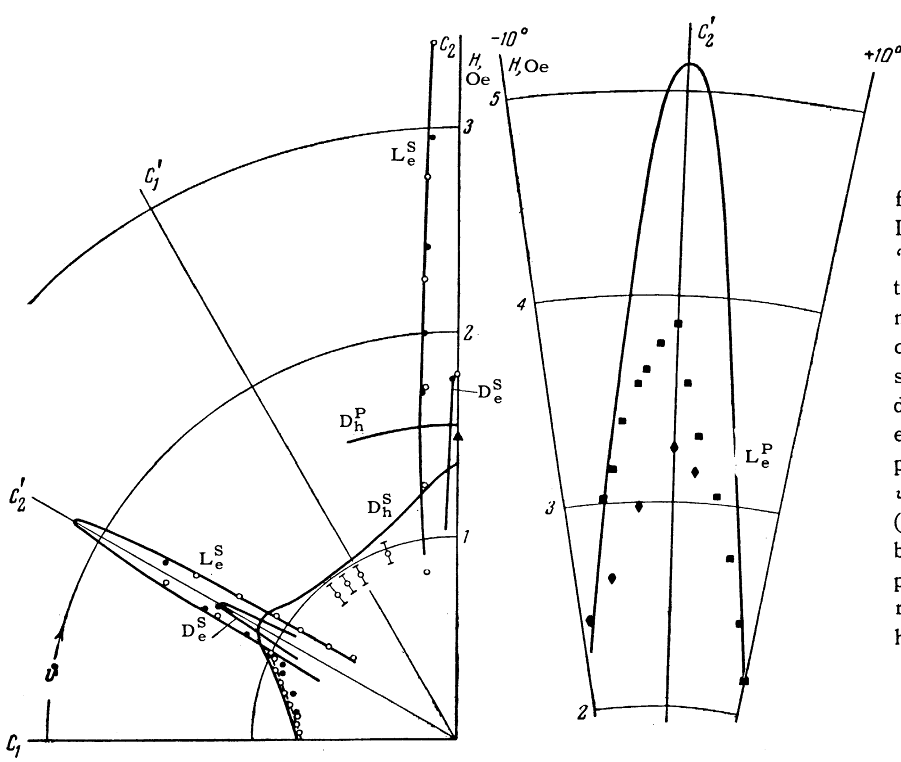

FIG. 2. Values of the limiting magnetic field H at which the Doppler damping D or a Landau damping L appears; the subscripts "h" and "e" indicate holes and electrons; the superscripts S and P indicate the magnetoplasma wave mode. The continuous curves represent calculated results. Near some points, the experimental errors are indicated, in other cases the error does not exceed the dimensions of the point. The positions of the black dots at low values of ϑ should be compared with the dashed curve (the shift is due to different frequencies being used in different experiments). The points ■, ○ represent the experimental results from the sample with thickness h = 1 mm; ●, ♦, ▲ – h = 0.47 mm.

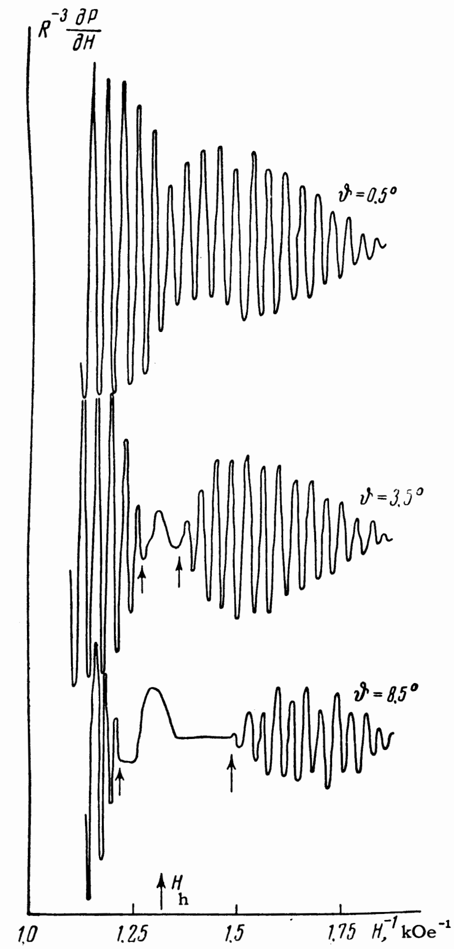

FIG. 3. The recordings from experiments intended to measure the limits of the Doppler damping (Fig. 2, D_h^S), indicated by arrows. H_h is the first-order cyclotron resonance field for holes when $\vartheta = 0$.

polarization conditions which ensure the absence of resonance at $\vartheta = 0$ are not satisfied[11] and the waves are strongly damped (Fig. 3) over the whole range of fields given by Eq. (3). It follows from (3) that

$$v_{Fh}\sin\vartheta = \frac{e}{m_h{}^*c}\frac{H_1 - H_2}{k_1 + k_2} = \frac{eh}{m_h{}^*\pi c}\frac{H_1 - H_2}{n_1 + n_2}, \quad (5)$$

where H_1, n_1 and H_2, n_2 are, respectively, the field and order of a standing wave for the limits of the absorption region on both the strong and weak field sides. The values of H_1, H_2, and n_1 are determined directly from the recordings such as those shown in Fig. 3.

When $\vartheta \lesssim 12°$, the value of n_2 may be found by linear extrapolation, which is justified by the fact that $(H_1 - H_2)/H_h < 1$ and by the similarity of the oscillation periods for $H > H_1$ and $H < H_2$. The results of the analysis of the experiments using Eq. (5) are shown in Fig. 4. Since the Fermi surface of holes has axial symmetry with respect to

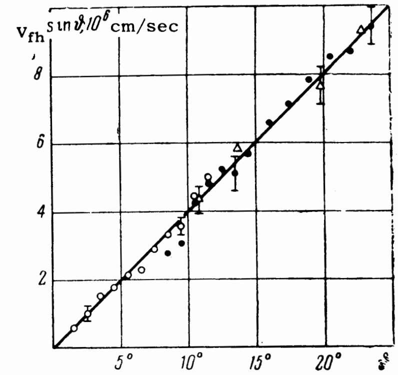

FIG. 4. Determination of the Fermi velocity of holes and of the effective mass at a limiting point. The points o, •, represent the experimental results from the sample which was h = 1 mm thick; △ – h = 0.47 mm. The points represented by o were calculated using Eq. (5); •, △ – using Eq. (6). The probable absolute experimental error is indicated for some of the points.

the C_3 axis, the points for $v_{Fh}\sin\vartheta$ fit a straight line whose slope gives the value $v_{Fh} = (2.3 \pm 0.15) \times 10^7$ cm/sec (the straight line in Fig. 4 is drawn using the least squares method).

When $\vartheta > 12°$, it is impossible to measure H_2 because the limit of the resonance damping on the weak-field side shifts to a region where the oscillations are not observed because of the relaxation damping. In this case, it follows from Eq. (3) that

$$v_{Fh}\sin\vartheta = \left(\frac{eH_1}{m_h{}^*c} - \omega\right)\frac{1}{k_1}. \quad (6)$$

Since $eH_1/m_h^*c - \omega = 0.1\omega$, these measurements are convenient for the determination of a more accurate value of the effective mass of holes at a special point, m_h^*, from the value of v_{Fh} found for $\vartheta \lesssim 12°$. The value of m_h^* is found from the coincidence of the results obtained using Eqs. (5) and (6), which occurs at $m_h^* = (0.220 \pm 0.002)m_0$ (Fig. 4).

Figure 4 shows only the points for $\vartheta \lesssim 25°$, since for $\vartheta \gtrsim 25°$, the limit of the oscillations is governed by the Landau damping of the electrons (Fig. 2, L_e^S). When $\vartheta \approx 50-70°$ (**H** close to C_1') and the electron velocities are low, the resonance damping of holes predominates. However, this damping is not as strong as that for $\vartheta \lesssim 25°$, which may be judged by the strong broadening of the region of oscillation damping. For this reason, the field H_1 can be determined only approximately, yielding the estimate $v_{Fh} = (2 \pm 0.5) \times 10^7$ cm/sec.

B. **J** ∥ C_1. In this case, the P-wave having the spectrum $\omega \propto$ **kH** is excited (the S-wave is weakly excited[3]). When **H** ∥ C_2, the observed P-wave limit is governed by the relaxation damping but

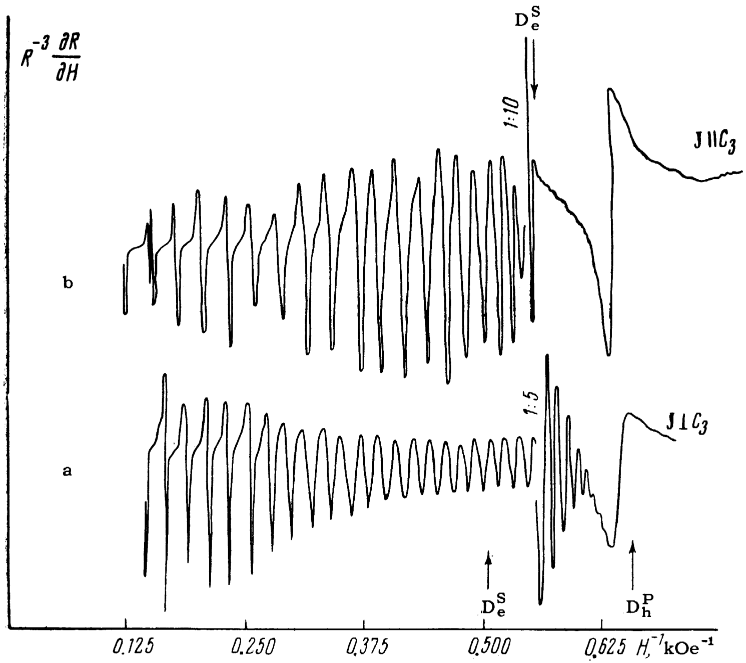

FIG. 5. The recordings from the experiments intended to measure the limits of the Doppler damping D_h^P (curve a) and D_e^S (curve b). The arrow D_e^P represents the calculated damping field of the P-wave for electrons. The symbols 1:5 and 1:10 represent the points where the gain of the circuit was altered.

the oscillations are observable up to the shifted peak of the hole resonance (Fig. 5a), which makes it possible to measure k and, consequently, to determine v_{Fh}: $v_{Fh} = (2.35 \pm 0.1) \times 10^7$ cm/sec.

2. Cyclotron Resonance of Electrons Shifted by the Doppler Effect

The Landau damping of the S-waves is weak for $H \parallel C_2$ and $H \parallel C_2'$, and the oscillation limit is governed by the Doppler-shifted resonance of electrons (Fig. 2, D_e^S; Fig. 5b). The Fermi velocity of electrons, for $H \parallel C_2$ to within $\approx 10'$, is found to be $v_{Fe} = (11.3 \pm 0.5) \times 10^7$ cm/sec. The values of v_{Fe} for directions close to the C_2 axis are shown in Fig. 6. Since the resonance is observed at $\vartheta = 30°$ and $\vartheta = 90°$, we can determine independently the two parameters m_e^* and v_{Fe} using a formula analogous to Eq. (6). The agreement between the values of v_{Fe} obtained for these two cases using the effective mass $m_e^* = 0.137 m_0$, measured in an investigation of the cyclotron resonance,[5] shows that we are dealing with the Doppler-shifted cyclotron resonance of electrons.

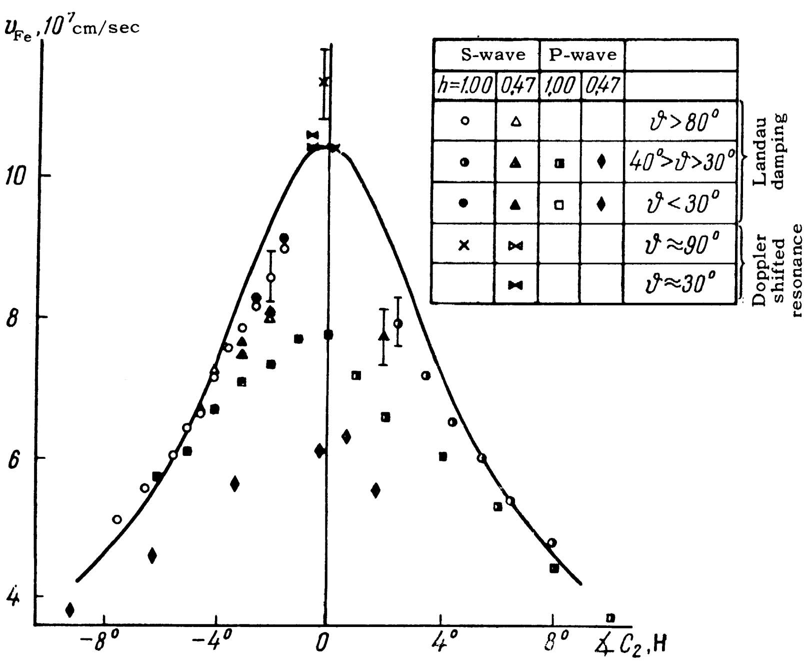

FIG. 6. Determination of the Fermi velocity of electrons. The continuous curve represents the results of calculations based on the Fermi surface model; the error may reach ≈ 5%. The experimental error is indicated for some of the points.

3. Landau Damping

A. S-waves. For directions of **H** close to C_2 or C_2', the limit of oscillations is given by Eq. (4), since the corresponding values of the field are $L_e^S > H_1$ (Fig. 2). A sharp limit of oscillations, which is due to the Landau damping (Fig. 7), makes it possible to use Eq. (4) to determine the Fermi velocity of electrons. The results are shown in Fig. 6. The values of v_{Fe} obtained for $\measuredangle$ **H**, $C_2 \gtrsim 3°$, $\measuredangle$ **H**, $C_2' \gtrsim 3°$ ($\vartheta = 90°, 30°$) for samples of different thicknesses are all equal and the values of v_{Fe} for the angles $\measuredangle$ **H**, $C_2 \lesssim 10°$, $\measuredangle$ **H**, $C_2' \lesssim 10°$ have sixfold symmetry, as required by the crystal symmetry. Therefore, we may assume that the experiments described give the correct value of the electron velocity.

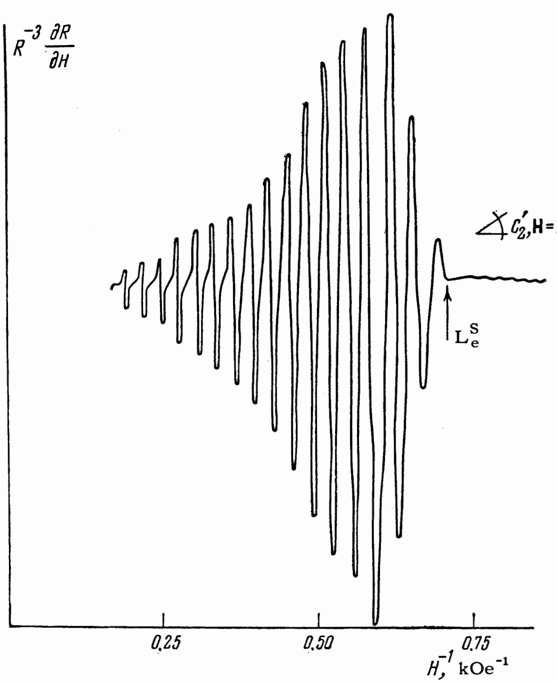

FIG. 7. The recordings from an experiment intended to measure the Landau damping limit L_e^S. The reduction in the oscillation amplitude as the field increases is not due to the damping of the waves (the width of the peaks decreases).

As the field direction H approaches the axes C_2 and C_2', the Landau damping decreases, the oscillation limit shifts toward the region of fields $H < L_e^S$ and the values of v_{Fe} calculated from the limits of the fields are found to be too low. For a thinner sample, the shift is greater and this explains the systematic differences (Fig. 6) in the positions of the points representing samples of different thicknesses.

B. P-waves. For the P-waves, the sharp oscillation limit, which may be ascribed to the Landau damping, is observed only for **H** close to C_2' (Fig. 2). For a sample 1 mm thick, the position of the limit satisfies the condition of Eq. (4) when **H**, $C_2' \gtrsim 5°$; a calculation of v_{Fe} gives values which agree with those obtained in the experiments on the S-waves (Fig. 6). In the $\measuredangle$ **H**, $C_2' < 5°$ region, in which the Landau damping is weak, the points obtained in the experiments on the P-waves lie below the points corresponding to the S-waves.

The systematic difference between the measured values of the fields L_e^P (Fig. 2) and the results of the calculation of v_{Fe} from the experiments for samples of various thicknesses using the P-waves (Figs. 2 and 6) is explained in the same way as was done in the experiments on the S-waves.

DISCUSSION OF RESULTS

According to the results obtained in [3], the S-wave spectrum for **H** $\parallel C_1$ and **k** $\parallel C_2$ is governed by holes, the contribution of electrons to the corresponding conductivity tensor components being small. Therefore, the interaction of holes with the wave field is strong, which should lead to strong damping of the wave in the shifted cyclotron resonance of holes. This qualitative conclusion is confirmed by the experiments.

When **H** $\parallel C_2$, electrons make a contribution to the S-wave spectrum which is greater than that from holes. This causes a strong interaction between electrons and the S-wave in a field **H** close to the C_2 axis, leading to the Landau damping and the damping in the Doppler-shifted cyclotron resonance of electrons. It is interesting that the Landau damping of the S-waves is weak when **H** $\parallel C_2$, C_2'. The absence of the Landau damping for **H** $\parallel C_2$ for both wave modes follows from the theory developed in [3]. From Eqs. (5) and (6), which give the components of the conductivity tensor governing the wave spectrum, it follows that the terms having a singularity at $\omega = k_H v_H$ are identically equal to zero when **H** $\parallel C_2$, which means the absence of the Landau damping in the approximation considered. When **H** $\parallel C_2'$, an estimate shows that the Landau damping has the same order of magnitude as the relaxation damping.

The spectrum of the P-waves for **k** $\parallel C_2$ and any direction of the field **H** in the basal plane is governed primarily by holes, the contribution of electrons being small. Consequently, the Landau damping for the P-waves is weaker than that for the S-waves, and the P-waves may be propagated when $H < L_e^P$ (Figs. 2 and 6). For the same reasons, the Doppler-shifted resonance of electrons, which should take place in fields $D_e^P > D_h^P$, is not observed for the P-waves when $H \parallel C_2$.

1. HOLE FERMI SURFACE

The hole Fermi surface has axial symmetry,[5] which allows us, using the known values of $S_{extr} = (6.75 \pm 0.25) \times 10^{-42}$ cgs,[9] $m^* = (0.063 \pm 0.001)m_0$,[5] and the formula

$$m^* = \frac{1}{2\pi} \oint \frac{dl}{v_\perp},$$

to calculate the Fermi velocity of holes at the limiting point when $H \perp C_3$: $v_{Fh} = (2.55 \pm 0.15) \times 10^7$ cm/sec. This value agrees with the measured values: $(2.3 \pm 0.15) \times 10^7$ cm/sec (S-waves) and $(2.35 \pm 0.1) \times 10^7$ cm/sec (P-waves).

The value $v_{Fh} = 3.4 \times 10^7$ cm/sec was reported by Kirsch;[4] the difference between this value and our results is obviously due to the use of a linear approximation for the wave velocity in the range of fields where, as indicated by experiments, it is markedly nonlinear. Thus, Fig. 5a shows a considerable reduction in the oscillation period when the field is decreased. Kirsch did not explain the absence of the anisotropy of the shifted resonance in the basal plane. This absence was due to the fact that the observed resonance was not associated with an S-wave but with a weakly anisotropic P-wave. It should be mentioned that a peak of D_h^P is observed, as shown in Fig. 5, at the same value of the field for all polarizations of the current **J**.

According to the measurements of the Doppler shift in the cyclotron resonance for $H \perp C_3$, the hole mass at the limiting point is $m_h^* = (0.220 \pm 0.002)m_0$, while in an earlier investigation[5] we observed a resonance corresponding to the mass $(0.210 \pm 0.004)m_0$. The difference between these two values is greater than the experimental error and is obviously due to the fact that the cyclotron resonance is not excited at the limiting point in the hole Fermi surface because of the small diameter of the orbit and great depth of the skin layer; it is also possible that the same reasons account for a considerable shift in the resonance fields, which is different for different resonance orders. In any case, the value of m_h^* obtained in the present investigation seems more reliable. Consequently, we may conclude that the hole spectrum is not quadratic because the central-cross-section mass was found to be $m^* = (0.203 \pm 0.004)m_0$,[5] which is considerably less than m_h^*. It is interesting to note that the calculation of the same masses, carried out using the constants of the bismuth spectrum found by Fal'kovskiĭ,[12] gives, respectively, $0.196m_0$ and $0.210\ m_0$. It is evident that the difference between the calculated values of the masses is practically identical with the difference between the measured values given above.

2. ELECTRON FERMI SURFACE

Figure 6 shows, in addition to the values of v_{Fe} found in our experiments, a calculated curve representing the ellipsoidal model of the Fermi surface with the parameters reported in [5,10]. The electron velocity along the C_2 axis, determined from the Doppler shifted resonance, $v_{Fe} = (11.3 \pm 0.5) \times 10^7$ cm/sec is equal, within the limits of the experimental error, with the model value $v_{Fe} = (10.4 \pm 0.6) \times 10^7$ cm/sec. In [5] we proposed a qualitative model for the electron Fermi surface of bismuth, constructed on the basis of an analysis of the cyclotron resonance data. We assumed that the increase in the mass at limiting points $m^* = (v_F \sqrt{K})^{-1}$ was due to a departure of the Gaussian curvature K of the surface from the ellipsoidal model curvature, and the velocities in the middle part of the Fermi surface were assumed to be the same as for the ellipsoidal model. The direct measurements of the electron velocity along the C_2 axis, carried out in the present investigation, confirmed this assumption.

The velocities of electrons traveling at an angle to the C_2 axis, found from the investigation of the Landau damping, were in good agreement with the ellipsoidal model of the Fermi surface in all those cases when the same values were obtained in the experiments carried out under different conditions.

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