ON THE THEORY OF SOUND ABSORPTION IN SOLIDS

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The equation of motion of an isotropic elastic medium, describing the propagation and absorption of sound interacting with thermal phonons, is obtained in a linear approximation. Sound scattering on thermal vibrations as well as decay processes are taken into account. An expression for the damping constant of the sound is derived for arbitrary values of the parameter $\hbar\Omega/T$, and different particular cases are analyzed.

1. INTRODUCTION

A large number of papers, dealing with various particular cases, have been dedicated to the problem of sound absorption in solids. Landau and Rumer[1] calculated the damping constant γ_t of transverse acoustic waves, which is due to their coalescence with thermal transverse or longitudinal phonons, under the conditions $\hbar\Omega \ll \kappa T^\circ \equiv T$ and $\Omega\tau \gg 1$, and found

$$\gamma_t \sim T^4 q . \tag{1}$$

Here Ω and q are the frequency and wave number of the propagating sound, κ is the Boltzmann constant, and τ is the relaxation time of the thermal phonons. Slonimskiĭ[2] studied the absorption of longitudinal sound due to the disintegration of the propagating wave into two waves, either both transverse or one longitudinal and one transverse, in the high-frequency region $\hbar\Omega \gg T$ and $\Omega\tau \gg 1$. This problem was discussed recently by Orbach and Vredevoe[3], who considered, alongside with the three-phonon processes, also four-phonon interactions which make an insignificant contribution to absorption. Akhiezer[4] dealt with the low-frequency region, $\hbar\Omega \ll T$ and $\Omega\tau \ll 1$; he found the frequency dependence of the damping constant, and, as in various limiting cases, also the temperature dependence.

Herring[5] investigated sound absorption with account taken of the anisotropy of the solid, and showed that the damping constant is given by

$$\gamma_t \sim T^4 q, \tag{2}$$

for transverse sound and by

$$\gamma_l \sim T^3 q^2 \tag{3}$$

for longitudinal sound.

Bömmel and Dransfeld[6] studied experimentally the temperature dependence of γ_l and γ_t and found that both constants are proportional to T^4, in disagreement with Eq. (3). Neither does the relation $\gamma_l \propto T^3 q^2$ conform with the experiments on thermomagnetic effects in a quantizing magnetic field. Guseva and Zyryanov have shown[7] that such experiments yield information about the dependence of γ on q and T.

Puri and Geballe[8] measured the differential thermal emf due to the nonequilibrium of phonons (drag thermal emf) in n-type germanium in the presence of a quantizing magnetic field. The drag thermal emf and its dependence on temperature and magnetic field intensity contradict Eq. (3). In the experiments of [8], an essential role in the interaction with electrons is played by long-wave phonons, the damping of which is due to scattering by short-wave thermal phonons.

In the interesting work of Simons[9] it was shown that at low temperatures, when $\Omega\tau \gg 1$ and $\hbar\Omega \ll T$,

$$\gamma_l \sim \gamma_t \sim T^4 q. \tag{4}$$

Using this relationship we can explain the experiments described in [6,7] (see [7]). However, the introduction of the damping constant with allowance for collisions between thermal phonons and impurities or with one another is not trivial. The difficulties here are similar to those which arise in the study of ultrasound absorption by electrons in metals. The problem can be solved in a way shown by Silin[10], who derived the equation of motion of a lattice interacting with electrons. A similar equation will be derived below for a lattice interacting with thermal phonons. Such an equation describes the temperature dependence of the elastic constants, their space-time dispersion, and, of course, the propagation and absorption of sound. Unlike Simons' results [9], our results for sound absorption have broader applicability, being valid not only when $\hbar\Omega \ll T$, but also in the opposite case.

2. THE INTERACTION ENERGY OF SOUND WITH THERMAL PHONONS

The interaction of elastic waves in solids is connected with the anharmonicity of the vibrations. For not too large displacement amplitudes only the third-order anharmonicity is substantial. In an isotropic elastic medium one can represent the energy connected with this anharmonicity (see, e.g., [11]) by

$$W = \int d\mathbf{r}\left\{\left(\mu + \frac{A}{4}\right)u_{\alpha\beta}u_{\gamma\alpha}u_{\gamma\beta} + \left(\frac{B+K}{2} - \frac{\mu}{3}\right)u_{\alpha\alpha}(u_{\gamma\beta})^2 + \frac{A}{12}u_{\alpha\beta}u_{\beta\gamma}u_{\gamma\alpha} + \frac{B}{2}u_{\alpha\beta}u_{\beta\alpha}u_{\gamma\gamma} + \frac{C}{3}(u_{\alpha\alpha})^3\right\}. \quad (5)$$

Here $u_{\alpha\beta} = \partial u_\alpha/\partial x_\beta$, $\mathbf{u}(\mathbf{r}, t)$ is the displacement vector, K and μ are the moduli of hydrostatic compression and shear, and A, B, and C are the anharmonic constants.

At low temperatures the thermal vibrations of the medium obey the laws of quantum mechanics, while the propagating external sound can be described classically, assuming the density of its quanta is sufficiently large. Correspondingly, we present the displacement vector in the form

$$\mathbf{u} = \mathbf{u}^0 + \mathbf{u}' + \mathbf{u}'',$$

and assume that $\mathbf{u}^0$ is caused by the propagating sound, while $\mathbf{u}'$ and $\mathbf{u}''$ are due to the thermal motion of the medium. Further, we will make use of the correspondence principle and express $\mathbf{u}'$ and $\mathbf{u}''$ through Bose operators of second quantization, e.g.,

$$\mathbf{u}' = \sum_{\mathbf{k}\lambda}\left(\frac{\hbar}{2\omega_\lambda(\mathbf{k})\rho V}\right)^{-1/2}\mathbf{e}_\lambda\{b_{\mathbf{k}\lambda}e^{i\mathbf{k}\mathbf{r}} + b_{\mathbf{k}\lambda}^+e^{-i\mathbf{k}\mathbf{r}}\}. \quad (6)$$

Here

$$b_{\mathbf{k}\lambda}b^+_{\mathbf{k}'\lambda'} - b^+_{\mathbf{k}'\lambda'}\delta_{\mathbf{k}\lambda} = b_{\mathbf{k}\mathbf{k}'}\delta_{\lambda\lambda'},$$

$\mathbf{e}_\lambda$ is the unit polarization vector, satisfying the conditions

$$\mathbf{e}_\lambda\mathbf{e}_{\lambda'} = \delta_{\lambda\lambda'}, \qquad e_\lambda^\alpha e_\lambda^\beta = \delta_{\alpha\beta},$$

λ is the polarization index, $\mathbf{k}$ is the wave vector, $\omega_\lambda(\mathbf{k})$ is the vibration frequency, ρ is the substance density, and V is the normalizing volume.

With the aid of Eqs. (6) and (5) we find

$$W = \frac{\hbar}{2V}\int d\mathbf{r}\sum_{\substack{\mathbf{k}\lambda\\ \mathbf{k}'\lambda'}} M_{\alpha\beta}(\mathbf{k}\lambda, \mathbf{k}'\lambda')\{e^{i(\mathbf{k}-\mathbf{k}')\mathbf{r}}(b^+_{\mathbf{k}'\lambda'}b_{\mathbf{k}\lambda} + b_{\mathbf{k}\lambda}b^+_{\mathbf{k}'\lambda'}) - e^{-i(\mathbf{k}+\mathbf{k}')\mathbf{r}}(b^+_{\mathbf{k}'\lambda'}b^+_{\mathbf{k}\lambda} + b_{\mathbf{k}'\lambda'}b_{\mathbf{k}\lambda})\}u_{\alpha\beta}; \quad (7)$$

$$M_{\alpha\beta}(\mathbf{k}\lambda, \mathbf{k}'\lambda') = (\rho^2\omega_\lambda(\mathbf{k})\omega_{\lambda'}(\mathbf{k}'))^{-1/2}\{[(B+K-{}^2/_3\mu)(\mathbf{k}\mathbf{k}')\delta_{\alpha\beta} + (\mu + {}^1/_4A)(k_\alpha k_\beta' + k_\alpha' k_\beta)](\mathbf{e}\mathbf{e}') + (\mu + {}^1/_4A)(e_\alpha e_\beta' + e_\alpha' e_\beta)(\mathbf{k}\mathbf{k}') + [(\mu + {}^1/_4A)(e_\alpha' k_\beta) + {}^1/_4Ae_\beta' k_\alpha + {}^1/_2B(\mathbf{e}'\mathbf{k})\delta_{\alpha\beta}](\mathbf{e}\mathbf{k}') + [(\mu + {}^1/_4A)(e_\alpha k_\beta') + {}^1/_4Ae_\beta k_\alpha' + {}^1/_2B(\mathbf{e}\mathbf{k}')\delta_{\alpha\beta}](\mathbf{e}'\mathbf{k}) + [(B+K-{}^2/_3\mu)e_\alpha' k_\beta' + Be_\beta' k_\alpha'](\mathbf{e}\mathbf{k}) + [(B+K-{}^2/_3\mu)e_\alpha k_\beta + Be_\beta k_\alpha](\mathbf{e}'\mathbf{k}') + 2Ck_\beta k_\beta' e_\alpha e_\alpha' \delta_{\alpha\beta}\}, \quad (8)$$

Here $\mathbf{e}$ and $\mathbf{e}'$ are the polarization vectors of phonons with indices λ and λ'.

3. THE DENSITY MATRIX OF THERMAL PHONONS

To describe the thermal phonons we use the expression for a single-particle statistical operator

$$\frac{\partial}{\partial t}\hat{\rho} + \frac{i}{\hbar}[H_0 + W, \hat{\rho}] = I(\hat{\rho}), \quad (9)$$

where

$$H_0 = \sum_\nu \hbar\omega_\nu b_\nu^+ b_\nu; \qquad \nu \equiv \mathbf{k}, \lambda; \qquad [A, B] = AB - BA,$$

and $I(\hat{\rho})$ is the collision operator of the thermal phonons with impurities and with one another. These collisions become essential and must be taken into account when the sound frequency Ω is comparable to or less than the effective collision frequency $1/\tau$ of the thermal phonons. The local-equilibrium density matrix is given by

$$[H_0 + W, \hat{\rho}_0] = 0. \quad (10)$$

From Eq. (10) we obtain for the $\hat{\rho}_0$ matrix elements in the H_0 representation, in an approximation linear with respect to W,

$$\frac{1}{2}\mathrm{Sp}[\hat{\rho}_0,\ (b_{\nu'}^+ b_\nu + b_\nu b_{\nu'}^+)] = N_{\nu'}^0\delta_{\nu\nu'} + \frac{N_{\nu'}^0 - N_\nu^0}{\omega_{\nu'} - \omega_\nu}\Phi(\nu\nu'), \quad (11)$$

$$\frac{1}{2}\mathrm{Sp}[\hat{\rho}_0,\ (b_{\nu'}^+ b_\nu^+ + b_{\nu'} b)] = -\frac{1 + N_{\nu'}^0 + N_\nu^0}{\omega_{\nu'} + \omega_\nu}\Phi'(\nu\nu'); \quad (12)$$

$$N_\nu^0 = (e^{\hbar\omega_\nu/T} - 1)^{-1},$$

$$\Phi(\nu\nu') = \frac{1}{V}\int d\mathbf{r}\, e^{i(\mathbf{k}-\mathbf{k}')\mathbf{r}} M_{\alpha\beta}(\mathbf{k}\lambda, \mathbf{k}'\lambda')u_{\alpha\beta},$$

$$\Phi'(\nu\nu') = \frac{1}{V}\int d\mathbf{r}\, e^{-i(\mathbf{k}+\mathbf{k}')\mathbf{r}} M_{\alpha\beta}(\mathbf{k}\lambda, \mathbf{k}'\lambda')u_{\alpha\beta}. \quad (13)$$

Small deviations from the local equilibrium are described by an addition $\delta\hat{\rho}$, which, according to Eq. (9), is given in linear approximation by

$$\frac{\partial}{\partial t}\delta\hat{\rho} + \frac{i}{\hbar}[H_0, \delta\hat{\rho}] + \frac{\partial}{\partial t}\hat{\rho}_0 = I(\delta\hat{\rho}). \quad (14)$$

Approximating the relaxation time by

$$I(\delta\hat{\rho}) \sim -\delta\hat{\rho}/\tau$$

for the case $\delta\hat{\rho} \sim u^0_\alpha \sim \exp(-i\Omega t)$ we obtain with the aid of Eqs. (10)—(14),

$$\tfrac{1}{2}\,\mathrm{Sp}[\hat{\rho},\ (b_{\nu'}^{+}b_\nu + b_\nu b_{\nu'}^{+})] = N_{\nu}{}^{0}\delta_{\nu\nu'} + \left(1 - \frac{\Omega}{\omega_\nu - \omega_{\nu'} + \Omega + i/\tau}\right) \times \frac{N_{\nu'}{}^{0} - N_\nu}{\hbar(\omega_{\nu'} - \omega_\nu)}\Phi(\nu\nu'),$$

$$\tfrac{1}{2}\,\mathrm{Sp}[\hat{\rho},\ (b_{\nu'}^{+}b_\nu^{+} + b_\nu b_{\nu'})] = -\left(1 - \frac{\Omega}{\omega_{\nu'} + \omega_\nu + \Omega + i/\tau} + \frac{\Omega}{\omega_{\nu'} + \omega_\nu - \Omega - i/\tau}\right)\frac{1 + N_{\nu'}{}^{0} + N_\nu}{\hbar(\omega_{\nu'} + \omega_\nu)}\Phi'(\nu\nu'), \quad (15)$$

where $\hat{\rho} = \hat{\rho}_0 + \delta\hat{\rho}$.

4. THE EQUATION OF MOTION OF THE LATTICE

The self-consistent interaction between the sound and the thermal phonons leads in the equation of motion of the medium to an additional force, which can be easily found by variation of W with respect to $u_{\alpha\beta}$. Simple calculations yield for the average value of this force the expression:

$$F_\alpha = \frac{\hbar}{2}\frac{\partial}{\partial x_\beta}\sum_{\nu\nu'} M_{\alpha\beta}(\nu\nu')\{e^{i(\mathbf{k}-\mathbf{k}')\mathbf{r}}\,\mathrm{Sp}[\hat{\rho},\ (b_{\nu'}^{+}b_\nu + b_\nu b_{\nu'}^{+})] - e^{-i(\mathbf{k}-\mathbf{k}')\mathbf{r}}\,\mathrm{Sp}[\hat{\rho},\ (b_{\nu'}^{+}b_\nu^{+} + b_{\nu'}b_\nu)]\}. \quad (16)$$

Besides this force, allowance for the collision integral leads to another force, which appears in the equation of motion when the phonon collisions transfer momentum to the lattice (scattering from impurities and Umklapp processes). When the main contribution to τ is made by the normal collisions between thermal phonons, only the F_α determined by Eq. (6) will appear in the equation of motion of the lattice, and this is the case we shall deal with in what follows.

Assuming further $\mathbf{u} = \mathbf{u}(q)\exp(-i\Omega t + i\mathbf{q}\cdot\mathbf{r})$ and taking (15) and (16) into account, we find the equation of motion for the Fourier component $\mathbf{u}(q)$:

$$-\rho\Omega^2 u_\alpha + \lambda_{\alpha\gamma\delta\beta}q_\gamma q_\delta u_\beta = 0, \quad (17)$$

where

$$\lambda_{\alpha\gamma\delta\beta} = \lambda^0_{\alpha\gamma\delta\beta} + \hbar\sum_{\nu\nu'} M_{\alpha\gamma}(\nu\nu')M_{\beta\delta}(\nu'\nu) \times \left\{\left(1 - \frac{\Omega}{\omega_\nu - \omega_{\nu'} + \Omega + i/\tau}\right)\frac{N_{\nu'}{}^{0} - N_\nu{}^{0}}{\omega_{\nu'} - \omega_\nu}\delta_{\mathbf{k}-\mathbf{k}'+\mathbf{q}} + \frac{1}{2}\left(2 + \frac{\Omega}{\omega_{\nu'} + \omega_\nu - \Omega - i/\tau} - \frac{\Omega}{\omega_{\nu'} + \omega_\nu + \Omega + i/\tau}\right) \times \frac{1 + N_{\nu'}{}^{0} + N_\nu{}^{0}}{\omega_{\nu'} + \omega_\nu}\delta_{\mathbf{k}+\mathbf{k}'-\mathbf{q}}\right\}$$

is the elastic-modulus tensor renormalized as a result of the interaction between the sound and the thermal motion of the lattice. Because $\lambda_{\alpha\gamma\delta\beta}$ is complex, dissipative processes appear, which lead, in particular, to sound absorption.

5. THE DAMPING DECREMENT OF THE SOUND

The equation of motion of the lattice (17) allows one to express the damping decrement of the sound vibrations in the general case without the limitations due to the relation between $\hbar\Omega$ and T. Assuming the damping decrement of the sound $\gamma = \mathrm{Im}\,\Omega$ to be small in comparison with the vibration frequency $\omega_q = \mathrm{Re}\,\Omega$, we can easily find from Eq. (17) the following expression:

$$\gamma = \frac{\hbar}{\rho V}\sum_{\nu\nu'} M_{\alpha\gamma}(\nu\nu')q_\gamma M_{\beta\delta}(\nu'\nu)q_\delta\bar{u}_\beta\bar{u}_\alpha \times \left\{\frac{N_\nu{}^{0} - N_{\nu'}{}^{0}}{\omega_{\nu'} - \omega_\nu}\frac{1/\tau}{(\omega_{\nu'} - \omega_\nu - \omega_q)^2 + 1/\tau^2}\delta_{\mathbf{k}'-\mathbf{k}-\mathbf{q}} + \frac{1}{2}\frac{1 + N_{\nu'}{}^{0} + N_\nu{}^{0}}{\omega_{\nu'} + \omega_\nu}\frac{1/\tau}{(\omega_{\nu'} + \omega_\nu - \omega_q)^2 + 1/\tau^2}\delta_{\mathbf{k}+\mathbf{k}'-\mathbf{q}} - \frac{1}{2}\frac{1 + N_{\nu'}{}^{0} + N_\nu{}^{0}}{\omega_{\nu'} + \omega_\nu}\frac{1/\tau}{(\omega_{\nu'} + \omega_\nu + \omega_q)^2 + 1/\tau^2}\delta_{\mathbf{k}+\mathbf{k}'-\mathbf{q}}\right\};$$

$$\bar{u}_\beta\bar{u}_\alpha = u_\beta u_\alpha/|u|^2. \quad (18)$$

Let us consider the particular cases.

A. $\hbar\omega_q \ll T$. In this limiting case the last two terms in (18) are small in comparison with the first one. This becomes evident at $\tau \to \infty$, because the energy and momentum cannot be conserved when sound interacts with the dominating thermal phonons having energy $\hbar\omega_q \sim T$.

Disregarding the two last terms in (18) and recognizing that only the scattering at equal polarizations λ and λ' is significant in the first term, we obtain

$$\gamma = \frac{\hbar}{\rho}\sum_\nu\int\frac{d\mathbf{k}}{(2\pi)^3}M_{\alpha\gamma}(\nu\nu)q_\gamma M_{\beta\delta}(\nu\nu)q_\delta\bar{u}_\beta\bar{u}_\alpha \times \frac{\partial N_\nu{}^{0}}{\partial\omega_\nu}\frac{1/\tau}{[(\partial\omega_\nu/\partial\mathbf{k})\mathbf{q} - \omega_q]^2 + 1/\tau^2}. \quad (19)$$

This expression agrees essentially with Simons' result [9]. A more consistent introduction of the damping decrement yields a different angular dependence of the integrand in Eq. (19), and, unlike Simons' result, γ is a function of all three anharmonic constants. In particular, as pointed out in [9], when $1/\tau \ll \hbar\omega_q \ll T$, Eq. (19) yields for longitudinal and transverse phonons

$$\gamma_l \sim T^4 q; \qquad \gamma_t \sim T^4 q. \quad (20)$$

The last expression was first obtained by Landau and Rumer [1]. It was shown earlier [7] that expressions (20) fit well the experimental measurements of the drag thermal emf in n-type germanium.

B. $\hbar\omega_q \gg T$, $\omega_q\tau \gg 1$. In this case the expression for γ can be written

$$\gamma = \frac{\pi\hbar}{\omega_q\rho}\sum_{\lambda\lambda'}\int\frac{d\mathbf{k}}{(2\pi)^3}\{M_{\alpha\gamma}(\mathbf{k}\lambda, \mathbf{k}+\mathbf{q}\lambda')\,q_\gamma M_{\beta\delta}(\mathbf{k}+\mathbf{q}\lambda', \mathbf{k}\lambda)$$
$$\times\, q_\delta\bar{u}_\beta\bar{u}_\alpha e^{-\hbar\omega_k/T}\,\delta(\omega_{\lambda'}(|\mathbf{k}+\mathbf{q}|)-\omega_\lambda(k)-\omega_q)$$
$$+\,{}^1/_2 M_{\alpha\gamma}(\mathbf{k}\lambda, \mathbf{q}-\mathbf{k}\lambda')\,q_\gamma M_{\beta\delta}(\mathbf{q}-\mathbf{k}\lambda', \mathbf{k}\lambda)\,q_\delta\bar{u}_\beta\bar{u}_\alpha$$
$$\times\,\delta(\omega_\lambda(|\mathbf{q}-\mathbf{k}|)+\omega_\lambda(k)-\omega_q)\}. \tag{21}$$

The limits of integration with respect to **k** are determined by the energy and momentum conservation.

Thus, for example, for the processes

$$t + l \to l$$

(a transverse sound quantum t coalesces with a longitudinal one l, and a longitudinal sound quantum l is formed), the integration over the angle ϑ between the vectors **q** and **k** is with the aid of a δ-function with a pole defined by equation

$$\cos\vartheta = {}^1/_2[(c^2-1)\,q/k + 2c]$$

($c = c_t/c_l$, where c_t and c_l are the velocities of the transverse and longitudinal sound). This equation yields the integration limits with respect to the absolute value of k:

$$k_{min} = (1-c)\,q/2, \quad k_{max} = \infty.$$

An estimate of the integral yields for the damping constant

$$\gamma_{t+l\to l} = \frac{\hbar}{\rho}\left(\frac{Q_1^2}{\rho^2 c_l^4}\right)(1-c^2)\left[\exp\left\{\frac{-\hbar\omega_q(1-c)}{2Tc}\right\}\right]q^5,$$
$$Q_1 = A + 2B + K + 7\mu/3. \tag{22}$$

This result with a slightly different pre-exponential factor was first found by Orbach and Vredevoe[3], although it is already contained in the work of Landau and Rumer[1].

The coalescence of transverse sound with the transverse thermal quanta with the formation of a longitudinal quantum, i.e.,

$$t + t \to l$$

also yields an exponentially small damping decrement of the transverse sound

$$\gamma_{t+t\to l} \sim \exp\left\{-\frac{\hbar\omega_q}{T}\frac{1-c}{1+c}\right\}.$$

The damping of the longitudinal sound with $\hbar\omega_q \gg T$ turns out to be independent of temperature and is described by the second term in Eq. (21), which takes account of the disintegration of the longitudinal sound quantum into two transverse ones or into one transverse and one longitudinal quantum. Such processes were first considered by Slonimskiĭ[2]. In this case the damping decrement has the form:

$$\gamma_{l\to t+l} \sim \gamma_{l\to t+t} \sim \hbar q^5/\rho. \tag{23}$$

Thus, when sound interacts with thermal phonons, the damping decrements of longitudinal and transverse waves are substantially different when $\hbar\omega_q \gg T$. However, if the phonon dispersion is negligible in comparison with the damping (with the level width), then the conservation laws allow the decay processes

$$t \to t + t, \qquad l \to l + l,$$

which yield the following dependence of the damping decrement on q:

$$\gamma_l \sim \gamma_t \sim \hbar q^5/\rho. \tag{24}$$

Equation (24) shows that the damping decrements of the longitudinal and transverse sound waves may contain temperature-independent terms.

C. For the processes $l + l \to l$ (possible only when phonon dispersion is neglected), γ can be found at any value of $\hbar\omega_q/T$. Such dispersion processes are described by the first term in Eq. (18). In this case

$$\gamma = \left(\frac{\hbar}{4\pi\rho}\right)\left(\frac{Q_2^2}{\rho^2 c_l^4}\right)\int_0^\infty dk\,k^2(k+q)^2\left\{\left[\exp\left(\frac{\hbar c_l k}{T}\right)-1\right]^{-1}\right.$$
$$\left.-\left[\exp\left(\frac{\hbar c_l(k+q)}{T}\right)-1\right]^{-1}\right\},$$
$$Q_2 = 2A + 6B + 3K + 4\mu + 2C. \tag{25}$$

Computation of the integral gives

$$\gamma = \left(\frac{\hbar}{4\pi\rho}\right)\left(\frac{Q_2^2}{\rho^2 c_l^4}\right)\left(\frac{T}{\hbar c_l}\right)^5\sum_{n=1}^\infty\left\{\frac{24}{n^5}+\frac{6}{n^3}\left(\frac{\hbar\omega_q}{T}\right)\right.$$
$$\left.+\frac{2}{n^2}\left(\frac{\hbar\omega_q}{T}\right)^2\right\}\left[1-\exp\left(-\frac{n\hbar\omega_q}{T}\right)\right].$$

For $\hbar\omega_q \ll T$ we have

$$\gamma = \left(\frac{\hbar}{4\pi\rho}\right)\left(\frac{Q_2^2}{\rho^2 c_l^4}\right)\left(\frac{T^4}{\hbar c_l}\right)q. \tag{26}$$

If $\hbar\omega_q \gg T$, then

$$\gamma = \frac{\pi}{12}\,\frac{\hbar}{\rho}\left(\frac{Q_2^2}{\rho^2 c_l^4}\right)\left(\frac{T}{\hbar c_l}\right)^3 q^2, \tag{27}$$

i.e., unlike the case B, the damping constant is not exponentially small. This is connected with the fact that the energy-momentum conservation law allows all k values in the integral (25).

In conclusion we note that a direct experimental measurement of the sound absorption in the high-frequency region $\hbar\omega_q \gg T$ is difficult at present. Therefore it seems advisable to investigate the differential drag thermal emf in super-strong magnetic fields $H > 10^5 (T°)^2$. Such experiments may yield quite valuable information on the dependence of the damping decrement in the high-frequency region.

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