RELAXATION OF ONSAGER-FEYNMAN VORTICES ON HEATING OF ROTATING HELIUM II ABOVE THE PHASE-TRANSITION TEMPERATURE

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The decay of vortex filaments in the superfluid component of rotating helium II, which occurs on transition to helium I, is investigated by the oscillating-disk method. The decay occurs in the form of a three-stage relaxation process, in the first stage of which the damping of the disk oscillations remains completely unchanged; this indicates that in helium I the vortex pattern of the rotating fluid characteristic of helium II and responsible for the additional damping of the disk oscillations is completely conserved. The vortices subsequently become detached from the disk surface and the damping becomes even lower than would be expected for helium I under the given conditions. Finally, during the third stage of the relaxation, the expected stable damping value is gradually attained. The duration of the first of these processes was measured as a function of the velocity of rotation and temperature and also of the degree of roughness of the disk. Possible factors responsible for the prolonged existence of the vortices in helium I and their subsequent decay are discussed.

As is well known, one of the manifestations of the quantum properties of helium II is the generation in it (at velocities above the critical) of vortices of the superfluid component.[1,2] The circulation around each of these small vortices is quantized[1,2]:

$$\Gamma = 2\pi \frac{\hbar}{m} n \tag{1}$$

(m is the mass of the helium atom, n is an integer) while the energy, which coincides numerically with the tension of the vortex filament, is expressed as[2]

$$\varepsilon = \pi \rho_s \frac{\hbar^2}{m^2} \ln \frac{b}{a}, \tag{2}$$

where ρ_s is the density of the superfluid component, b is the outer radius of the vortex, and a is the radius of the vortex core.

For the case of uniform rotation of helium II together with the cylindrical container in which it is placed, the number of vortices is determined by the formula[2]

$$N = m\omega_0 / \pi \hbar, \tag{3}$$

where ω_0 is the angular velocity of the container. These vortices are formed parallel to the axis of rotation of the vessel.

In the presence of quantized vortices, dissipation processes are clearly developed, owing to which the rotating helium II possesses anisotropic viscous properties.[3] In the special case of a disk which, being immersed in helium II, executes harmonic axial-torsional oscillations with frequency Ω and simultaneously rotates with the same angular velocity ω_0 as the container, the dependence of the logarithmic damping decrement δ on ω_0 has a characteristic maximum.[4] This dependence is a specific vortex effect. It determines the energy which the oscillating disk loses for a given angular velocity, not in overcoming internal friction but in the excitation of elastic oscillations[3] in the system of vortices (the analog of radiation damping, in which the energy of the source is carried away by the wave). So far as the maximum is concerned, it always corresponds to that velocity $\tilde{\omega}_0$ for which the oscillations of the separate vortices cease to be independent, and the vortices begin to form a single vortex chain. As computer calculations show, for these angular velocities, the vortices are attached to the disk in the best fashion and their slipping over the surface of the disk is minimal.[3]

It is natural that the vortex damping for any given angular velocity which determines the number of vortices must depend first of all on the vortex tension, given by Eq. (2). It should be obvious that at the phase-transition temperature, for which ρ_s vanishes, the vortex damping must disappear.

Yet our experiments have shown that the damping of the disk remains unchanged in its character for a rather prolonged time interval even above this temperature, providing the helium II–helium I phase transition is accomplished in a rotational state.

The results of measurements are shown in Fig. 1. It is seen from this that the disk damping at an angular velocity $\omega_0 = \tilde{\omega}_0$ remains unchanged for 18 minutes, after which it falls off sharply and then increases with time, gradually reaching that value which is characteristic for helium I. As has already been made clear, the total damping of the disk in helium II, brought about on the one hand by viscous forces and on the other by vortex effects, exceeds the damping of the disk in helium I. Of course, all attempts to observe the vortex damping in liquid helium set into motion above the λ point and not cooled thereafter to temperatures corresponding to helium II have not met with success. The shift of the phase-transition temperature in the liquid helium that would have been the result of rotation[5] has also not been observed.

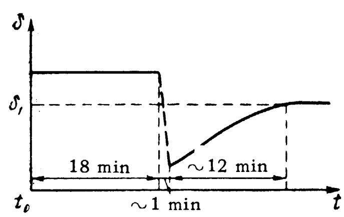

FIG. 1. Change in the damping decrement of disk oscillations in the relaxation process (t is the time measured from the instant of the phase transition).

It was natural to assume that in the transition through the λ point the entire superfluid component confined in the vortex, having been transformed to the normal state, begins to be slowed down rather rapidly. This should inevitably have manifested itself in the energy of the vortex which determines its tension.

Thus if the number of vortices were to remain constant for some time, then, because of the decrease in the tension, the vortex damping should decrease. In Fig. 1, this would have led to a departure from linearity in the first portion of the graph. Therefore, this portion was studied by us in considerable detail. In particular, the second derivatives were obtained for various points of the first portion by graphical differentiation. However, the sensitivity of the experiment did not make it possible to distinguish the shape of this portion from a straight line. Thus, the change in the vortex damping turned out at this stage to be beyond the limits of experimental error. This gave us the right to assume that during the first part of the time period both the number of vortices and their tension remained constant.

In the second part, an interval of the order of 1 minute, the sudden detachment of the vortices from the disk takes place. This leads to a sharp decrease in the decrement which drops even below the value which one would expect for helium I rotating as a whole. The latter circumstance can be understood if we recall that the logarithmic damping decrement of the disk oscillations in rotating helium II is expressed by a formula whose principal terms are of the form[3]

$$\delta - \frac{\Omega_0}{\Omega}\delta_0 \sim \sqrt{\frac{\eta_n\rho_n}{2}}\left(\sqrt{1+\frac{2\omega_0}{\Omega}} + \sqrt{1-\frac{2\omega_0}{\Omega}}\right) + \sqrt{\eta_s\rho_s\left(1-\frac{2\omega_0}{\Omega}\right)}, \quad (4)$$

while in helium I we should have

$$\delta - \frac{\Omega_0}{\Omega}\delta_0 \sim \sqrt{\frac{\eta\rho}{2}}\left(\sqrt{1+\frac{2\omega_0}{\Omega}} + \sqrt{1-\frac{2\omega_0}{\Omega}}\right). \quad (5)$$

In these formulas δ and Ω are the decrement and the oscillation frequency, δ_0 and Ω_0 are the vacuum values of these quantities, η and η_n are the dynamic viscosity of helium I and of the normal component of helium II, and ρ and ρ_n are the corresponding densities.

The first term in Eq. (4) is associated with the usual nature of the interaction of a viscous liquid with an oscillating surface, which takes place between the disk and the normal component. The same holds for formula (5). The second term in Eq. (4) is associated with the above-mentioned mechanism of energy loss of the disk to waves traveling along the vortex filaments. In the metastable region of rotation of helium I, when the system of vortices still continues to be attached to the disk, its previous value is preserved by Eq. (4), a fact which is also reflected by the first part of Fig. 1. At the instant of detachment of the vortices, the second term of Eq. (4) falls rapidly to zero; however, that part of the liquid which "entered into the composition" of the vortex, does not immediately transform to the rotational regime of the liquid as a whole. Therefore, beginning with the instant of detachment of the vortices, the decrement is expressed by Eq. (5), with this difference, however, that in place of the total density of the liquid ρ, some effective density $\rho' < \rho$ enters into it.

The increase of ρ' from values close to the value of ρ_n (prevalent at the instant of onset of heating of the helium) up to the total density ρ corresponding to the final temperature of the experiment, constitutes the physical meaning of the third portion of the curves under consideration, the duration of which is of the order of 12 minutes.

What factors can cause a sudden detachment of the vortices from the surface of the disk? Experiment shows that the vortices of helium I stretched between two surfaces are very stable and weakly damped formations right up to the moment of detachment. Some decrease in the rate of motion of the liquid inside the vortex, sufficiently weak to escape direct measurement, leads to an increase in the pressure along the vortex core. As a result of this increase in pressure, the vortex is drawn off from the solid surface and, becoming free, disintegrates. Thus the most stable are those vortex systems for which the slippage coefficient is minimal.

This point of view is confirmed by the behavior of vortices for $\omega_0 > \tilde{\omega}_0$. The relaxation time becomes less as the slippage coefficient increases, growing, as has been noted, with increasing ω_0 in the region $\omega_0 > \tilde{\omega}_0$.[3] Correspondingly, for small $\omega_0 < \tilde{\omega}_0$, when the slippage coefficient also lies above its minimum, the relaxation time again falls off with decreasing ω_0.

Our confidence in the validity of the proposed treatment is supported by observations on the relaxation times of vortices attached to a smooth surface: for $\omega_0 = \tilde{\omega}_0$, the relaxation time for a smooth surface falls off by more than four-fold compared with a roughened one.

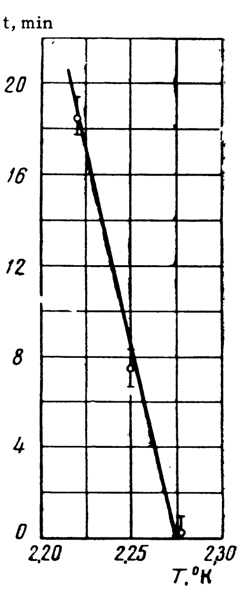

FIG. 2. Temperature dependence of the relaxation time (duration of the time interval of the first, horizontal portion of the graph in Fig. 1) for $\omega_0 = \tilde{\omega}_0$

The comments above pertain to experiments carried out at T = 2.22°K. As the temperature increases, the relaxation time, all other conditions being equal, falls off (see Fig. 2). This effect is probably associated with the temperature dependence of the viscosity.

One can also conjecture as to the conservation of the system of vortices in the transition from rotating helium II to rotating helium I by noting the following fact. The shape of the meniscus of helium II for high angular velocities differs from the shape of the meniscus of helium I for the same velocities in the presence of a small conical depression at the vertex of the parabola.[6] As M. P. Kemoklidze and Yu. G. Mamaladze have shown,[7] such a shape of the meniscus is most certainly a quantum effect. In the transition through the λ point, the conical depression which was discussed above must disappear. Nevertheless, for heating of rotating helium from temperatures below the λ point to temperatures ~ 3°K the conical depression at the vertex of the parabola continues to be maintained for a considerable time, relaxing slowly.[8]

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