OPTICAL BREMSSTRAHLUNG IN AN ABSORBING MEDIUM

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The angular distribution and the degree of polarization of radiation generated by an electron moving in an absorbing dielectric and impinging normally on the interface with a ferromagnetic dielectric are calculated with account taken of multiple scattering. The radiation intensity is averaged over all possible trajectories with allowance for the relative phase shifts, resulting from scattering, of the waves emitted from different parts of the path, as is required for a relativistic particle. The results are valid for a thin non-transparent plate, in which the probability of scattering through obtuse angles is small under the condition that the rms scattering angle along the wave absorption path in the medium is smaller than unity.

1. A charged particle crossing the interface between media generates transition radiation,[1] and also bremsstrahlung as a result of scattering. The intensity of the radiation produced when an electron is incident from vacuum unto an absorbing medium, with allowance for multiple scattering, was calculated earlier[2]. When averaging over all possible trajectories, the authors of that paper neglected the relative phase shifts, due to scattering, of the waves emitted from different sections of the path. In this paper we carry out the averaging with allowance for this relative phase shift, which turns out to be essential if the particle velocity is not too small compared with that of light.

In connection with the possibility of realizing in a ferromagnetic dielectric conditions under which the group velocity can be negative ($\epsilon \neq 1$ and $\mu \neq 1$)[3], it is of interest to obtain and discuss the results also in the case of non-unity magnetic permeability of the medium. Since the introduction of the magnetic permeability does not complicate the calculations, we shall not assume from the very outset that $\mu = 1$.

2. We start from formulas (4.4)–(4.9) of[4], which determine the radiation field in the presence of a plane interface between the media, by using directly the law of motion of the radiating charge. It follows from these formulas, in particular, that when a particle is incident from vacuum on a medium in a direction normal to the interface (along the negative z axis), and for arbitrary motion in the medium, the spherical wave of the radiation field in the vacuum is determined by the sum of the following Hertz vectors

$$\Pi_{2\omega\parallel} = -\frac{ev\mathbf{k}\exp\left(i\frac{\omega}{c}R\right)}{\pi\omega^2 R} \times \frac{(\varepsilon - \beta\sqrt{\varepsilon\mu - \sin^2\vartheta_z})\cos\vartheta_z}{(1-\beta^2\cos^2\vartheta_z)(\varepsilon\cos\vartheta_z + \sqrt{\varepsilon\mu - \sin^2\vartheta_z})}, \tag{1}$$

$$\Pi_{1\omega\parallel} = \frac{ie\mathbf{k}\exp\left(i\frac{\omega}{c}R\right)\cos\vartheta_z}{\pi\omega R(\varepsilon\cos\vartheta_z + \sqrt{\varepsilon\mu - \sin^2\vartheta_z})} \times \int_0^T \left[v_z - (v_x\cos\vartheta_x + v_y\cos\vartheta_y)\frac{\sqrt{\varepsilon\mu - \sin^2\vartheta_z}}{\sin^2\vartheta_z}\right] \times \exp\left[i\omega t - i\frac{\omega}{c}(x\cos\vartheta_x + y\cos\vartheta_y + z\sqrt{\varepsilon\mu - \sin^2\vartheta_z})\right]dt, \tag{2}$$

$$\Pi_{1\omega\perp} = \frac{ie\exp\left(i\frac{\omega}{c}R\right)\mu\cos\vartheta_z}{\pi\omega R(\mu\cos\vartheta_z + \sqrt{\varepsilon\mu - \sin^2\vartheta_z})} \times \int_0^T \left[\mathbf{i}v_x + \mathbf{j}v_y + \mathbf{k}(v_x\cos\vartheta_x + v_y\cos\vartheta_y)\frac{\sqrt{\varepsilon\mu - \sin^2\vartheta_z}}{\sin^2\vartheta_z}\right] \times \exp\left[i\omega t - i\frac{\omega}{c}(x\cos\vartheta_x + y\cos\vartheta_y + z\sqrt{\varepsilon\mu - \sin^2\vartheta_z})\right]dt, \tag{3}$$

where $\cos\vartheta_x$, $\cos\vartheta_y$, and $\cos\vartheta_z$ are the direction cosines of the wave vector; x, y, z and v_x, v_y, v_z are the coordinates and velocity components of the particle in the medium ($z < 0$) in a rectangular coordinate system with the z axis directed into the vacuum and the xy plane coinciding with the interface; **i**, **j**, and **k** are unit vectors in the direction of the coordinate axes: R is the distance from the origin to the point of observation; T is the time of particle motion in the medium. The vectors (1) and (2) describe waves polarized in the incidence plane, and the vector (3) waves polarized in the perpendicular plane.

The electromagnetic field is defined in terms of the Hertz vector by means of the formulas

$$\mathbf{E}_\omega = \frac{\omega^2}{c^2}[\mathbf{\Pi}_\omega - \mathbf{n}(\mathbf{n}\mathbf{\Pi}_\omega)], \quad \mathbf{H}_\omega = \frac{\omega^2}{c^2}[\mathbf{n}\mathbf{\Pi}_\omega], \quad (4)^*$$

where **n** is a unit vector in the observation direction.

The formulas for the spectral energy density of the waves polarized in the plane of incidence and in the perpendicular plane, per unit solid angle, which are derivable from (1)—(4), can be conveniently written in the form

$$W_{\mathbf{n}\omega\|} = \frac{e^2\omega^2}{\pi^2c^3}\frac{\sin^2\vartheta_z\cos^2\vartheta_z}{|\varepsilon\cos\vartheta_z + \sqrt{\varepsilon\mu-\sin^2\vartheta_z}|^2} \times\Big\{\frac{v^2|\varepsilon-\beta\sqrt{\varepsilon\mu-\sin^2\vartheta_z}|^2}{\omega^2(1-\beta^2\cos^2\vartheta_z)^2} + \frac{2v}{\omega(1-\beta^2\cos^2\vartheta_z)}\operatorname{Im}(\varepsilon-\beta\sqrt{\varepsilon\mu-\sin^2\vartheta_z})^* \times\int_0^T\Big[v_z-(v_x\cos\vartheta_x+v_y\cos\vartheta_y)\frac{\sqrt{\varepsilon\mu-\sin^2\vartheta_z}}{\sin^2\vartheta_z}\Big] \times\exp\Big[i\omega t - i\frac{\omega}{c}(x\cos\vartheta_x + y\cos\vartheta_y + z\sqrt{\varepsilon\mu-\sin^2\vartheta_z})\Big]dt + 2\operatorname{Re}\int_0^T\Big[v_z-(v_x\cos\vartheta_x+v_y\cos\vartheta_y)\frac{\sqrt{\varepsilon\mu-\sin^2\vartheta_z}^*}{\sin^2\vartheta_z}\Big] \times\exp\Big(2z\frac{\omega}{c}\operatorname{Im}\sqrt{\varepsilon\mu-\sin^2\vartheta_z}\Big)dt\int_0^{T-t}\Big[v_z'-(v_x'\cos\vartheta_x + v_y'\cos\vartheta_y)\frac{\sqrt{\varepsilon\mu-\sin^2\vartheta_z}}{\sin^2\vartheta_z}\Big] \times\exp\Big[i\omega\tau - i\frac{\omega}{c}(\chi\cos\vartheta_x+\eta\cos\vartheta_y + \zeta\sqrt{\varepsilon\mu-\sin^2\vartheta_z})\Big]d\tau\Big\}, \quad (5)$$

$$W_{\mathbf{n}\omega\perp} = \frac{2e^2\omega^2}{\pi^2c^3}\frac{|\mu|^2\cos^2\vartheta_z}{|\mu\cos\vartheta_z+\sqrt{\varepsilon\mu-\sin^2\vartheta_z}|^2\sin^2\vartheta_z} \times\operatorname{Re}\int_0^T(v_x\cos\vartheta_y - v_y\cos\vartheta_x) \times\exp\Big(2z\frac{\omega}{c}\operatorname{Im}\sqrt{\varepsilon\mu-\sin^2\vartheta_z}\Big) \times dt\int_0^{T-t}(v_x'\cos\vartheta_y - v_y'\cos\vartheta_x) \times\exp\Big[i\omega\tau - i\frac{\omega}{c}(\chi\cos\vartheta_x+\eta\cos\vartheta_y + \zeta\sqrt{\varepsilon\mu-\sin^2\vartheta_z})\Big]d\tau, \quad (6)$$

where x, y, z are the coordinates of the particle at the instant t; χ, η, ζ is the change in the coordinates within t and $t+\tau$; v and v′ are the particle velocities at the instant t and at the later instant $t+\tau$, respectively.

3. The problem consists of averaging the wave radiation intensity of all polarizations over all possible particle trajectories. To this end we use the distribution function $w(\mathbf{r}, \theta, t)$, which satisfies the usual kinetic equation, in a Fokker-Planck approximation having the form

$$\frac{\partial w}{\partial t} + \mathbf{v}\frac{\partial w}{\partial \mathbf{r}} = q\Delta_\theta w. \quad (7)$$

Since the velocity is not changed by the scattering, we can, within the framework of applicability of (7), i.e., in scattering through small angles, go over to the angle vectors

$$\theta = \frac{\mathbf{v}-\mathbf{v}_0}{v_0}, \quad \theta' = \frac{\mathbf{v}'-\mathbf{v}_0}{v_0}. \quad (8)$$

The radiation intensity of the waves with different polarizations is averaged over all possible trajectories in the following manner:

$$W_{\mathbf{n}\omega\|} = \frac{e^2\omega^2\beta^2}{\pi^2c}\frac{\sin^2\vartheta\cos^2\vartheta}{|\varepsilon\cos\vartheta+\sqrt{\varepsilon\mu-\sin^2\vartheta}|^2} \times\Big\{\frac{|\varepsilon-\beta\sqrt{\varepsilon\mu-\sin^2\vartheta}|^2}{\omega^2(1-\beta^2\cos^2\vartheta)^2} - \frac{2\operatorname{Im}(\varepsilon-\beta\sqrt{\varepsilon\mu-\sin^2\vartheta})^*}{\omega(1-\beta^2\cos^2\vartheta)} \times\int_0^T dt\int u_1(\theta,t)\Big[1+(\theta\varkappa)\frac{\sqrt{\varepsilon\mu-\sin^2\vartheta}}{\sin^2\vartheta}\Big]d\theta + 2\operatorname{Re}\int_0^T dt\int_0^{T-t}d\tau\int u_0(\theta,t)\,u_2(\theta,\theta',\tau) \times\Big[1+(\theta\varkappa)\frac{\sqrt{\varepsilon\mu-\sin^2\vartheta}^*}{\sin^2\vartheta} + (\theta'\varkappa)\frac{\sqrt{\varepsilon\mu-\sin^2\vartheta}}{\sin^2\vartheta} + (\theta\varkappa)(\theta'\varkappa)\frac{|\varepsilon\mu-\sin^2\vartheta|}{\sin^4\vartheta}\Big]d\theta d\theta'\Big\}, \quad (9)$$

$$W_{\mathbf{n}\omega\perp} = \frac{2e^2\omega^2\beta^2|\mu|^2\operatorname{ctg}^2\vartheta}{\pi^2c|\mu\cos\vartheta+\sqrt{\varepsilon\mu-\sin^2\vartheta}|^2}\operatorname{Re}\int_0^T dt\int_0^{T-t}d\tau\int u_0(\theta,t) \times u_2(\theta,\theta',\tau)[\theta\varkappa][\theta'\varkappa]\,d\theta\,d\theta', \quad (10)$$

*$(\mathbf{n}\Pi_\omega) \equiv \mathbf{n}\cdot\Pi_\omega$, $[\mathbf{n}\Pi_\omega] \equiv \mathbf{n}\times\Pi_\omega$.

where

$$u_0(\theta, t) = \int w_1(\mathbf{r}, \theta, t) \exp\left(2z\frac{\omega}{c}\operatorname{Im}\sqrt{\varepsilon\mu - \sin^2\vartheta}\right) d\mathbf{r}, \qquad (11)$$

$$u_1(\theta, t) = \int w_1(\mathbf{r}, \theta, t) \exp\left[i\omega t - i\frac{\omega}{c}(\mathbf{r}\varkappa + z\sqrt{\varepsilon\mu - \sin^2\vartheta})\right] d\mathbf{r}, \qquad (12)$$

$$u_2(\theta, \theta', \tau) = \int w_2(\rho, \theta, \theta', \tau) \times \exp\left[i\omega t - i\frac{\omega}{c}(\rho\varkappa + \zeta\sqrt{\varepsilon\mu - \sin^2\vartheta})\right] d\rho, \qquad (13)$$

$$\mathbf{r} = \mathbf{i}x + \mathbf{j}y,\ \rho = \mathbf{i}\chi + \mathbf{j}\eta,\ \varkappa = \mathbf{i}\cos\vartheta_x + \mathbf{j}\cos\vartheta_y,\ \vartheta \equiv \vartheta_z, \qquad (14)$$

$w_1(\mathbf{r}, \theta, t)$ and $w_2(\rho, \theta, \theta', \tau)$ are distribution functions with initial conditions

$$w_1(\mathbf{r}, \theta, 0) = \delta(\mathbf{r})\delta(\theta),$$
$$w_2(\rho, \theta, \theta', 0) = \delta(\rho)\delta(\theta - \theta'). \qquad (15)$$

The corresponding initial conditions for the functions $u_0(\theta, t)$, $u_1(\theta, t)$, and $u_2(\theta, \theta', \tau)$ are

$$u_0(\theta, 0) = \delta(\theta),\quad u_1(\theta, 0) = \delta(\theta),$$
$$u_2(\theta, \theta', 0) = \delta(\theta - \theta'). \qquad (16)$$

From the kinetic equation (7) we obtain the following equations for the functions $u_0(\theta, t)$, $u_1(\theta, t)$, and $u_2(\theta, \theta', \tau)$:

$$\frac{\partial u_0}{\partial t} + 2\omega\beta\left(1 - \frac{\theta^2}{2}\right)\operatorname{Im}\sqrt{\varepsilon\mu - \sin^2\vartheta}\, u_0 = q\Delta_\theta u_0; \qquad (17)$$

$$\frac{\partial u_1}{\partial t} - i\omega\left[1 - \beta\theta\varkappa + \beta\sqrt{\varepsilon\mu - \sin^2\vartheta}\left(1 - \frac{\theta^2}{2}\right)\right] u_1 = q\Delta_\theta u_1; \qquad (18)$$

$$\frac{\partial u_2}{\partial \tau} - i\omega\left[1 - \beta\theta'\varkappa + \beta\sqrt{\varepsilon\mu - \sin^2\vartheta}\left(1 - \frac{\theta'^2}{2}\right)\right] u_2 = q\Delta_{\theta'} u_2. \qquad (19)$$

Solutions of these equations, satisfying the initial conditions (16), are

$$u_0(\theta, t) = \exp\left(-\frac{\eta_1\theta^2}{4q}\operatorname{ctg}\eta_1 t - \frac{\eta_1^2 t}{2q} - \ln\sin\eta_1 t + \ln\frac{\eta_1}{4\pi q}\right), \qquad (20)$$

$$u_1(\theta, t) = \exp\left[-\frac{\eta\theta^2}{4q}\operatorname{cth}\eta t + \frac{\eta(1 - \operatorname{ch}\eta t)}{2q\sqrt{\varepsilon\mu - \sin^2\vartheta}\operatorname{sh}\eta t} \times \left(\theta\varkappa + \frac{\sin^2\vartheta}{\sqrt{\varepsilon\mu - \sin^2\vartheta}}\right) + \frac{i\omega\beta t}{2}\frac{\sin^2\vartheta}{\sqrt{\varepsilon\mu - \sin^2\vartheta}} + i\omega t(1 + \beta\sqrt{\varepsilon\mu - \sin^2\vartheta}) - \ln\operatorname{sh}\eta t + \ln\frac{\eta}{4\pi q}\right], \qquad (21)$$

$$u_2(\theta, \theta', \tau) = \exp\left[-\frac{\eta(\theta^2 + \theta'^2)}{4q}\operatorname{cth}\eta\tau + \frac{\eta\theta\theta'}{2q\operatorname{sh}\eta\tau} + \frac{\eta(1 - \operatorname{ch}\eta\tau)}{2q\sqrt{\varepsilon\mu - \sin^2\vartheta}\operatorname{sh}\eta\tau}\left(\theta\varkappa + \frac{\sin^2\vartheta}{\sqrt{\varepsilon\mu - \sin^2\vartheta}} + \theta'\varkappa\right) + i\omega\tau(1 + \beta\sqrt{\varepsilon\mu - \sin^2\vartheta}) + \frac{i\omega\beta\tau}{2}\frac{\sin^2\vartheta}{\sqrt{\varepsilon\mu - \sin^2\vartheta}} - \ln\operatorname{sh}\eta\tau + \ln\frac{\eta}{4\pi q}\right], \qquad (22)$$

where

$$\eta_1 = \sqrt{4\omega\beta q \operatorname{Im}\sqrt{\varepsilon\mu - \sin^2\vartheta}},\quad \eta = \sqrt{2i\omega\beta q\sqrt{\varepsilon\mu - \sin^2\vartheta}}. \qquad (23)$$

Substituting the functions $u_0(\theta, t)$, $u_1(\theta, t)$, and $u_2(\theta, \theta', \tau)$ in (5) and (6), and integrating over θ and θ', we get

$$W_{n\omega\parallel} = \frac{e^2\omega^2\beta^2}{\pi^2 c}\frac{\sin^2\vartheta\cos^2\vartheta}{|\varepsilon\cos\vartheta + \sqrt{\varepsilon\mu - \sin^2\vartheta}|^2} \times \left\{\frac{|\varepsilon - \beta\sqrt{\varepsilon\mu - \sin^2\vartheta}|^2}{\omega^2(1 - \beta^2\cos^2\vartheta)^2} - \frac{2\operatorname{Im}(\varepsilon - \beta\sqrt{\varepsilon\mu - \sin^2\vartheta})^*}{\omega(1 - \beta^2\cos^2\vartheta)} \times \int_0^T \frac{dt}{\operatorname{ch}^2\eta t}\exp\left[\frac{\eta}{4q}\frac{\sin^2\vartheta}{\varepsilon\mu - \sin^2\vartheta}(\eta t - \operatorname{th}\eta t) + i\omega t(1 + \beta\sqrt{\varepsilon\mu - \sin^2\vartheta})\right]\right.$$
$$+ 2\operatorname{Re}\int_0^T dt\int_0^{T-t} d\tau\frac{\eta_1^2\cos\eta_1 t}{p^2\operatorname{ch}^2\eta\tau\sin^2\eta_1 t} \times \left(2i\frac{\operatorname{Im}\sqrt{\varepsilon\mu - \sin^2\vartheta}}{\sqrt{\varepsilon\mu - \sin^2\vartheta}}\operatorname{ch}\eta\tau + \frac{\eta_1\operatorname{ctg}\eta_1 t}{p}\frac{|\varepsilon\mu - \sin^2\vartheta|}{\varepsilon\mu - \sin^2\vartheta} + \frac{2q}{\eta_1}\frac{|\varepsilon\mu - \sin^2\vartheta|}{\sin^2\vartheta}\operatorname{tg}\eta_1 t\right)\exp\left[\frac{\eta}{4q}\frac{\sin^2\vartheta}{\varepsilon\mu - \sin^2\vartheta}\right.$$
$$\left.\left.\times\left(\eta\tau - \frac{\eta_1}{p}\operatorname{ctg}\eta_1 t\operatorname{th}\eta\tau\right) + i\omega\tau(1 + \beta\sqrt{\varepsilon\mu - \sin^2\vartheta}) - \frac{\eta_1^2 t}{2q}\right]\right\}, \qquad (24)$$

$$W_{n\omega\perp} = \frac{4e^2\omega^2\beta^2 q|\mu|^2\cos^2\vartheta}{\pi^2 c|\mu\cos\vartheta + \sqrt{\varepsilon\mu - \sin^2\vartheta}|^2} \times \operatorname{Re}\int_0^T dt\int_0^{T-t} d\tau\frac{\eta_1}{p^2\operatorname{ch}^2\eta\tau\sin\eta_1 t} \times \exp\left[\frac{\eta}{4q}\frac{\sin^2\vartheta}{\varepsilon\mu - \sin^2\vartheta}\left(\eta\tau - \frac{\eta_1}{p}\operatorname{ctg}\eta_1 t\operatorname{th}\eta\tau\right) + i\omega\tau(1 + \beta\sqrt{\varepsilon\mu - \sin^2\vartheta}) - \frac{\eta_1^2 t}{2q}\right], \qquad (25)$$

where

$$p = \eta\operatorname{th}\eta\tau + \eta_1\operatorname{ctg}\eta_1 t, \qquad (26)$$

we use the notation of (23), and we assume that

$$\operatorname{Im}\varepsilon \geqslant 0,\quad \operatorname{Im}\mu \geqslant 0,\quad \operatorname{Im}\sqrt{\varepsilon\mu - \sin^2\vartheta} \geqslant 0. \qquad (27)$$

In an earlier paper [5] we calculated the angular distribution of the intensity of the bremsstrahlung at small angles to the initial direction of motion of a relativistic charged particle moving through a plate, at frequencies much higher than optical. Let us compare the consequences resulting from (24) and (25) with the results (22) and (21) obtained there. To this end we put $\mu = 1$, $1 - \epsilon = \omega_0^2/\omega^2 \ll 1$, $\vartheta \ll 1$, and $1 - \beta^2 \ll 1$. We recall that here ϑ is the angle that the wave vector makes with the positive z direction ($\vartheta < \pi/2$). But the particle moves in the negative z direction. Therefore the comparison must be made after first changing the direction of particle motion, i.e., replacing β by $-\beta$. Then the angle between the quantum emission direction and that of the particle motion will be acute, just as in [5]. We note that reversal of the sign of the velocity corresponds to motion of a particle along the realized trajectory in the opposite direction. At the instant t = 0, the particle is emitted from the medium into the vacuum and the conditions (15) are no longer initial but final. The radiation intensity is in this case the average over all possible trajectories relative to the realized direction of particle motion in vacuum. Neglecting the transition radiation, the integration with respect to τ should be extended to infinite, and the first two terms in (24) should be neglected. The results then coincide with those obtained in [5]. Thus, averaging with respect to the realized direction of particle motion in vacuum leads to the same results as the averaging with respect to the initial direction of motion.

The results (24) and (25) can be of interest, for example, in connection with the possibility of realizing conditions under which the group velocity is negative when $\epsilon \neq 1$ and $\mu \neq 1$ [3]. It is known that a medium in thermodynamic equilibrium absorbs electromagnetic waves. Accordingly, the inequalities (27) should be satisfied. If the imaginary part of the product $\epsilon\mu$ is negative, then the wave that carries energy away from the radiating system continuously lags the radiation source in phase. In this case, when the damping is small, the propagation velocity of the wave packet, the group velocity, is called negative. When the damping is appreciable, the wave packet spreads out rapidly and there is no velocity of energy propagation as such here. It is possible, however to define an energy propagation direction, at any damping, for both a wave packet and for a monochromatic wave. Therefore, besides the concept "negative group velocity," it is convenient to introduce a different concept, the application of which would not be limited by the value of the damping, for example "negative direction of energy propagation."

It follows from the last inequality of (27) that $\mathrm{Re}\,(\epsilon\mu - \sin^2\vartheta)^{1/2} < 0$ in the region of frequencies with negative direction of energy propagation ($\mathrm{Im}\,\epsilon\mu < 0$). The author has shown elsewhere [6] that the Vavilov-Cerenkov radiation generated in this region of frequencies makes an obtuse angle with the particle velocity, and when the particle moves from vacuum into the medium the radiation passes through the interface into the vacuum. The sign of the energy propagation direction greatly influences all radiation processes, including bremsstrahlung. The radiation singularities predicted by the theory can be used to observe the regions of frequencies with negative energy propagation direction.

At frequencies of the order of optical and lower, the results (24) and (25) are valid when a non-transparent ferrodielectric plate is used, i.e., if the thickness is much larger than the wave absorption path:

$$d > c/\omega\,\mathrm{Im}\sqrt{\varepsilon\mu - \sin^2\vartheta}. \qquad (28)$$

Inasmuch as $T = d/v$, it follows therefore that $\omega\beta T\,\mathrm{Im}\,(\epsilon\mu - \sin^2\vartheta)^{1/2} > 1$, and the results of integration with respect to time do not depend on T. It is assumed, in addition, that during the time that the wave traverses the absorption path the rms multiple-scattering angle is small compared with unity

$$4qT_0 \ll 1, \quad T_0 = 1/\omega\beta\,\mathrm{Im}\sqrt{\varepsilon\mu - \sin^2\vartheta}. \qquad (29)$$

Satisfaction of this inequality ensures, within a wide range, sufficiently rapid convergence of the expansion of formulas (24) and (25) in powers of q. Confining ourselves to the linear terms of the expansion of the integrands and integrating with respect to time, we obtain

$$\begin{aligned} W_{n\omega\parallel} &= \frac{e^2\beta^2}{\pi^2 c}\frac{\sin^2\vartheta\cos^2\vartheta}{|\varepsilon\cos\vartheta + \sqrt{\varepsilon\mu - \sin^2\vartheta}|^2} \\ &\times \Bigg\{ \frac{|(\varepsilon - 1)(1 + \beta\sqrt{\varepsilon\mu - \sin^2\vartheta}) - \beta^2(\varepsilon\mu - 1)|^2}{(1 - \beta^2\cos^2\vartheta)^2\,|1 + \beta\sqrt{\varepsilon\mu - \sin^2\vartheta}|^2} \\ &+ \frac{4\beta q\,|\varepsilon - \beta\sqrt{\varepsilon\mu - \sin^2\vartheta}|^2}{\omega(1 - \beta^2\cos^2\vartheta)} \\ &\times \mathrm{Im}\,\frac{2\sqrt{\varepsilon\mu - \sin^2\vartheta}(1 + \beta\sqrt{\varepsilon\mu - \sin^2\vartheta}) + \beta\sin^2\vartheta}{(\varepsilon - \beta\sqrt{\varepsilon\mu - \sin^2\vartheta})(1 + \beta\sqrt{\varepsilon\mu - \sin^2\vartheta})^4} \\ &+ \frac{q}{\omega J P^4}\Bigg[P^3\left(2 + \frac{|\varepsilon\mu - \sin^2\vartheta|}{\sin^2\vartheta}\right) \\ &- 2P^2(1 + R) + 16PJ^2R(1 + R) + \\ &+ (P^2 + 12PJ^2 - 16J^4)\beta^2\sin^2\vartheta\Bigg]\Bigg\}, \end{aligned} \qquad (30)$$

$$J = \beta \operatorname{Im} \sqrt{\varepsilon\mu - \sin^2\vartheta}, \quad R = \beta \operatorname{Re} \sqrt{\varepsilon\mu - \sin^2\vartheta}, \quad P = J^2 + (1+R)^2:$$

$$W_{n\omega\perp} = \frac{e^2\beta q\,|\mu|^2\cos^2\vartheta}{\pi^2\omega c\,|\mu\cos\vartheta + \sqrt{\varepsilon\mu - \sin^2\vartheta}\,|^2\,|1+\beta\sqrt{\varepsilon\mu-\sin^2\vartheta}\,|^2 \operatorname{Im}\sqrt{\varepsilon\mu - \sin^2\vartheta}}. \tag{31}$$

If condition (29) is satisfied, the next terms of the expansion results (24) and (25) in powers of q can be neglected, with the exception of a small region of angles in the vicinity of the Vavilov-Cerenkov refraction angle

$$\vartheta_r = \arc\sin\left(\left|\sqrt{\varepsilon\mu\beta^2 - 1}\right|\right)/\beta \tag{32}$$

(with $\beta^2|\epsilon\mu| > 1$, $0 < |\sqrt{\epsilon\mu\beta^2 - 1}| < \beta$, and $\operatorname{Im}\sqrt{\epsilon\mu - \sin\vartheta_r} \ll \operatorname{Re}|\sqrt{\epsilon\mu - \sin^2\vartheta_r}|$) at frequencies with negative group velocity ($\operatorname{Im}\epsilon\mu < 0$). To consider the angular distribution and the degree of polarization of the radiation in the vicinity of this angle, it would be necessary to carry out in (24) and (25) numerical integration with respect to time.

In the nonrelativistic approximation, the results (30) and (31) become much simpler and take the form

$$W_{n\omega\parallel} = \frac{e^2}{\pi^2\omega c}\frac{\cos^2\vartheta}{|\varepsilon\cos\vartheta + \sqrt{\varepsilon\mu - \sin^2\vartheta}\,|^2}\Big[\omega\beta^2|\varepsilon - 1|^2\sin^2\vartheta + 8\beta^3 q|\varepsilon|^2\sin^2\vartheta \operatorname{Im}\frac{\sqrt{\varepsilon\mu - \sin^2\vartheta}}{\varepsilon} + \frac{\beta q\,|\varepsilon\mu - \sin^2\vartheta|}{\operatorname{Im}\sqrt{\varepsilon\mu - \sin^2\vartheta}}\Big], \tag{33}$$

$$W_{n\omega\perp} = \frac{e^2\beta q\,|\mu|^2\cos^2\vartheta}{\pi^2\omega c\,|\mu\cos\vartheta + \sqrt{\varepsilon\mu - \sin^2\vartheta}\,|^2 \operatorname{Im}\sqrt{\varepsilon\mu - \sin^2\vartheta}}. \tag{34}$$

We note that the coefficient q in the kinetic equation (7) is equal to one-quarter of the rms multiple-scattering angle per unit time, and for a nonrelativistic particle it is inversely proportional to the cube of its velocity. Thus, the last term in formula (33) for the intensity of radiation of waves polarized in the plane of incidence is inversely proportional to the particle energy (bremsstrahlung), and the second term does not depend on the energy. It takes into account the interference of the field generated on the path in vacuum with that part of the radiation field which is produced on the path in the medium and is perturbed by the multiple scattering. The first term is proportional to the particle energy and corresponds to transition radiation.

Boersch et al.[7] investigated recently the angular and spectral distributions of the radiation intensity of electrons with energy V = 30 keV ($\beta = 0.33$) incident on a metallic target. By varying the accelerating potential in the vicinity of 30 keV, they ascertained that the observed radiation consists of two parts. The radiation intensity of one of them is proportional to the electron energy and can be satisfactorily explained by transition-radiation theory. The second part of the intensity is inversely proportional to the electron energy, and was therefore, naturally, identified with bremsstrahlung. It was observed that when a silver target is bombarded with electrons this part of the radiation intensity forms a sharp peak at $\lambda = 3250$ Å, where the silver has relatively small absorption. Comparison with the results of a theoretical paper [2] has shown that the spectral and angular distributions of that part of the activity which is inversely proportional to the particle energy is well described by the theory, and that the experimentally observed radiation intensity is appreciably larger than the theoretical one. Thus, for example, if we use the value of the rms multiple-scattering angle per unit path, calculated in [8], namely,

$$\langle\theta\rangle^2 = \frac{N\cdot 8\pi e^4 Z(Z+1)(1-\beta^2)}{m^2c^4\beta^4} \times \left[\ln\frac{241\beta}{Z^{1/3}(1-\beta^2)} - 1 - \frac{\beta^2}{4}\right], \tag{35}$$

and start from the measured optical properties of silver reported in [9], then the discrepancy is by a factor of five. If we start from data of other measurements of the optical properties of silver [10], the discrepancy turns out to be even larger.

In the energy region under consideration, formula (35) agrees qualitatively with the empirical formula for the most probable multiple-scattering angle for thin plates, as given by Bothe [11]:

$$\theta_\lambda = \frac{8}{V}\frac{V+511}{V+1022}Z\left(\frac{\rho x}{A}\right)^{1/2}, \tag{36}$$

where V is the electron kinetic energy in keV, x the layer thickness in microns, Z the atomic number of the element, A the atomic weight, and ρ the density in g/cm^3. However, formula (36) corresponds to somewhat larger values of the rms multiple-scattering angle. For 30-keV electrons, the theory [8] yields $\langle\theta^2\rangle = 4.8$ rad/μ, whereas the empirical formula gives a value 1.7 times larger, $\langle\theta^2\rangle = 2\theta_\lambda = 8$ rad/μ. Thus, if we start from the experimental value of the rms multiple-scattering angle, then the discrepancy is smaller, and the observed radiation intensity turns out to be three times larger than that calculated from the results of [2].

In comparing the experimental data of [7] with our results, it is necessary first to put $\mu = 1$ in (30) and (31). We then get

$$W_{n\omega\parallel} = \frac{e^2\beta^2}{\pi^2 c} \frac{\sin^2\vartheta\cos^2\vartheta}{|\varepsilon\cos\vartheta + \sqrt{\varepsilon - \sin^2\vartheta}|^2}$$

$$\times\left\{\frac{|(\varepsilon-1)(1+\beta\sqrt{\varepsilon-\sin^2\vartheta}-\beta^2)|^2}{(1-\beta^2\cos^2\vartheta)^2|1+\beta\sqrt{\varepsilon-\sin^2\vartheta}|^2}\right.$$

$$+\frac{4\beta q|\varepsilon-\beta\sqrt{\varepsilon-\sin^2\vartheta}|^2}{\omega(1-\beta^2\cos^2\vartheta)}$$

$$\times \operatorname{Im}\frac{2\sqrt{\varepsilon-\sin^2\vartheta}(1+\beta\sqrt{\varepsilon-\sin^2\vartheta})+\beta\sin^2\vartheta}{(\varepsilon-\beta\sqrt{\varepsilon-\sin^2\vartheta})(1+\beta\sqrt{\varepsilon-\sin^2\vartheta})^4}$$

$$+\frac{q}{\omega J P^4}\left[P^3\left(2+\frac{|\varepsilon-\sin^2\vartheta|}{\sin^2\vartheta}\right)\right.$$

$$-2P^2(1+R)+16PJ^2R(1+R)$$

$$\left.\left.+(P^2+12PJ^2-16J^4)\beta^2\sin^2\vartheta\right]\right\}, \tag{37}$$

$$J = \beta\operatorname{Im}\sqrt{\varepsilon-\sin^2\vartheta}, \qquad R = \beta\operatorname{Re}\sqrt{\varepsilon-\sin^2\vartheta},$$
$$P = \beta^2(\operatorname{Im}\sqrt{\varepsilon-\sin^2\vartheta})^2 + (1+\beta\operatorname{Re}\sqrt{\varepsilon-\sin^2\vartheta})^2;$$

$$W_{n\omega\perp} = \frac{e^2\beta q\cos^2\vartheta}{\pi^2\omega c|\cos\vartheta+\sqrt{\varepsilon-\sin^2\vartheta}|^2|1+\beta\sqrt{\varepsilon-\sin^2\vartheta}|^2\operatorname{Im}\sqrt{\varepsilon-\sin^2\vartheta}}, \tag{38}$$

with

$$q \ll \omega\beta\operatorname{Im}\sqrt{\varepsilon-\sin^2\vartheta}. \tag{39}$$

For waves polarized in a plane perpendicular to the incidence plane, and in the nonrelativistic approximation for waves polarized in the plane of incidence, our results coincide with those obtained in [2] (apart from a relatively small interference term). For the reason mentioned at the beginning of this article, however, formula (37) gives values that differ noticeably from the corresponding formula (8) of [2] if the particle velocity is not very small compared with that of light. A comparison shows that for a silver target the maximum intensity of the bremsstrahlung (at $\lambda = 3250$ Å) of waves polarized in the incidence plane should be, for $\beta = 0.33$, larger by 1.72 times than obtained from the results of [2], and the summary intensity should be 1.4 times larger. Thus, if we start from the empirical formula (36) and the measured optical properties of silver given in [9], then the discrepancy between the theory and experiments on bremsstrahlung is by a factor of two. On the other hand, if we start from other measurement data [10] on the properties of silver, then the experimental value of the intensity exceeds the theoretical even more.

We note that an electromagnetic wave with wavelength $\lambda = 3250$ Å in vacuum is absorbed in silver on a path of approximately one-tenth of a micron. On this path, the rms multiple-scattering angle of 30-keV electrons is, according to the empirical formula (36), 0.8 rad/μ. This value is at the borderline of the applicability of the results of (37) and (38), where the theory gives only a qualitative description of the experiments. Since the rms multiple-scattering angle decreases with increasing particle energy, one should expect better agreement with the experimental data for electrons with higher energies. However, if the target is so thick that the electrons become scattered in it into obtuse angles (for example, such as the 5-μ target used in [7]), then the radiation observed in this case should be more intense than called for by formulas (37) and (38), since the latter were obtained in the small-scattering-angle approximation and therefore describe only a part of the intensity. Since such electrons constitute a noticeable fraction in a thick target, and since they lose energy on returning to the surface, so that the bremsstrahlung per unit path becomes more intense, their contribution to the observed bremsstrahlung is apparently not small. To check on these assumptions, one could use, for example, targets several tenths of a micron thick and electrons with energy 40—60 keV.

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