

VISCOSITY AND MELTING POINT OF ALUMINUM, LEAD, AND SODIUM CHLORIDE  
SUBJECTED TO SHOCK COMPRESSION

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The viscosity of aluminum, lead, and sodium chloride was determined at pressures of 100–2000 kbar by an experimental investigation of the development of disturbances in the shock-wave front. It is shown that, when complete geometrical similarity is observed in the experimental arrangements with respect to the wavelength of the disturbances  $\lambda$ , the influence of the viscosity reduces to a phase shift between the curves (disturbance amplitude–time) showing the development of disturbances having various  $\lambda$ . However, under certain thermodynamic conditions it is found that the law of the development of a disturbance is independent of  $\lambda$ . This is associated with the melting of the substance behind the shock-wave front. The experimental material obtained makes it possible to estimate the melting curve of the investigated substances up to pressures of 1000–2000 kbar. The estimated melting curve of sodium chloride, obtained by the proposed method, coincides with the results of independent measurements.

## INTRODUCTION

A method for investigating the viscosity of substances behind a shock-wave front has been proposed in<sup>[1]</sup>; it is based on an experimental investigation of the development of small disturbances in the front. Experiments have shown<sup>[1]</sup> that, when complete geometrical similarity is observed in the experimental arrangements with respect to the wavelength of disturbances  $\lambda$ , the influence of the viscosity reduces to a phase shift between the curves (amplitude–time) showing the development of perturbations having various  $\lambda$ . The geometrical similarity in the experimental arrangements guarantees also the absence of possible systematic errors. The first results of an investigation of the viscosity of some substances behind a shock-wave front have been reported in<sup>[1]</sup>.

We report here the results of an experimental investigation of the viscosity of aluminum, lead, and sodium chloride in a wide range of densities and temperatures.

## EXPERIMENTAL RESULTS

The experiments were carried out using an arrangement similar to that described in<sup>[1]</sup>, and shown schematically in Fig. 1. We investigated mainly the development of disturbances of two wavelengths— $\lambda_1 = 1$  cm and  $\lambda_2 = 2$  cm—of relative initial amplitude  $2\pi a_0/\lambda = 0.63$ –1.89 in two geo-

metrically similar, with respect to  $\lambda$ , experimental arrangements.

The dynamic viscosity  $\eta$  was calculated from the formula

$$\eta = \rho D \Delta x / k (1/\lambda_1 - 1/\lambda_2),$$

where  $\rho$  is the density of the substance behind an unperturbed front of a shock wave;  $D$  is the wave velocity of an unperturbed shock-wave front;  $k$  is a constant calculated as described in<sup>[1]</sup>;  $\Delta x$  is the phase shift of the curves showing the development of disturbances with  $\lambda_1$  and  $\lambda_2$ , plotted in the coordinates  $y = a(t)/a_0$  and  $x = S(t)/\lambda$  [ $a(t)$  and  $a_0$  are, respectively, the running and initial disturbance amplitude,  $S$  is the path traveled by a shock wave]. This formula is taken from<sup>[1]</sup>,

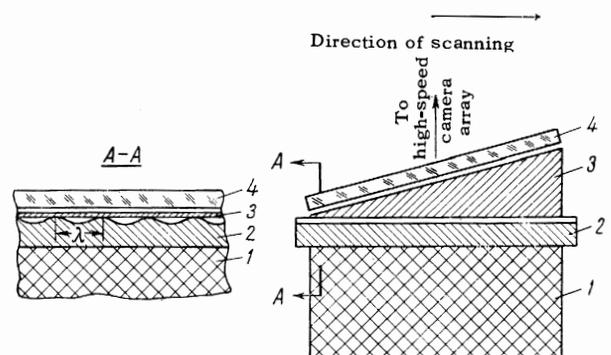


FIG. 1. Experimental arrangement: 1) high-explosive charge; 2) 3) a disk and a wedge of the material under investigation; 4) Perspex plate.

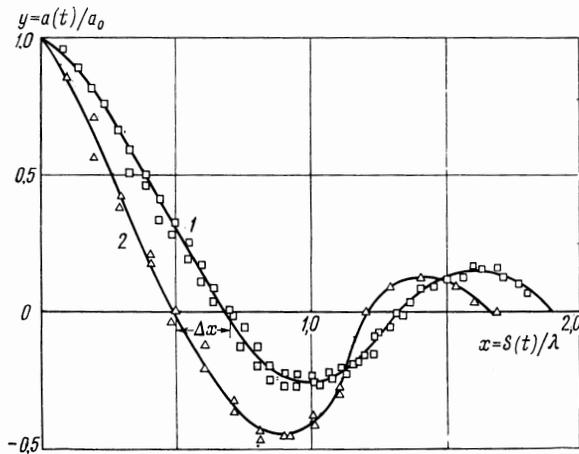


FIG. 2. Experimental curves showing the development of disturbances in a shock-wave front in aluminum ( $m = 1$ ) at  $p = 310$  kbar: 1)  $\lambda = 2$  cm,  $a_0/\lambda = 0.139$ ; 2)  $\lambda = 1$  cm,  $a_0/\lambda = 0.139$ .  $\Delta x$  is the phase shift between the experimental curves.

where it was found that the phase shift between the curves showing the development of disturbances in a shock-wave front, calculated with and without allowance for the viscosity, is inversely proportional to the Reynolds number.

Figures 2 and 3 give typical curves showing the development of disturbances in a shock-wave front. The experimentally observed value of  $\Delta x$  did not exceed 0.3 for the investigated  $\lambda$ .

The table (rows 1, 2) lists some of the parameters ( $p$  is the pressure,  $\sigma$  is the compression, and  $T$  is the temperature) of aluminum and lead shock-compressed by pressures of 310 and 350 kbar, respectively; the values of the dynamic viscosity  $\eta$  are also included in the table. The values of  $\eta$  are given together with the maximum error, due to inaccuracy of the determination of the value of  $\Delta x$ . In the calculation of the parameters of a given substance behind a shock-wave front, we used the equations of state for aluminum and lead given by Kormer et al.<sup>[2]</sup> and Al'tshuler et al.<sup>[3]</sup>

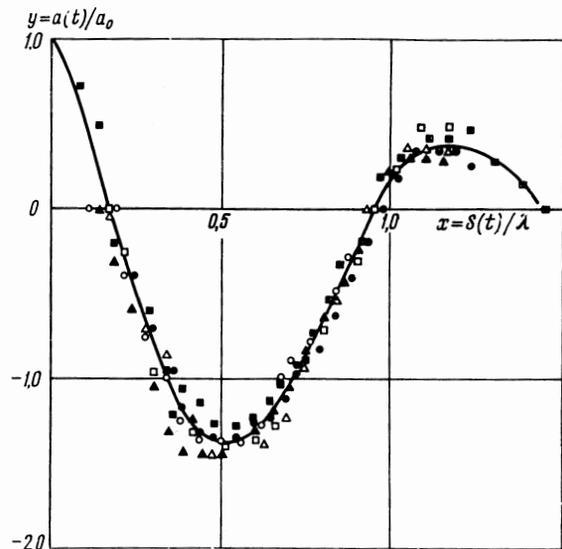


FIG. 3. Experimental curves showing the development of disturbances in a shock-wave front in aluminum ( $m = 1.43$ ) at  $p = 240$  kbar. Symbols representing the plots of the experimental data: open squares, circles and triangles for  $\lambda = 1$  cm,  $a_0/\lambda = 0.06$ ; black ones for  $\lambda = 2$  cm,  $a_0/\lambda = 0.06$ .

The table also contains the results of our investigation of the viscosity of aluminum and lead at higher temperatures and densities. Aluminum was investigated more fully and in some experiments the initial density was less than the normal density by a factor  $m$  ( $m = \rho_0/\rho_{00}$ ). This made it possible to investigate the viscosity of aluminum in the density range from 2.38 to 4.98 g/cm<sup>3</sup> at temperatures from 630 to 10 100°K; in these experiments, the pressure behind the shock-wave front ranged from 120 to 2020 kbar.

The decay of disturbances in the shock-wave front in sodium chloride was investigated in two cases:  $p = 240$  kbar ( $\sigma = 1.44$ ,  $T = 1550^\circ\text{K}$ ,  $m = 1.05$ ), and  $p = 205$  kbar ( $\sigma = 1.31$ ,  $T = 2620^\circ\text{K}$ ,  $m = 1.3$ ). To calculate the parameters of sodium chloride behind the shock-wave front, we used the equation of state given in<sup>[4]</sup>. Experiments showed

Substance	$m$	$p$	$\sigma$	$T, ^\circ\text{K}$	$\eta \cdot 10^{-4}, \text{P}$	Point in Fig. 4.
Al	1	310	1,26	630	$2 \pm 0,5$	1
Pb	1	350	1,36	1400	$3,7 \pm 1,4$	—
Al	1	680	1,42	1600	$10 \pm 4$	2
	1	1050	1,56	3500	$7 \pm 2$	3
	1	2020	1,83	10100	$< 2$	4
	1,23	275	1,18	1700	$1 \pm 0,5$	5
	1,43	245	1,13	2600	$< 0,2$	6
	4	120	0,87	4600	$< 0,2$	7
Pb	1	410	1,38	1700	$15 \pm 2$	—
	1	1240	1,65	7000	$< 30$	—
	1	2500	2,06	20000	$< 13$	—

that, in the former case,  $\eta = (2 \pm 1) \times 10^5$  Poises (curves of the type shown in Fig. 2), while in the second case  $\eta < 10^4$  P (curves of the type shown in Fig. 3).

## DISCUSSION OF RESULTS

It follows from the data given in the table that the values of  $\eta$  for aluminum and lead behind the shock-wave front are practically equal at pressures of 310 and 350 kbar and they amount to  $(2-4) \times 10^4$  P. The weak dependence of the viscosity on the nature of the substance has been reported earlier by Il'yushin<sup>[5]</sup> and Popov.<sup>[6]</sup> They have shown that, under dynamic loading conditions, characterized by deformation rates  $\dot{\epsilon} = 6 \times 10^2 - 6 \times 10^3 \text{ sec}^{-1}$ , aluminum and steels of various grades behave as viscous liquids with  $\eta = (3-4) \times 10^5$  P. It can be shown that, in our case, the substance is deformed behind the disturbed shock-wave front at a rate of the order of  $\dot{\epsilon} = 4\pi^2 a_0 D / \lambda^2$ . In our experiments, the rate of deformation was  $\dot{\epsilon} \approx 10^7 \text{ sec}^{-1}$ . Thus, the viscosities of aluminum and lead were practically identical at a deformation rate  $\dot{\epsilon} \approx 10^7 \text{ sec}^{-1}$  at pressures of 310-350 kbar behind a shock-wave front.

It was reported in<sup>[1]</sup> that the dynamic viscosity of aluminum remained constant, within the limits of the experimental scatter of results, at a pressure of 310 kbar when the deformation rate  $\dot{\epsilon}$  varied from  $4 \times 10^5$  to  $8 \times 10^6 \text{ sec}^{-1}$ . Although it would be desirable to extend the range of deformation rates in further investigations—and, particularly, to increase the range to the rates used in<sup>[5,6]</sup>, the constancy of the viscosity of aluminum within the limits of the experimental error, in the range of deformation rates used by us, does suggest that the viscosity of aluminum, calculated from the decay of disturbances in a shock-wave front, is independent of the rate of deformation, the viscosity is Newtonian.

The same conclusion can be arrived at in a different way. In the viscoplastic body model,<sup>[5-7]</sup> the viscosity of a substance in the plastic state is independent of the deformation rate. We shall show that the turbulent flow behind a shock-wave front considered here makes aluminum plastic.

The order of magnitude of the deformation behind a disturbed shock-wave front is given by  $\epsilon = 2\pi a_0 / \lambda$ . In the present experiments, and those reported in<sup>[1]</sup>, the deformation ranged from 29 to 174%. It is known that, even at very high values of  $\dot{\epsilon}$ , metals can support elastic deformation not higher than 1%.<sup>[8]</sup> Consequently, there is every reason to assume that, in the turbulent flow behind

a shock-wave front, aluminum becomes plastic (if the viscoplastic body model is valid), and therefore its viscosity is independent of the deformation rate.

Hence, we may conclude that the viscosity of aluminum at 310 kbar is an order of magnitude smaller than the viscosity reported in<sup>[5,6]</sup> solely because of a considerable difference between temperatures used in the experiments being compared.

The data presented in the table show that when the shock-wave pressure is increased from 310 to 1050 kbar, the viscosity of aluminum does not change greatly and does not exceed the value of  $10^5$  P. The most interesting result is that, under certain thermodynamic conditions behind the shock-wave front, the law of development of disturbances in the front is independent of the wavelength of disturbances. In this case, as mentioned in<sup>[1]</sup>, the curves showing the development of disturbances are identical, within the limits of the experimental error, for different  $\lambda$  (cf., for example, Fig. 3), which indicates a strong change in the viscosity of the substance. It was difficult to give the exact value of the viscosity since the apparatus used in our experiments did not resolve a value of  $\Delta x$  smaller than 0.02-0.03, which corresponded, in the experiments carried out (in the case of aluminum), to a viscosity not higher than  $10^4$  P or, in some experiments, not higher than  $10^3$  P.

Whenever the experimental curves for different values of  $\lambda$  coincided, the depth of the first minimum in the experimental curves was greater than unity (cf. Fig. 3). We were unable to explain quantitatively this observation, which was evidently related to the existence, right from the beginning, of large pressure gradients behind a disturbed shock-wave front due to large initial disturbances.

This phenomenon was not very important in the present case because the experiments showed that the coincidence of the curves for different  $\lambda$  was independent of the initial disturbance amplitude when this amplitude changed by a factor of at least 2 (the experiments were carried out on aluminum for  $m = 4$ ).

The fall in the viscosity of a substance behind a shock-wave front may be due to the melting behind the front.<sup>[1]</sup> Assuming such melting and using the results given in the table, we shall estimate the phase equilibrium (solid-liquid) curve of aluminum behind a shock-wave front. Figure 4 shows, in the T-p plane, the shock adiabats of aluminum of different initial densities;<sup>[2]</sup> the points which represent the experimental conditions. When the transition is made from points 1

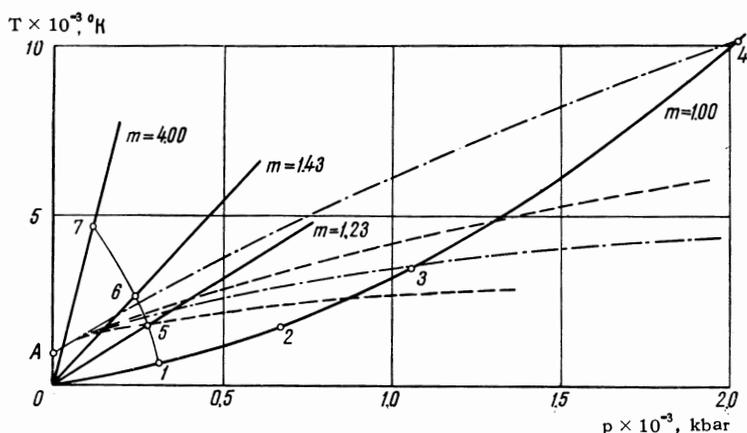


FIG. 4. T-p diagram for shock-compressed aluminum of various initial densities  $m$ : 1)-7) curve obtained using a single high-explosive charge; [2] A is the melting point of aluminum under normal pressure; the chain curves represent the limits of the range of possible positions of the melting curve in accordance with the data reported in the present paper, while dashed curves delimit the range of possible positions of the melting curve in accordance with calculations reported in [11].

and 5 to points 6 and 7, and also from point 3 to 4, the curves showing the development of disturbances having various  $\lambda$  are found to coincide; the viscosity decreases, at least by a factor of 5. Consequently, we may assume that the melting curve of aluminum in the T-p plane intersects curves 1-7 and 1-4 between 5 and 6 and between 3 and 4, respectively.

We shall now use the semi-empirical equation of Simon,<sup>[9]</sup> which relates the melting temperature and pressure:

$$p/a' = (T/T_0)^c - 1,$$

where  $T_0$  is the melting point under normal pressure, and  $T$  is the melting under a pressure  $p$ ;  $a'$  and  $c$  are some constants. The product  $ca'$  determines the initial slope of the melting curve in the T-p plane and, according to Gonikberg et al.,<sup>[10]</sup>  $ca' = 145$  kbar for aluminum. Using this value and bearing in mind that at  $m = 1$  aluminum melts at a pressure not lower than  $p = 1050$  kbar (point 3 in Fig. 4) and not higher than  $p = 2020$  kbar (point 4 in Fig. 4), we can determine the unknown constants  $c$  and  $a'$ , corresponding to the two extreme positions of the melting curve of aluminum in the T-p plane. Calculations show that, in this case,  $c$  lies within the limits  $1.2 < c < 2.1$  and correspondingly  $a'$  lies within the range  $121$  kbar  $> a' > 69.1$  kbar.

The region through which the melting curve passes thus agrees with points 5 and 6.<sup>1)</sup>

Figure 4 shows also the results of theoretical estimates of the melting curve of aluminum, taken from Urlin's work.<sup>[11]</sup> The experimental and calculated results overlap (Fig. 4).

Using the values obtained for  $\eta$  for nonporous lead, we may assume that the melting of lead be-

hind a shock-wave front takes place in the pressure range 410-1240 kbar.

To determine the melting curve more exactly, further theoretical and experimental investigations are needed and, in particular the accuracy of the experimental values of the viscosity must be higher and estimates of viscosity of molten metals at pressures of the order of 1000 kbar and higher will have to be obtained. Nevertheless, the proposed method of deducing the phase state of a substance behind a shock-wave front is quite reliable. This is indicated by the agreement of the estimates of the phase state of sodium chloride behind a shock-wave front, obtained from the decay of disturbances, with the results of independent experiments.<sup>[4]</sup> Figure 5 shows the phase equilibrium curve of sodium chloride (curve CBD) in the T-p plane and the dynamic adiabats for  $m = 1, 1.05, \text{ and } 1.3$ .<sup>[4]</sup> The phase equilibrium curve of sodium chloride was plotted in<sup>[4]</sup> using Simon's formula with coefficients found by an ex-

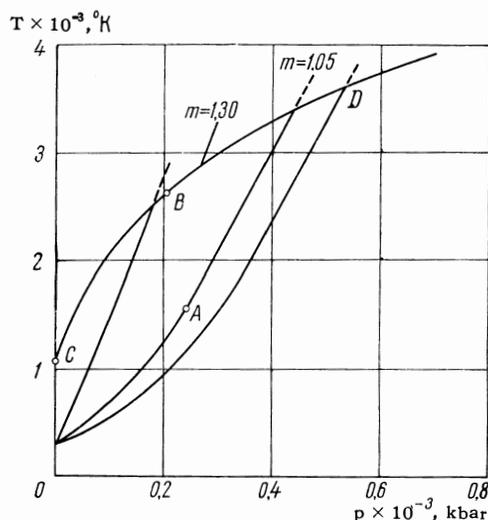


FIG. 5. T-p diagram for shock-compressed sodium chloride of various initial densities.

<sup>1)</sup>A similar plot, using points 5 and 6 as the starting points, disagrees with the position of the points 3 and 4.

perimental determination of the melting point of single-crystal sodium chloride in a shock-wave front. We measured the viscosity of sodium chloride in the solid (point A in Fig. 5) and liquid (point B) phases. In the latter case the curves representing the development of disturbances of various wavelengths  $\lambda$  in a shock-wave front coincide within the limits of the experimental error.

In conclusion, we shall represent all the experimental data on the viscosity of aluminum up to the melting point by an analytic dependence. To this end, we shall use the theory of Frenkel' and Eyring,<sup>[12,13]</sup> developed for a liquid, since there is as yet no theory of the viscosity of solids. According to this theory, the viscosity of a liquid depends in the following way on temperature and pressures:

$$\eta = \eta_0 \rho \exp(E_{\text{act}}/RT),$$

where  $\rho$  is the density and  $E_{\text{act}}$  is the activation energy.

This formula has been derived on the assumption that the viscosity is associated with the motion of vacancies of molecular size and that the formation of vacancies requires work to be done in opposition to internal pressure ( $p$ ) forces. Therefore,  $E_{\text{act}} = E_{\text{visc}} + p\Delta V$ , where  $\Delta V$  is the increase in the volume due to the formation of a vacancy. The value of  $\Delta V$  should be some fraction of the total volume  $V$  and hence  $\Delta V \approx V/n_1$ . Then,  $E_{\text{act}} = E_{\text{visc}} + pV/n_1$ .<sup>[13]</sup> For aluminum, compressed by a shock wave at pressures of about 1000 kbar, we have  $p \propto \rho^4$ <sup>[14]</sup> and consequently  $pV \propto \rho^3$ .

Thus the Frenkel'-Eyring formula, applied to the experimental data being considered, can be written in a general form as follows,

$$\eta = \eta_0 \sigma \exp\left(\frac{A + B\sigma^3}{T}\right).$$

Using the experimental data given in the table (rows 1, 3, and 4), we can find the values of  $\eta_0$ ,  $A$ , and  $B$ . Thus, we obtain

$$\eta = 4 \cdot 10^9 \sigma \exp\left(\frac{4350\sigma^3 - 7830}{T}\right).$$

This dependence describes satisfactorily the viscosity at point 5 in Fig. 4 but it does not describe at all the viscosity of aluminum under the conditions obtaining in<sup>[5,6]</sup>. Moreover, when  $\sigma < 1.13$ , the above formula gives negative activation energies for viscous flow, which indicates that the Frenkel'-Eyring theory is inapplicable to the phenomena considered. Nevertheless, we must mention that in the region close to the melting point, i.e., when  $\sigma = 1.56$  ( $p = 1050$  kbar), the

activation energy for viscous flow of aluminum behind a shock-wave front is  $E_{\text{act}} = 15.35$  kcal/mole, which is of the same order of magnitude as the self-diffusion energy of aluminum atoms under normal conditions ( $E = 33$  kcal/mole<sup>[15]</sup>).

## CONCLUSIONS

The experimental data obtained on the viscosity of aluminum, lead, and sodium chloride behind a shock-wave front lead to the following conclusions:

1. The viscosities of aluminum and lead at pressures of 310–350 kbar behind a shock-wave front are practically equal and amount to  $(2-4) \times 10^4$  P. Under these conditions, aluminum and lead are deformed by 174% at a deformation rate of  $10^7$  sec<sup>-1</sup>.
2. The viscosity of aluminum at a pressure of 310 kbar is independent of the deformation rate, at least in the range from  $4 \times 10^5$  to  $8 \times 10^6$  sec<sup>-1</sup>; it has been shown that aluminum behind a disturbed shock-wave front goes over to the plastic state.
3. The method for investigating the viscosity of substances behind a shock-wave front, proposed in<sup>[1]</sup>, makes it possible to deduce sufficiently reliably the phase state of a substance at high pressures and temperatures. In the case of aluminum, the experiment shows that this metal melts in the pressure range 1050–2020 kbar along a shock adiabat for which  $m = 1$ .

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