FUSION OF PHOTONS IN A UNIFORM ELECTROMAGNETIC FIELD

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The effect of "fusion" of two photons into one in an external uniform electromagnetic field, is considered in the Schwinger formulation of quantum electrodynamics. An expression is derived for the probability of this process.

In this note we consider the fusion of two photons into a single photon in the presence of an external homogeneous electromagnetic field. It is well known that such a field cannot be considered as a perturbation, and therefore it is impossible to treat this problem in the Feynman formulation of quantum electrodynamics. We shall make use of Schwinger's formulation (where the presence of the external field is taken into account in the electron Green's function [1]). Other processes involving photons in an external field have been considered in [2] (elastic scattering of the photon) and in [3] (decay of a photon into two photons).

The S-matrix element describing the fusion of two photons can be written in the form

$$S=\frac{e^3}{3!}\int d^4x_1d^4x_2d^4x_3\,\mathrm{Sp}\,\hat{A}(x_1)S^c(x_2-x_1)\times\hat{A}(x_2)S^c(x_3-x_2)\hat{A}(x_3)S^c(x_1-x_3). \tag{1}$$

As in the case of elastic scattering of a photon [2], the fusion amplitude becomes infinite if all the photons are represented as plane waves. Therefore one of the incident photons will be represented as a superposition with different frequencies (this is always justifiable)

$$A_{1\alpha}(x_1)=e_\alpha{}^1\int\frac{d\omega_1}{\sqrt{2\omega_1}}f(\omega_1)e^{ik_1x_1},$$

whereas the second incident photon, as well as the one resulting from the process will be taken in the form of plane waves

$$A_{2\alpha}(x_2)=\frac{e_\alpha{}^2}{\sqrt{2\omega_2}}e^{ik_2x_2},\quad A_{3\alpha}(x_3)=\frac{e_\alpha{}^3}{\sqrt{2\omega_3}}e^{-ik_3x_3}.$$

In the approximation $eF/m^2 \ll 1$, the momentum representation $S^c(p)$ of the electron Green's function has the form [1]

$$S^c(p)=\frac{m-\hat{p}}{m^2+p^2-i\varepsilon}+\frac{e}{4}\left\{\sigma_{\alpha\beta}F_{\alpha\beta},\frac{m-\hat{p}}{(m^2+p^2-i\varepsilon)^2}\right\},\varepsilon>0, \tag{2}$$

where $F_{\alpha\beta}$ is the field-strength tensor of the external electromagnetic field, $\{a, b\} = ab + ba$.

The fusion matrix element can be represented in the form

$$M=\frac{e^3}{3!}\frac{f(\omega_1)}{\sqrt{8\omega_1\omega_2\omega_3}}e_\mu{}^1e_\nu{}^2e_\rho{}^3\Delta_{\mu\nu\rho}\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3), \tag{3}$$

$$\Delta_{\mu\nu\rho}=\int d^4p\,\mathrm{Sp}\,\gamma_\mu S^c(p)\gamma_\nu S^c(p+k_2)\gamma_\rho S^c(p-k_1). \tag{4}$$

Computations yield the following expression for $\Delta_{\mu\nu\rho}$

$$\Delta_{\mu\nu\rho}=\frac{4\pi^2e}{3m^2}[k_{1\alpha}F_{\alpha\mu}\delta_{\nu\rho}+k_{2\alpha}F_{\alpha\nu}\delta_{\mu\rho}-k_{3\alpha}F_{\alpha\rho}\delta_{\mu\nu}]. \tag{5}$$

The conservation law $k_1 + k_2 = k_3$ implies that the process can occur only if $\mathbf{k}_1 \uparrow\uparrow \mathbf{k}_2 \uparrow\uparrow \mathbf{k}_3$. The total probability for fusion of two polarized photons is

$$w=\frac{8}{81}\frac{\alpha^4\pi^3}{m^4}\frac{|f(\omega_1)|^2}{\omega_1\omega_2(\omega_1+\omega_2)}[(k_1Fe_1)(e_2e_3)+(k_2Fe_2)(e_1e_3)-(k_3Fe_3)(e_1e_2)]^2, \tag{6}$$

where $\alpha = e^2/4$ and $k_3 = k_1 + k_2$. Averaging over polarizations yields

$$w=\frac{4}{81}\frac{\alpha^4\pi^3}{m^4}\frac{|f(\omega_1)|^2}{\omega_1\omega_2(\omega_1+\omega_2)}[(k_1F)^2+(k_2F)^2+(k_1F)(k_2F)]. \tag{7}$$

For the case $k_1 = k_2 = k$ we have

$$w=\frac{2}{27}\frac{\alpha^4\pi^3}{m^4\omega}|f(\omega)|^2(n_\mu F_{\mu\nu})^2, \tag{8}$$

where $n_\mu = k_\mu/\omega$. In a homogeneous electric field $(n_\mu F_{\mu\nu})^2 = E^2\sin^2\theta$, and in a uniform magnetic field $(n_\mu F_{\mu\nu})^2 = H^2\sin^2\theta$, where θ is the angle between **k** and **E** (or **k** and **H**, respectively).

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[2] S. S. Sannikov, JETP 52, 467 (1967), Soviet Phys. JETP 25, 306 (1967).

[3] V. G. Skobov, JETP 35, 1315 (1958), Soviet Phys. JETP 8, 919 (1959).

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