SOVIET PHYSICS JETP VOLUME 25, NUMBER 5 NOVEMBER, 1967

THE RESISTANCE OF METALS LOCATED IN STRONG MAGNETIC FIELDS

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Submitted to JETP editor December 2, 1966

J. Exptl. Theoret. Phys. (U.S.S.R.) 52, 1312–1316 (May, 1967)

The effect of magnetic breakdown on the resistance of metals is investigated. An analytic expression for the magnetic resistance is given for metals with closed Fermi surfaces. The expression is valid for strong magnetic fields ($\gamma \ll 1$). It is shown that the transverse resistance changes noticeably if the electron undergoes magnetic breakdown at least once on traversing one mean free path length. The resistance may fall off with increase in magnetic field strength in metals with equal numbers of electrons and holes, owing to magnetic breakdown. In contrast, in metals with uncompensated numbers of electrons and holes, an increase in the resistance is characteristic. In the region of magnetic fields of intermediate strength, where the probability of magnetic breakdown is not close to unity, the resistance increases with the magnetic field more slowly than H^2, by somewhat more rapidly than linearly.

THE character of the behavior of the resistance of a metal in a strong magnetic field[1)] is essentially determined by the dynamic motion of the conduction electrons in the magnetic field.[1–3] This enables us, by means of the experimentally observed sharp anisotropy of the transverse resistance of polycrystals, to establish the topology of the Fermi surface for the electrons in the metal. Such a method of study of the Fermi surface is more effective if the studied samples are sufficiently large so as to exclude size effects, if the magnetic field is not too large, and if quantum effects do not need to be taken into acoount. However, these conditions are not always easily established experimentally, in view of the appreciable difficulties in obtaining pure single crystals of large size, while an increase in the magnetic field (i.e., decrease of r) is connected with the appearance of the quantum tunnel effect in many metals—"magnetic breakdown."[4–6] This effect obliterates somewhat the boundary between open and closed Fermi surfaces. As the result of magnetic breakdown, electrons in a metal with a closed Fermi surface can go through many cells of the reciprocal lattice in momentum space; conversely, an electron situated on an open cross section of the Fermi surface can move along a closed orbit.

At first glance, it appears that the investigation of galvanomagnetic phenomena in a very strong magnetic field becomes a less sensitive "instrument" of study of the topology of Fermi surfaces. However, the probability of magnetic breakdown $p = \exp(-H_0/H)$ is practically independent of the free path length of the electrons[5,6] (H_0 is the characteristic field at which magnetic breakdown is established). Therefore, it suffices to study the resistance of samples of a single metal of different purity in order to make completely clear which resistance singularities are associated with the "openness' of the Fermi surface, and which are associated with the appearance of magnetic breakdown. The latter singularities (connected with magnetic breakdown) also turn out to be quite important.[7]

We are interested in the effect of magnetic breakdown on the resistance of metals with closed Fermi surfaces. As an example, we consider the case in which the magnetic breakdown can take place in practice only in a single direction of momentum space. In this direction, which we take to be the p_x axis, the electron can pass through many cells of the reciprocal lattice with a definite probability.

For the calculation of the electrical conductivity of such a metal, we mentally disregard the existence of magnetic breakdown and assume that the "fictitious" Fermi surface is such that all possible electron orbits are realized on it. These orbits may be located in a single cell or may be

1)The Larmor radius r is much smaller than the length of the free path of the electron l_0: $\gamma = r/l_0 \ll 1$.

spread out over many cells of the reciprocal lattice. Inasmuch as we are not interested in quantum effects in the resistance, i.e., it is enough for us to calculate the resistance as a smoothed function of the magnetic field H, it follows that such an approach is sufficiently rigorous, and we can compute the electrical conductivity tensor σ_{ik} classically.[1-3]

The electron velocity v(t) is generally not a periodic function of t, the dimensionless time of motion of the electron in the magnetic field (in these variables, the period of revolution of the electron over the smallest orbit is equal to 2π). Therefore, averaging in σ_{ik} over all possible electron orbits is extremely cumbersome.

We assume that the p_x axis is an axis of symmetry of the crystal, and that the electron orbits are symmetric about the p_x axis. These assumptions, which do not violate the generality of our discussions, greatly simplify the calculation of the tensor σ_{ik}. Then, at the instant of magnetic breakdown $t_m = \pi M$ (M is an integer, $t_0 = 0$), the argument of the function $v_i(t)$, specified in a single cell of momentum space, undergoes an increase by π and, consequently,

$$\sigma_{ik}(p,\gamma)=\frac{ec}{Hh^3}\int m^{*2}dp_z\sum_{l=0}^{2N}p^lq^{2N-l}\sum_{\{P_l\}}\frac{1}{N}\times\sum_{m=0}^{l}\int_{t_m}^{t_{m+1}}e^{-\gamma t}v_i(t+\pi m)\left\{\int_{t_m}^{t}e^{\gamma t'}v_k(t'+\pi m)\,dt'+\sum_{n=0}^{m-1}\int_{t_n}^{t_{n+1}}e^{\gamma t'}v_k(t'+\pi n)\,dt'\right\}dt;$$

$$p+q=1,\qquad t_{l+1}=2\pi N. \tag{1}$$

Here e, m^*, and p_z are the charge, effective mass and projection of the momentum of the electron in the direction of the magnetic field; $p^l q^{2N-l}$ is the probability that the electron undergoes magnetic breakdown l times within a time $2\pi N$, and is "restrained" from magnetic breakdown $2N-l$ times in favorable situations. In addition, one must sum over all equally probable but different orbits $\{P_l\}$, the number of which is equal to $C_{2N}^{l} = 2N!/l!(2N-l)!$.

The transverse components of the electric conductivity tensor depend in a most significant way on the probability of magnetic breakdown:

$$\sigma_{\alpha\beta}(p,\gamma)=\sigma_{\alpha\beta}(0,\gamma)+\frac{ec}{Hh^3}\int\frac{m^{*2}dp_z}{(1+e^{-\pi\gamma})(1+e^{\pi\gamma})}\times\int_0^{\pi}e^{-\gamma t}v_\alpha(t)\,dt\int_0^{\pi}e^{\gamma t'}v_\beta(t')\,dt'\Big[\frac{1}{N}\sum_{l=1}^{2N}2lp^lq^{2N-l}C_{2N}^{l}+\sum_{l=2}^{2N}p^lq^{2N-l}\sum_{\{P_l\}}\frac{1}{N}\sum_{m=1}^{l}\sum_{n=0}^{m-1}(-1)^{n+m}\exp\{\gamma(t_n-t_m)\}\cos t_n\cos t_m\Big],\quad \alpha,\beta=x,y. \tag{2}$$

The first component in the square brackets is equal to 4p, and the second component can be transformed to the following:

$$\sum_{l=2}^{2N}p^lq^{2N-l}\sum_{\{P\}}\frac{1}{N}\sum_{m=1}^{l}\sum_{n=0}^{m-1}(-1)^{n+m}\exp\{\gamma(t_n-t_m)\}\cos t_n\cos t_m$$

$$=\sum_{l=2}^{2N}p^lq^{2N-l}\frac{1}{N}\sum_{M=0}^{2N-2}(2N-M)\exp\{-\pi\gamma(M+1)\}\times\sum_{k=0}^{\min\{M,l\}}(-1)^kC_M^kC_{2N-2-M}^{l-2-k}=2p^2e^{-\pi\gamma}\sum_{M=0}^{2N-2}\left(1-\frac{M}{2N}\right)\times(p-q)^Me^{-\pi\gamma M}=\frac{2p^2}{e^{\pi\gamma}+q-p}+O\left(\frac{1}{N}\right). \tag{3}$$

In what follows, we shall use only the asymptotic expression for the tensor σ_{ik} as $N\to\infty$. Omitting numerical factors of the order of unity, we obtain an expression for the entire electric conductivity tensor:

$$\sigma_{ik}=\begin{pmatrix}\gamma^2a_{xx} & \frac{(n_1-n_2)ec}{H}+\frac{p\gamma^2}{q+\gamma}a'_{xy} & \gamma a_{xz}\\ \frac{(n_2-n_1)ec}{H}+\frac{p\gamma^2}{q+\gamma}a'_{yx} & \gamma^2a_{yy}+\frac{p\gamma}{q+\gamma}a'_{yy} & \gamma a_{yz}\\ \gamma a_{zx} & \gamma a_{zy} & a_{zz}\end{pmatrix}, \tag{4}$$

where n_1 and n_2 are the numbers of electrons and holes. The components of the matrices a_{ik} and a_{ik}' are the same, in order of magnitude, as the electrical conductivity of metals σ_0 in the absence of the magnetic field. We shall assume that the ratio δ of the number of electrons free to undergo magnetic breakdown to the total number of electrons is not a small parameter ($a_{ik}' \approx \delta a_{ik} \approx \sigma_0$). The components σ_{yz} and σ_{zy} are seen to be small because of the symmetry of the electron orbits relative to the p_x axis. In the general case, $\sigma_{yz} = \gamma a_{yz}'/(q+\gamma)$; however, this circumstance brings about no change in the resistance.

Magnetic breakdown has practically no effect on the longitudinal resistance of metals, while the transverse resistance is materially changed if the electron undergoes magnetic breakdown at least once during the time of free flight. The significant effect of magnetic breakdown on the resistance of metals is seen from the graphs presented by Falicov and Sievert,[7] who computed the resistance of metals with account of the magnetic breakdown with an electronic computer.

One must distinguish between three regions of magnetic fields, in which the resistance of metals with closed Fermi surfaces depends in different ways on the magnetic field.

1. The transverse resistance does not "notice" the magnetic breakdown and is isotropic if $p \ll \gamma$.

2. If $p \gtrsim \gamma$ and $H \lesssim H_0$, the transverse resistance has the following form:2)

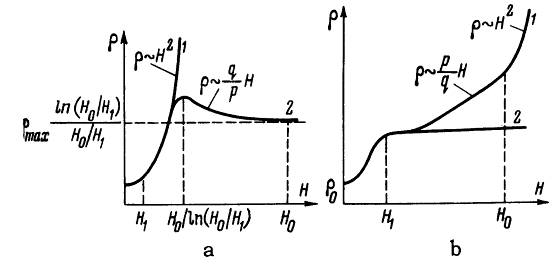
$$\rho = \begin{cases} Al_0H^2\cos^2\alpha + A_1\dfrac{q}{p}H, & n_1 = n_2 \\ B\dfrac{p}{q}H\cos^2\alpha + B_1 l_0^{-1}, & n_1 \neq n_2 \end{cases}.$$

The quantities A, B, A_1, and B_1 are characteristics of the electron system, while B and A_1 are generally independent of the form of the collision integral. The resistance becomes strongly anisotropic (α is the angle between the electric current and the x axis), but its dependence on the magnetic field is different than in metals with open Fermi surfaces for $p = 0$.

3. When the probability of magnetic breakdown is close to unity ($H \gg H_0$), the magnetic field H is a very large parameter of the problem and all the quantities are expanded in a series of powers of H^{-1}. Therefore, when $H \gg H_0$ the resistance can either increase quadratically with the magnetic field or reach saturation. The asymptote of the resistance in this case is the same as if there were open electron orbits along the p_x axis.

However, in this range of magnetic fields, the condition of quasiclassical behavior can easily be violated and for the calculation of the resistance it is necessary to use the methods of quantum theory. In magnetic fields $H \lesssim H_0$, the structure of the energy levels scarcely shows any effect on the non-oscillating part of the resistance, and classical consideration in the presence of magnetic breakdown is valid to the same extent as in the investigation of the resistance of metals with open Fermi surfaces.[1-3]

2)The asymptote of the resistance of metals with open Fermi surfaces has the same form, only the roles of the parameters p and q are interchanged.



Dependence of the resistance on the intensity of the magnetic field in metals with closed Fermi surfaces: a – number of electrons and holes compensated ($n_1 = n_2$) b – number of electrons not equal to the number of holes ($n_1 \neq n_2$). Curves 1 – for $\alpha \neq \pi/2$, curves 2 – for $\alpha = \pi/2$.

A sample plot of the resistance against a strong magnetic field is shown in the drawing. In metals with equal numbers of electrons and holes ($n_1 = n_2$), the resistance ρ_{yy} has a maximum for $H = H_0/\ln(H_0/H_1)$ (H_1 is the field for which $\gamma = 1$), and then falls off with increase in the magnetic field, reaching saturation (curve a). A similar curve was shown by Falicov and Sievert.[7]

In metals with uncompensated numbers of electrons and holes, the resistance begins to increase after saturation with increase in the magnetic field (except for the case $\alpha = \pi/2$). In extraordinarily pure metals ($H_0 \gg H_1$), the increase over sections of the field $\Delta H \sim H_1$ is excellently approximated by a linear dependence of the resistance on the magnetic field. The transverse resistance of pure aluminum, investigated by Borovik and Volotskaya, behaves in similar fashion.[8] We also note that a strong derivative from Kohler's rule[3] takes place in the case of magnetic breakdown.

Evidently the anomalous behavior of the resistance, in a magnetic field, of certain metals[8] (beryllium, thallium, aluminum, etc.) is due to magnetic breakdown.

I take this opportunity to thank V. P. Galaĭko and M. I. Kaganov for discussions.

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Translated by R. T. Beyer
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