DISPERSION LAW AND NONLINEAR INTERACTION OF LANGMUIR WAVES IN A WEAKLY TURBULENT PLASMA

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The correction that must be introduced to the Langmuir-wave frequency as a result of the nonlinear coupling of waves with different wave numbers in a weakly turbulent plasma is determined. It is shown that under certain conditions the allowance for this correction is important for the description of the time variation of the energy spectrum of Langmuir waves.

1. ONE of the causes of nonlinear interaction of Langmuir waves in a plasma is induced scattering of the waves by particles.[1-4] If the particles have an isotropic momentum distribution, the probability of this process is proportional to the difference of the frequencies of the interacting waves. Since the wave frequencies differ little from the Langmuir frequency, it is important in the description of the scattering process to take into account small corrections to the frequency. Usually one takes into account only the "kinetic" correction, which is proportional to the square of the ratio of the Debye radius of the electrons to the wavelength.[1-4] However, in a weakly turbulent plasma the nonlinear interaction of the waves leads not only to a time variation of their spectrum, but also to a change in the dispersion law (i.e., to a change in the group velocity of the plasmons). It is obvious that within the framework of the theory of weak nonlinearity this change is small compared with the frequency, but it may turn out to be far from small compared with the "kinetic" correction. Therefore under certain conditions the "nonlinear" correction to the frequency is very important for a correct description of the nonlinear interaction of the Langmuir waves.

The dependence of the frequency of the Langmuir waves on the amplitude, with the ion motion completely neglected, was considered by a number of authors.[5-7] It was found that the correction to the frequency is proportional to the ratio of the mean square of the displacement of the electrons in the field of the waves to the square of the wavelength. It is obvious that this "nonlinear" correction is larger than the "kinetic" correction only if the Debye radius of the electrons is smaller than the mean displacement of the electrons in the field of the waves. But under this condition, as shown by Silin,[8] the plasma is unstable.

It will be shown below that in the case when the motion of the ions is important in the wave interaction, the "nonlinear" correction to the frequency of the Langmuir waves is proportional to the ratio of the total energy of the waves per unit volume to the density of the thermal energy of the particles. This value is much larger than the correction which would result from neglect of the ion motion, and can be larger than the "kinetic" correction. In this case, allowance for the "nonlinear" correction leads to a transfer of energy from the waves with lower energy to the waves with larger energy (and not from the shorter to the longer waves, as was the case when only the "kinetic" correction was taken into account[2]).

2. The nonlinear equation for the field in a homogeneous and stationary medium can be written in the form (see [9]):

$$\frac{c^2k^2}{\omega^2}\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)E_j(\omega,\mathbf{k})=\sum_{n=1}^{\infty}\int d\omega_1\,d\mathbf{k}_1\ldots d\omega_{n-1}\,d\mathbf{k}_{n-1}$$
$$\times\varepsilon_{ij(1)\ldots j(n)}(\omega,\mathbf{k},\omega_1,\mathbf{k}_1,\ldots,\omega_{n-1},\mathbf{k}_{n-1})E_{j(1)}(\omega-\omega_1,\mathbf{k}-\mathbf{k}_1)$$
$$\ldots E_{j(n)}(\omega_{n-1},\mathbf{k}_{n-1}), \qquad (1)$$

where $\mathbf{E}(\omega, \mathbf{k})$ is the Fourier component of the electric field intensity, and $\varepsilon_{ij(1)\ldots j(n)}$ are the tensors of the dielectric constant.

To obtain the law of dispersion of waves with random initial phases (i.e., for noise) we multiply equation (1) by $E_i^*(\omega, \mathbf{k})$ and average over the statistical ensemble. We confine ourselves here to terms up to the fourth power in the field and use the relation

$$\langle E_i^*(\omega,\mathbf{k})E_j(\omega',\mathbf{k}')\rangle=\delta(\omega-\omega')\delta(\mathbf{k}-\mathbf{k}')(E_iE_j)_{\omega,\mathbf{k}}.$$

Following [9], the average of the product of four field components is expressed in terms of the average of the pair products, and in averaging the terms containing products of three field components we use the nonlinear equation (1) and express also the terms through the products of the four components. As a result we obtain the dispersion equation in the form

$$(E_iE_j)_{\omega,\mathbf{k}}\left[\frac{c^2k^2}{\omega^2}\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)-\varepsilon_{ij}^{H}(\omega,\mathbf{k})\right]$$
$$=\mathrm{Re}\left\{(E_iE_m)_{\omega,\mathbf{k}}\int d\omega'\,d\mathbf{k}'(E_lE_j)_{\omega',\mathbf{k}'}Q_{ijlm}(\omega,\mathbf{k},\omega',\mathbf{k}')\right.$$
$$\left.-\frac{1}{2}\int d\omega'\,d\mathbf{k}'(E_sE_l)_{\omega',\mathbf{k}'}(E_pE_j)_{\omega'',\mathbf{k}''}P_{slpj}(\omega,\mathbf{k},\omega',\mathbf{k}')\right\}, \qquad (2)$$

where $\varepsilon_{ij}^{H}(\omega, \mathbf{k})$ is the Hermitian part of the dielectric tensor, $\omega'' = \omega - \omega'$, $\mathbf{k}'' = \mathbf{k} - \mathbf{k}'$:

$$Q_{ijlm}(\omega,\mathbf{k},\omega',\mathbf{k}')=V_{ijlm}(\omega,\mathbf{k},\omega',\mathbf{k}')$$
$$-S_{irj}(\omega,\mathbf{k},\omega',\mathbf{k}')A_{rp}(\omega'',\mathbf{k}'')S_{plm}(\omega'',\mathbf{k}'',\omega,\mathbf{k}),$$
$$P_{slpj}(\omega,\mathbf{k},\omega',\mathbf{k}')=A_{iq}^*(\omega,\mathbf{k})S_{ijl}(\omega,\mathbf{k},\omega',\mathbf{k}')S_{qps}^*(\omega,\mathbf{k},\omega',\mathbf{k}'),$$
$$V_{ijlm}(\omega,\mathbf{k},\omega',\mathbf{k}')=\varepsilon_{ijml}(\omega,\mathbf{k},\omega'',\mathbf{k}'',-\omega',-\mathbf{k}')$$
$$+\varepsilon_{ijlm}(\omega,\mathbf{k},\omega'',\mathbf{k}'',\omega,\mathbf{k}),\quad S_{ijl}(\omega,\mathbf{k},\omega',\mathbf{k}')$$
$$=\varepsilon_{ijl}(\omega,\mathbf{k},\omega',\mathbf{k}')+\varepsilon_{ilj}(\omega,\mathbf{k},\omega'',\mathbf{k}''). \qquad (3)$$

For an isotropic medium, which we shall consider below, the tensor $A_{ij}(\omega, \mathbf{k})$ takes the form

$$A_{ij}(\omega,\mathbf{k})=\frac{k_ik_j}{k^2}\frac{1}{\varepsilon^l(\omega,k)}+\frac{\delta_{ij}-k_ik_j/k^2}{\varepsilon^{tr}(\omega,k)-k^2c^2/\omega^2}, \qquad (4)$$

where ε^l and ε^{tr} are respectively the longitudinal and transverse dielectric constants.

We confine ourselves to a consideration of longitudinal noise when

$$(E_iE_j)_{\omega,\mathbf{k}}=\frac{k_ik_j}{k^2}(E_l^2)_{\omega,\mathbf{k}}.$$

If the quantity $(E_l^2)_{\omega,\mathbf{k}}$ is real, we get from (2)

$$\left\{\varepsilon^{l'}(\omega,\mathbf{k})+\int d\omega'\,d\mathbf{k}'(E_l^2)_{\omega',\mathbf{k}'}\frac{k_i\,k_j'\,k_m\,k_l'}{k^2\,k'^2}\operatorname{Re}Q_{ijlm}(\omega,\mathbf{k},\omega',\mathbf{k}')\right\}(E_l^2)_{\omega,\mathbf{k}}$$
$$-\frac{1}{2}\int d\omega'\,d\mathbf{k}'(E_l^2)_{\omega',\mathbf{k}'}(E_l^2)_{\omega'',\mathbf{k}''}\frac{k_s'\,k_l'\,k_p''\,k_j''}{(k'k'')^2}\operatorname{Re}P_{slpj}(\omega,\mathbf{k},\omega',\mathbf{k}')=0, \qquad (5)$$

where $\varepsilon^{l'}$ is the real part of the longitudinal dielectric constant ε^l.

We confine ourselves to waves with frequencies close to the electron Langmuir frequency, and disregard the contribution made to the dispersion equation by the second term of (4) (this means that we confine ourselves to wave lengths shorter than c/ω_{Le}). Under these conditions, the last term in (5) must be discarded, since it is impossible to satisfy the energy and wave-vector conservation laws for three Langmuir waves. The dispersion equation (5) takes the form

$$\varepsilon^{l'}(\omega,k)+\int d\omega'\,d\mathbf{k}'(E_l^2)_{\omega',\mathbf{k}'}\operatorname{Re}Q(\omega,\mathbf{k},\omega',\mathbf{k}')=0, \qquad (6)$$

where, according to (3) and (4),

$$Q(\omega,\mathbf{k},\omega',\mathbf{k}')=\frac{k_i\,k_m\,k_j'\,k_l'}{(kk')^2}\Big\{V_{ijlm}(\omega,\mathbf{k},\omega',\mathbf{k}')$$
$$-\frac{k_r''\,k_p''}{k''^2\varepsilon^l(\omega'',k'')}S_{irj}(\omega,\mathbf{k},\omega',\mathbf{k}')S_{plm}(\omega'',\mathbf{k}'',\omega,\mathbf{k})\Big\}. \qquad (7)$$

Using the general expression for the dielectric tensors of the plasma, which is obtained by solving the kinetic equation by perturbation theory and which is contained in [9], we get

$$V(\omega,\mathbf{k},\omega',\mathbf{k}')=\frac{k_ik_mk_j'k_l'}{(kk')^2}V_{ijlm}(\omega,\mathbf{k},\omega',\mathbf{k}')$$
$$=-\frac{1}{(kk')^2}\sum_\alpha\frac{4\pi N_\alpha e_\alpha^4}{m_\alpha^3}\int d\mathbf{p}_\alpha\frac{1}{\omega-\mathbf{k}\mathbf{v}_\alpha}\mathbf{k}'\frac{\partial}{\partial\mathbf{v}_\alpha}\Big\{\frac{1}{\omega''-\mathbf{k}''\mathbf{v}_\alpha}$$
$$\times\Big[\mathbf{k}'\frac{\partial}{\partial\mathbf{v}_\alpha}\Big(\frac{\mathbf{k}}{\omega-\mathbf{k}\mathbf{v}_\alpha}\frac{\partial f_\alpha}{\partial\mathbf{v}_\alpha}\Big)-\mathbf{k}\frac{\partial}{\partial\mathbf{v}_\alpha}\Big(\frac{\mathbf{k}'}{\omega'-\mathbf{k}'\mathbf{v}_\alpha}\frac{\partial f_\alpha}{\partial\mathbf{v}_\alpha}\Big)\Big]\Big\}, \qquad (8)$$

$$S(\omega,\mathbf{k},\omega',\mathbf{k}')=\frac{k_ik_r''k_j'}{kk'k''}S_{irj}(\omega,\mathbf{k},\omega',\mathbf{k}')=$$
$$=-\frac{i}{kk'k''}\sum_\alpha\frac{4\pi N_\alpha e_\alpha^3}{m_\alpha^2}\int d\mathbf{p}_\alpha\frac{1}{\omega-\mathbf{k}\mathbf{v}_\alpha}\Big[\mathbf{k}''\frac{\partial}{\partial\mathbf{v}_\alpha}\Big(\frac{\mathbf{k}'}{\omega'-\mathbf{k}'\mathbf{v}_\alpha}\frac{\partial f_\alpha}{\partial\mathbf{v}_\alpha}\Big)$$
$$+\mathbf{k}'\frac{\partial}{\partial\mathbf{v}_\alpha}\Big(\frac{\mathbf{k}''}{\omega''-\mathbf{k}''\mathbf{v}_\alpha}\frac{\partial f_\alpha}{\partial\mathbf{v}_\alpha}\Big)\Big]$$

where f_α is the particle distribution function; N_α, e_α, m_α, $\mathbf{v}_\alpha$, and $\mathbf{p}_\alpha$ are the concentration, charge, mass, velocity, and momentum of the particles of type α.

We note that Eq. (6), in which we used explicit expressions for the quantities (8), was derived by Petviashvili[10] and is contained in the review of Kadomtsev.[1]

We solve the dispersion equation (6) by perturbation theory, taking as the zeroth approximation the usual dispersion equation for the longitudinal waves $\varepsilon^{l'}(\omega_0, \mathbf{k}) = 0$, i.e.,

$$\omega_0=\omega_L[1+{}^3/_2(kr_{De})^2], \qquad (9)$$

where $\omega_L=\sqrt{\omega_{Le}^2+\omega_{Li}^2}$, $\omega_{L\alpha}=\sqrt{4\pi N_\alpha e_\alpha^2/m_\alpha}$ is the Langmuir frequency of the particles of type α, and r_{De} is the Debye radius of the electrons.

Assuming that $(E_l^2)_{\omega,\mathbf{k}}$ differs from zero only when $\omega=\omega_0$, we introduce the noise spectral density (see [9]):

$$W^l(\mathbf{k})=\frac{(2\pi)^3}{4\pi}\int_0^\infty d\omega(E_l^2)_{\omega,\mathbf{k}}\frac{\partial}{\partial\omega}[\omega\varepsilon^{l'}(\omega,k)]. \qquad (10)$$

From (6) we obtain the correction to the frequency in the form

$$\Delta\omega=-\frac{\omega_0}{8\pi^2}\int d\mathbf{k}'W^l(\mathbf{k}')\operatorname{Re}[Q(\omega_0(\mathbf{k}),\mathbf{k},\omega_0(\mathbf{k}'),\mathbf{k}')$$
$$+Q(\omega_0(\mathbf{k}),\mathbf{k},-\omega_0(\mathbf{k}'),-\mathbf{k}')]. \qquad (11)$$

Owing to the presence of the mass in the denominator, the ionic terms in (8) are much smaller than the electronic terms, and will henceforth be disregarded. In addition, we take account of the fact that the phase velocities of the Langmuir waves are large compared with the thermal velocity of the particles $((|\mathbf{k}\cdot\mathbf{v}|/\omega) \ll 1$, $(|\mathbf{k}', \mathbf{v}|/\omega' \ll 1)$, and we expand in terms of these parameters in the electronic terms of (8). As a result we get from (11), in the lowest approximation in the quantity $(kr_{De})^2 \ll 1$,

$$\Delta\omega=-\frac{\omega_{Le}}{8\pi^2}\frac{e^2}{m^2\omega_{Le}^4}\int d\mathbf{k}'W(\mathbf{k}')\Big(\frac{\mathbf{k}\mathbf{k}'}{kk'}\Big)^2\Big\{r_{De}^{-2}\cdot$$
$$\times\Big[1-\frac{(k''r_{De})^{-2}}{1+(k''r_{De})^{-2}+\varepsilon_i'(\omega'',\mathbf{k}'')}\Big]+6k'^2-k''^2-\frac{4}{3}\frac{[\mathbf{k}\mathbf{k}']^4}{k''^2(\mathbf{k}\mathbf{k}')^2}\Big\}. \qquad (12)$$

we have assumed here that the particles have a Maxwellian distribution, and we used the notation (see [11])

$$\varepsilon_i'(\omega,k)=\frac{1}{(kr_{Di})^2}\operatorname{Re}\Big[1-\beta_i\exp\Big(\frac{-\beta_i^2}{2}\Big)\int_{+i\infty}^{\beta}d\tau\exp\Big(\frac{\tau^2}{2}\Big)\Big], \qquad (13)$$

$\beta_\alpha=\omega/kv_{T\alpha}$, $v_{T\alpha}=\sqrt{T_\alpha/m_\alpha}$, and $r_{D\alpha}=\sqrt{T_\alpha/4\pi N_\alpha e_\alpha^2}$ is the Debye radius of the particles of type α.

When the inequality $kr_{De} \gg \sqrt{m/m_i}$ is satisfied, the ionic dielectric constant in (12) can be neglected. Then owing to the fact that $(k''r_{De})^{-2} \gg 1$, the first term of (12) will be of the same order as the remaining terms, and the correction to the frequency can be written in the form

$$\Delta\omega=-\frac{\omega_{Le}}{8\pi^2}\frac{e^2}{m^2\omega_{Le}^4}\int d\mathbf{k}'W(\mathbf{k}')\Big(\frac{\mathbf{k}\mathbf{k}'}{kk'}\Big)^2\Big\{6k'^2-\frac{4}{3}\frac{[\mathbf{k}\mathbf{k}']^4}{(\mathbf{k}\mathbf{k}')^2k''^2}\Big\}. \qquad (14)$$

Thus, in accordance with the available literature data,[5-7] the correction to the frequency is proportional to the ratio of the mean square of the electron displacement in the wave field to the square of the wavelength.

Of much greater interest is the case when the ionic polarizability is appreciable. This occurs for sufficiently large wavelengths ($kr_{De} \ll \sqrt{m/m_i}$). From (12) we obtain with the aid of the asymptotic expression for (13)

$$\Delta\omega=-\frac{\omega_{Le}}{32\pi^3}\frac{1}{N_eT_e}\frac{r_{De}^2}{r_{De}^2+r_{Di}^2}\int d\mathbf{k}'\Big(\frac{\mathbf{k}\mathbf{k}'}{kk'}\Big)^2W(\mathbf{k}'). \qquad (15)$$

If the temperature of the electrons and the ions is of the same order, then the correction to the frequency (15) is larger than the correction (14) by a factor $(kr_{De})^{-2}$.

However, when the noise has an isotropic distribution with respect to the wave vectors, the correction (15) to the frequency does not depend on the wave vector **k** and is a small and insignificant correction to ω_L. On the other hand, if the noise distribution is anisotropic, then $\Delta\omega$ depends on the direction of **k**.

3. Under conditions when formula (15) is valid, the nonlinear interaction of the Langmuir waves due to the induced scattering by ions becomes important. The equation describing this process is [2,3]

$$\frac{\partial W(\mathbf{k},t)}{\partial t}=-\frac{1}{4(2\pi)^{5/2}}\frac{1}{N_e r_{De}^3}\frac{W(\mathbf{k},t)}{T_e}\frac{r_{Di}^2 r_{De}^4}{(r_{De}^2+r_{Di}^2)^2}\frac{v_{Te}}{v_{Ti}}\times\int d\mathbf{k}'\left(\frac{\mathbf{k}\mathbf{k}'}{kk'}\right)^2 W(\mathbf{k}',t)\frac{\omega(\mathbf{k})-\omega(\mathbf{k}')}{|\mathbf{k}-\mathbf{k}'|}. \tag{16}$$

For $\omega(\mathbf{k})$ we use formula (9) and the correction (15). We then get

$$\frac{\partial W(\mathbf{k},t)}{\partial t}=-\frac{3}{8(2\pi)^{5/2}}\frac{\omega_{Le}}{N_e r_{De}^3}\frac{W(\mathbf{k},t)}{T_e}\frac{r_{Di}^2 r_{De}^6}{(r_{Di}^2+r_{De}^2)^2}\frac{v_{Te}}{v_{Ti}}\times\int d\mathbf{k}' W(\mathbf{k}',t)\left(\frac{\mathbf{k}\mathbf{k}'}{kk'}\right)^2\frac{1}{|\mathbf{k}-\mathbf{k}'|}\Big\{(k^2-k'^2)-\frac{1}{48\pi^3}\frac{1}{T_e N_e}\frac{1}{r_{De}^2+r_{Di}^2}\int d\mathbf{k}'' W(\mathbf{k}'',t)\Big[\Big(\frac{\mathbf{k}\mathbf{k}''}{kk''}\Big)^2-\Big(\frac{\mathbf{k}'\mathbf{k}''}{k'k''}\Big)^2\Big]\Big\} \tag{17}$$

Equation (17) describes a nonlinear interaction process in which the noise energy is conserved

$$W_0=\int d\mathbf{k}\, W(\mathbf{k},t)=\text{const}$$

and the total number of quanta is conserved

$$N_0\equiv\int d\mathbf{k}\, N(\mathbf{k},t)=\int d\mathbf{k}\, W(\mathbf{k},t)/\hbar\omega_{Le}=\text{const}.$$

To reveal the effects resulting from the allowance for the "nonlinear" correction to the frequency, let us consider the interaction between two wave packets having a width small enough to be neglected:

$$W(\mathbf{k},\ t)=W_1(t)\,\delta(\mathbf{k}-\mathbf{k}_1)+W_2(t)\,\delta(\mathbf{k}-\mathbf{k}_2). \tag{18}$$

Equation (17) reduces to a system of two equations

$$dW_1/dt=-W_1W_2[a-b(W_1-W_2)],\qquad dW_2/dt=W_1W_2[a-b(W_1-W_2)], \tag{19}$$

where

$$a=\frac{3}{8(2\pi)^{5/2}}\frac{\omega_{Le}}{N_e r_{De}^3}\frac{1}{T_e}\frac{r_{Di}^2 r_{De}^6}{(r_{De}^2+r_{Di}^2)^2}\frac{v_{Te}}{v_{Ti}}\left(\frac{\mathbf{k}_1\mathbf{k}_2}{k_1k_2}\right)^2\frac{k_1^2-k_2^2}{|\mathbf{k}_1-\mathbf{k}_2|},$$

$$b=\frac{1}{128\pi^3(2\pi)^{5/2}}\frac{\omega_{Le}}{N_e r_{De}^3}\frac{r_{Di}^2 r_{De}^6}{(r_{De}^2+r_{Di}^2)^3}\times\frac{1}{N_eT_e^2}\frac{[\mathbf{k}_1\mathbf{k}_2]^2}{(k_1k_2)^2}\left(\frac{\mathbf{k}_1\mathbf{k}_2}{k_1k_2}\right)^2\frac{v_{Te}}{v_{Ti}}\frac{1}{|\mathbf{k}_1-\mathbf{k}_2|} \tag{20}$$

the solution of (19) with $\alpha\equiv bW_0/a\neq 1$ is

$$e^{-aW_0t}=\left(\frac{W_1(t)}{W_1(0)}\right)^{1/(1+\alpha)}\left(\frac{W_0-W_1(t)}{W_0-W_1(0)}\right)^{-1/(1-\alpha)}\times\left|\frac{a-b(2W_1(t)-W_0)}{a-b(2W_1(0)-W_0)}\right|^{2\alpha/(1-\alpha^2)}, \tag{21}$$

where $W_0=W_1(t)+W_2(t)=W_1(0)+W_2(0)$ is the total noise energy.

When $\alpha\ll 1$ we get from (21) the solution obtained in [2], which describes the transfer of energy from the short waves to the longer ones. The stationary state $(t\to\infty)$ occurs when the entire energy is concentrated in the waves with the smaller wave number.

If the "nonlinear" corrections in the dispersion law are larger than the "kinetic" corrections, then $\alpha>1$. The last factor on the right side of (21) can then become infinite if

$$W_1=W_{1,\,\mathrm{cr}}\equiv{}^1\!/_2W_0(1+\alpha^{-1})<W_0. \tag{22}$$

However, the function on the left side of the (21) is bounded. Therefore, the change in the quantity $W_1(t)$ can occur only in such a way that $W_1(t)\neq W_{1,\,\mathrm{cr}}$. It follows therefore that when $W_1(0)>W_{1,\,\mathrm{cr}}$ the stationary state will occur at $W_1(\infty)=W_0$ and $W_2(\infty)=0$. On the other hand, if $W_1(0)<W_{1,\,\mathrm{cr}}$, then the stationary state is reached at $W_1(\infty)=0$ and $W_2(\infty)=W_0$. In other words, in the case of large "nonlinear" corrections to the dispersion law, the direction of the noise-energy depends on the initial distribution of the noise: The transfer is from the wave packet with the lower energy to the wave packet with the higher energy. The time of such a transfer is obtained by writing the solution (21) for $\alpha\gg 1$ in the form

$$W_1(t)=\frac{W_0}{2}-\frac{W_0-2W_1(0)}{2}\left[\frac{W_0^2-4W_1(0)(W_0-W_1(0))\exp(-bW_0^2t)}{(W_0-2W_1(0))^2}\right]^{1/2}. \tag{23}$$

The characteristic time of establishment of the stationary distribution of the noise is

$$\tau_0\approx\frac{1}{bW_0^2}\sim 10^5\left(\frac{N_eT_e}{W_0}\right)^2\frac{\Delta k r_{De}}{\omega_{Le}}\sqrt{\frac{m}{m_i}}, \tag{24}$$

where $\Delta k=|\mathbf{k}_1-\mathbf{k}_2|$.

If $\Delta kr_{De}\sim 10^{-2}$, $\omega_{Le}\sim 10^{12}\ \sec^{-1}$ and $N_eT_e/W_0\sim 10$, then for a hydrogen plasma $\tau_0\sim 10^{-9}$ sec. We emphasize that formula (24) is valid at a sufficiently high noise level $((W_0/N_eT_e)>(kr_{De})^2)$ and for sufficiently long waves $((kr_{De})^2\ll m/m_i)$.

In order for the considered process of nonlinear interaction of Langmuir waves to actually determine the variation of their energy spectrum, it is necessary to verify that conditions exist under which the other nonlinear interaction processes occur more slowly. In particular, the characteristic time of the confluence of two Langmuir waves with formation of a transverse wave, obtained in [9,12], is equal to

$$\tau_{l\to t}\sim 10^3\frac{N_eT_e}{W_0}\frac{1}{\omega_{Le}}\left(\frac{c}{v_{Te}}\right)^5(kr_{De})^3,$$

and this time exceeds τ_0 under the condition

$$\frac{W_0}{NT}\gg 10^2\left(\frac{v_{Te}}{c}\right)^5\sqrt{\frac{m}{m}}\,(kr_{De})^{-2}. \tag{25}$$

When $v_{Te}/c\sim 10^{-2}$ and $(kr_{De})^{-2}\sim 10^4$ we get from (25) for a hydrogen plasma $(W_0/N_eT_e)\gg 10^{-6}$.

For Langmuir waves, processes of confluence and decay with four waves taking part are also possible. The "four-plasma" decays were investigated in a number of papers. In particular, Kovrizhnykh[13] and Liperovskiĭ and Tsytovich[14] obtained general expressions for the probability of this process with allowance for the ion motion; these expressions, however, do not agree. The characteristic time of the four-plasmon interaction under conditions when (24) is valid, as given in [14], is equal to

$$\tau_4\sim 10^2\frac{1}{\omega_{Le}}\left(\frac{\Delta k}{k}\right)^2(kr_{De})^{-2}\left(\frac{N_eT_e}{W_0}\right)^2.$$

Putting $\Delta k \sim k$, we find that this time is larger than τ_0 when the inequality $(kr_{De})^{-3} > 10^3\sqrt{m/m_i}$ is satisfied. If, on the other hand, we use the characteristic time corresponding to the probability obtained in [13] for the four-plasmon process

$$\tau_4 \sim 10^2 \frac{1}{\omega_{Le}} \left(\frac{\Delta k}{k}\right)^2 (kr_{De})^2 \left(\frac{N_e T_e}{W_0}\right)^2,$$

then $\tau_4 > \tau_0$ under the condition $(kr_{De})^2 > m/m_i$, which is not satisfied for formula (24).

The foregoing example has shown that small "nonlinear" corrections to the frequency of the Langmuir noise can radically change the entire character of the nonlinear interaction of the waves. It is obvious that in other cases, too, when the frequencies of the interacting waves differ little, allowance for the indicated corrections may be important for a correct description of the induced scattering of waves by particles.

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