A NEW TYPE OF OSCILLATIONS OF LONGITUDINAL MAGNETORESISTANCE

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The behavior of the longitudinal magnetoresistance $\rho_{zz}(H)$ in quantized magnetic fields is considered under conditions when the electron gas is degenerate. It is shown that oscillations of $\rho_{zz}(H)$ occur during inelastic scattering of the electrons by optical phonons. The positions of the oscillation peaks are determined by the condition $\zeta(H) \pm \hbar\omega_0 = (N + 1/2)\hbar\Omega$, where $\zeta(H)$ is the Fermi level in the magnetic field, ω_0 is the limiting optical-phonon frequency, and Ω is the cyclotron frequency. The longitudinal magnetoresistance in HgTe and n-GaSb was measured. The positions of the $\rho_{zz}(H)$ peaks were in satisfactory agreement with the calculated values.

IN a study of the inelastic scattering of electrons interacting with optical phonons, Gurevich and Firsov[1] have observed that the dependence of the magnetoresistance on the magnetic field in strong magnetic fields has a nonmonotonic character. Whenever the magnetic field is such that the distance between any two Landau levels coincides with the limiting frequency of the optical phonons, the transverse magnetoresistance has a maximum. The resonance condition is given by

$$M\Omega = \omega_0, \quad M = 1, 2, 3, \ldots, \tag{1}$$

where Ω is the cyclotron frequency and ω_0 is the limiting frequency of the optical phonons.

Gurevich and Firsov considered magnetophonon oscillations in the case of a nondegenerate electron gas both in crossed fields $E \perp H$[1] and in longitudinal fields $E \parallel H$[2]. The transverse magnetoresistance in the case of a degenerate electron gas was considered by Éfros[3]. In all three cases, the resonance condition is of the form (1). We shall consider below longitudinal magnetoresistance oscillations due to inelastic scattering of electrons by optical phonons in the case of a degenerate electron gas.

The expression for the density of the longitudinal current $E \parallel H \parallel z$ in a magnetic field is

$$j_z = \frac{e\hbar}{m} \sum_{\mu} k_z f_\mu(\varepsilon_\mu). \tag{2}$$

Here e is the electron charge, m the effective mass, $\hbar k_z$ the component of the quasimomentum of the electron in the field direction, $\mu \equiv \{N, k_y, k_z\}$ is the set of quantum numbers of the electron in the magnetic field, and f_μ is the distribution function of the electron in the state characterized by the set of quantum numbers μ.

The distribution function of the electrons satisfies the quantum kinetic equation. In the case when a relaxation time can be introduced, for example, when the Fourier transform of the electron-phonon interaction potential does not depend on the phonon quasimomentum [1)], the kinetic equation can be easily solved and the distribution function has in the approximation linear in the electric field the form

$$f_\mu = f_0(\varepsilon_\mu) - \frac{eE\hbar k_z}{m}\tau(\varepsilon_\mu)\frac{\partial f_0(\varepsilon_\mu)}{\partial \varepsilon_\mu}, \quad f_0 = \left[\exp\frac{\varepsilon_\mu - \zeta}{kT} + 1\right]^{-1}, \tag{3}$$

where f_0 is the Fermi distribution function, ζ the chemical potential of the electron system, and $\tau(\epsilon)$ the relaxation time:

$$\tau(\varepsilon) = \frac{\sqrt{2}\pi\alpha^2\hbar^2}{m^{1/2}|C|^2}\frac{1}{A(\varepsilon)},$$

$$A(\varepsilon) = \sum_N \left\{\frac{N_q + f_0(\varepsilon + \hbar\omega_0)}{[\varepsilon + \hbar\omega_0 - (N + 1/2)\hbar\Omega]^{1/2}} + \frac{N_q + 1 - f_0(\varepsilon - \hbar\omega_0)}{[\varepsilon - \hbar\omega_0 - (N + 1/2)\hbar\Omega]^{1/2}}\right\}, \tag{4}$$

$N_q = [\exp(\hbar\omega_0/kT) - 1]^{-1}$ is the Planck distribution function, C is the Fourier transform of the electron-phonon interaction potential, and $\alpha^2 = c\hbar/eH$.

Substituting (3) and (4) in (2), we obtain an expression for the current density in the form

$$j_z = -\frac{e^2\hbar E}{\pi|C|^2 m}\int d\varepsilon \frac{\partial f_0}{\partial \varepsilon}\frac{1}{A(\varepsilon)}\sum_N [\varepsilon - (N + 1/2)\hbar\Omega]^{1/2}. \tag{5}$$

The integral with respect to the energy can be easily estimated in the case of strong degeneracy, when the derivative of the Fermi function is approximated by a δ-function: $\partial f_0/\partial\epsilon = -\delta(\epsilon - \zeta)$.

Inasmuch as the longitudinal magnetoresistance is $\rho_{zz} = 1/\sigma_{zz}$, where σ_{zz} is the longitudinal component of the conductivity tensor, we have

$$\rho_{zz}(H) = \frac{-|C|^2 m}{e^2\hbar}\operatorname{sh}\left(\frac{\hbar\omega_0}{kT}\right) \cdot$$

$$\sum_N \{[\zeta + \hbar\omega_0 - (N + 1/2)\hbar\Omega]^{-1/2} + [\zeta - \hbar\omega_0 - (N + 1/2)\hbar\Omega]^{1/2}\}$$

$$\sum [\zeta - (N + 1/2)\hbar\Omega]^{1/2}$$

We see therefore that $\rho_{zz}(H)$ becomes infinite whenever the following condition is satisfied

$$\zeta \pm \hbar\omega_0 = \hbar\Omega(N + 1/2). \tag{7}$$

When $\zeta > \hbar\omega_0$, scattering with emission of an optical phonon and scattering with absorption of a phonon make contributions of the same order to ρ_{zz}. Since the probability of a transition with emission of an optical phonon is proportional to $[1 - f_0(\epsilon - \hbar\omega_0)](N_q + 1)$, and the

1)This takes place in nonpolar crystals [4]. The assumption C_q = const is not fundamental, but makes it possible to obtain an explicit analytic expression for τ in the entire electron-energy region. The character of the oscillations of the longitudinal magnetoresistance does not change if the dependence of C on q is taken into account.

probability of the transition with absorption is proportional to $[1 - f_0(\epsilon + \hbar\omega_0)]N_q$, it follows that in transitions in which the energy is $\epsilon \sim \zeta$ we have

$$[1 - f_0(\varepsilon - \hbar\omega_0)]\,(N_q + 1) \approx (1 - f_0(\varepsilon + \hbar\omega_0))\,N_q.$$

If $\zeta < \hbar\omega_0$, which is possible at not too high concentrations ($n < 3.8 \times 10^{15} \cdot (\hbar\omega_0 m/km_0)^{3/2}$) and at low temperatures, at which $\zeta \ll kT$, then obviously only transitions with absorption of an optical phonon take place. Then only the + sign is possible in the resonance condition (7).

It is seen from (7) that in a longitudinal magnetic field the resonance condition for a degenerate electron gas differs noticeably from the resonance condition in the case of a transverse field (1). Let us discuss the physical causes of such a difference. In the case of crossed fields, the motion of the electron in the absence of scattering occurs in a direction perpendicular to both the magnetic and electric field. All the electrons take part in this motion.

Scattering changes the state of electron motion and the center of the cyclotron orbit shifts along the electric field. Owing to the difference between the probabilities of the electron scattering along and against the field, a dissipative current is produced[5]. If the scattering is by optical phonons, then the electrons with energy $\zeta - \hbar\omega_0 \le \epsilon \le \zeta$ can go over, after scattering, into quantum states lying above the Fermi level. When the conditions in (1) are satisfied there take place, besides all other transitions, also transitions from the state characterized by the set of quantum numbers N, k_y, $k_z = 0$ into the state with the set of quantum numbers N′, k'_y, $k'_z = 0$. Since the density of the states has a singularity at the points $k_z = 0$, it follows that when $\Omega = \omega_0/M$ there are transitions in which the densities of the initial and final states have singularities. Therefore the probability of such transitions is anomalously large, as a result of which $\rho_{xx}(H)$ has a maximum at the value of the magnetic field corresponding to (1).

In the case of a longitudinal magnetic field E ∥ H ∥ z, the current is produced as a result of the acceleration of the electrons by the electric field. Then, owing to the Pauli principle, the electric field acts only on the electrons near the Fermi level. The electrons located deep under the Fermi level are in the equilibrium state. This means that in scattering by optical phonons the transitions of electrons from deep states to unoccupied states and vice versa are equally probable, unlike the situation in crossed fields. Therefore only the electrons lying near the Fermi level contribute to the current.

When an electron with a Fermi energy and having a set of quantum numbers N, k_y, k_z is scattered by an optical phonon, it can go over into a state with another set N′, k'_y, $k'_z = 0$. If condition (7) is satisfied, then the density of the final states has a singularity, and therefore $\rho_{zz}(H)$ has a maximum. As follows from (7), in the case of a simple parabolic band, the period of the oscillations depends both on the electron concentration and on the effective mass. Thus, the oscillations in question have features of both magnetophonon (MP) oscillations and Shubnikov–de Haas (SH) oscillations.

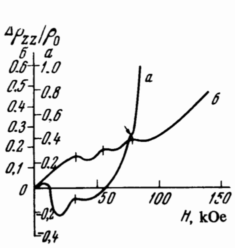

FIG. 1. Longitudinal magnetoresistance for HgTe sample. a – 20, b – 60°K.

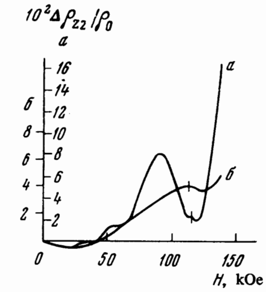

FIG. 2. Longitudinal magnetoresistance for GaSb sample with n = 2.5 × 10^{17} cm^{-3}, a – 20, b – 80°K.

We note one more feature of the foregoing oscillations of the longitudinal magnetoresistance (we call this the MPSH effect). As shown by Éfros[3], the oscillating part of the magnetoresistance in crossed fields, without allowance of the effects of cutoff, diverges logarithmically: $\rho_{xx}^{osc} \sim \ln\delta$, where $\delta = |N - \omega_0/\Omega|$.

In the case E ∥ H, as seen from (6), $\rho_{zz}^{osc} \sim \delta_{\pm}^{-1/2}$, where

$$\delta_{\pm} = |\zeta/\hbar\Omega \pm \omega_0/\Omega - N - 1/2|.$$

A similar singularity is possessed by the SH oscillations near resonance. In the case of SH oscillations, δ does not contain the term $\pm\omega_0/\Omega$, which reflects the inelastic character of the scattering. If the electrons experience, besides inelastic scattering by the optical phonons, also elastic scattering (for example, by acoustic phonons or by impurity ions), then $\rho_{zz}(H)$ always has a maximum at resonance, unlike the case of the nondegenerate electron gas.

We attempted to observe experimentally the MPSH oscillations on samples of HgTe and n-GaSb, in which the electron gas was degenerate. Figure 1 shows a plot of $\Delta\rho_{zz}/\rho_0 = (\rho_{zz}(H) - \rho_0)/\rho_0$ against H for an HgTe sample with electron density $n = 1.7 \times 10^{17}$ cm^{-3} at 20 and 60° K.

Maxima that could be related to electron transitions from the Fermi level to the Landau level were observed in magnetic fields H = 34, 55, and 78 kOe. In all the estimates, it is necessary to take into account the nonparabolicity of the conduction band. The energies of the Landau level and the Fermi energy in the absence of a magnetic field are determined in this case by the expressions

$$\varepsilon_{N^{\pm}} = -\frac{\varepsilon_g}{2} + \left\{\frac{\varepsilon_g^2}{4} + \varepsilon_g\left[\left(N + \frac{1}{2}\right)\hbar\Omega \pm \frac{1}{2}g\mu_B H\right]\right\}^{1/2}, \quad (8)$$

$$\zeta = -\frac{\varepsilon_g}{2} + \left\{\frac{\varepsilon_g^2}{4} + \varepsilon_g\frac{\hbar^2 k_F^2}{2m_n}\left(1 - \frac{m_n}{m_0}\right)\right\}^{1/2}, \quad (9)$$

where g—spectroscopic splitting factor at the bottom of the conduction band, ϵ_g—width of forbidden band, μ_B—Bohr magneton, $k_F^3 = 3\pi^2 n$, m_n—effective mass at the

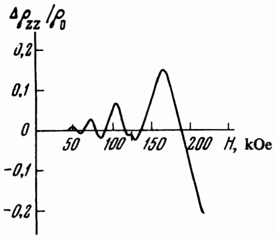

FIG. 3. Longitudinal magnetoresistance for GaSb sample with n = 6.2×10^{17} cm^{-3} at 20°K.

bottom of the conduction band, m_0—mass of free electron. Assuming for HgTe the values $\omega_0 = 2.17 \times 10^{13}$ sec^{-1} [6], $\epsilon_g = 0.15$ eV, $|g| = 40$, and $m_n = 0.02\, m_0$, we obtain from (8) the values of the magnetic fields satisfying the resonance condition $\zeta(H) \pm \hbar\omega_0 = \epsilon_N$.

All the calculated and experimental data are summarized in Table I. The symbol $\zeta - \hbar\omega_0 \rightarrow \epsilon_N^{\pm}$ denotes a transition from the Fermi level to one of the two spin-split Landau levels with quantum number N, with emission of an optical phonon, while $\zeta + \hbar\omega_0 \rightarrow \epsilon_N^{\pm}$ denotes a transition with absorption of a phonon.

As seen from Table I, the agreement between the calculated and experimental positions of the maxima is quite satisfactory. The calculated maxima at fields ~ 50 and 60 kOe are apparently not resolved under the experimental conditions, merging into one maximum at H ~ 55 kOe.

Table I

Transitions	H_{Theor} kOe	H_{exp}, kOe
$\zeta-\hbar\omega_0\rightarrow\epsilon_2^-$	33	34±3
$\zeta+\hbar\omega_0\rightarrow\epsilon_4^+$	34	
$\zeta+\hbar\omega_0\rightarrow\epsilon_4^-$	36	
$\zeta-\hbar\omega_0\rightarrow\epsilon_1^+$	48	55±5
$\zeta-\hbar\omega_0\rightarrow\epsilon_1^-$	60	
$\zeta+\hbar\omega_0\rightarrow\epsilon_3^-$	50	
$\zeta+\hbar\omega_0\rightarrow\epsilon_2^+$	63	
$\zeta+\hbar\omega_0\rightarrow\epsilon_2^-$	78	78±7

Figure 2 shows a plot (a) of $\Delta\rho_{zz}/\rho_0$ for a GaSb sample with n = 2.5×10^{17} cm^{-3} at 20°K. A maximum at H = 115 kOe is clearly seen against the background of the SH oscillations. At T = 80°K, the SH oscillations vanish (here $\hbar\Omega \sim kT$: it is known that SH oscillations are observed if $\hbar\omega \gg kT$), and the peak at 115 kOe increases in amplitude noticeably (curve b), this being due apparently to the greater role assumed by the electron scattering by optical phonons with increasing temperature.

Table II

Concentration, cm^{-3}	Transitions	H_{theor} kOe	H_{theor} kOe
$2.5\cdot10^{17}$	$\zeta+\hbar\omega_0\rightarrow\epsilon_2^+$	122	115±10
	$\zeta+\hbar\omega_0\rightarrow\epsilon_2^-$	127	
$6.2\cdot10^{17}$	$\zeta+\hbar\omega_0\rightarrow\epsilon_3^+$	117	123±11
	$\zeta+\hbar\omega_0\rightarrow\epsilon_3^-$	120	

The GaSb sample with n = 6.2×10^{17} cm^{-3} revealed maxima at H = 123 kOe (Fig. 3). Assuming for GaSb that $\omega_0 = 4.5 \times 10^{13}$ sec^{-1} [7], $\epsilon_g = 0.8$ eV, $|g| = 6$, and $m_n = 0.043 m_0$, we obtained the values of the magnetic field satisfying the resonance condition (Table II). As seen from Table II, the agreement between the calculated and experimentally observed positions of the maxima is satisfactory.

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158