

ULTRASHORT FLUCTUATION PULSES OF LIGHT IN A LASER

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It is shown that the spontaneous occurrence of picosecond pulses of light in a solid-state laser can be explained with the aid of the fluctuation intensity spikes produced by random interference of radiation in a large number of unphased axial modes. The most intense spikes, with an amplitude exceeding by one order of magnitude the average amplitude of the fluctuations, are registered as single picosecond pulses. The considered mechanism agrees well with experiment.

1. Recently much progress has been made in the generation of powerful light pulses of picosecond duration<sup>[1]</sup>. Interesting results were obtained in this direction recently by Duguay, Shapiro, et al.<sup>[2,3]</sup> In these investigations, spontaneous picosecond pulses were observed by the method of collision of light pulses in a medium with two-photon luminescence excitations<sup>[4]</sup> in the radiation of neodymium and ruby lasers operating in the Q-switched regime without a saturable solution or in the free-running regime. The mechanism of formation of such pulses is not clear. In<sup>[2]</sup>, their appearance is explained with the aid of coherent dynamic effects of the type of "180° pulse" in the amplifying medium<sup>[5-7]</sup>. However, this explanation is not very convincing, since the energy of the picosecond pulses is smaller by 2-3 orders of magnitude (and in the free-running regime<sup>[3]</sup> by 5-6 orders) than the gain saturation energy necessary for the formation of the "180° pulse."

It will be shown below that the spontaneous occurrence of picosecond pulses in a laser can be explained with the aid of fluctuation intensity spikes produced as a result of random interference between the radiation in a large number of axial modes. The most intense spikes are registered by the collision method as single picosecond pulses. Such a mechanism agrees well with the experiments of<sup>[2,3]</sup>. This mechanism is also responsible for the occurrence of the initial pulse in a laser with a saturable absorber.

2. The total radiation of the axial modes at any one point inside or outside the resonator is given by

$$E(t) = \sum_{n=1}^m A_n(t) \exp \{i[\omega_n t + \varphi_n(t)]\}, \quad (1)$$

where  $A_n$ ,  $\varphi_n$ , and  $\omega_n$  are the amplitude, phase, and frequency of the field in the n-th mode, and m is the number of modes. At the instant when the generation threshold is reached,  $A_n(t)$  and  $\varphi_n(t)$  are random functions of the time, determined by the statistical properties of the spontaneous emission in the mode. In the classical limiting case, when the average number of the photons in the mode is  $\langle n_0 \rangle \gg 1$ , the field in each mode  $E_n(t)$  is a Gaussian noise, whose random amplitude obeys the Rayleigh distribution, and the phase  $\varphi_n(t)$  is uniformly distributed in the interval  $(0, 2\pi)$ <sup>[8,9]</sup>. As the generation develops, the field in the mode is transformed into a coherent oscillation with a relatively

stable amplitude<sup>1)</sup>. In the stationary case, the field in the mode experiences only small fluctuations of intensity<sup>[11,12]</sup> and random drift of the phase. However, if there are no mechanisms for phasing the modes, then the phases of the different modes are independent and have a random distribution. Therefore the total field in a large number of axial modes remains as before a Gaussian noise, regardless of the coherence properties of the field in each mode<sup>[8,9]</sup>. Consequently, the fluctuations of the amplitude  $A(t)$  of the total field obey the Rayleigh distribution:

$$w(A) = \frac{2A}{\langle A^2 \rangle} \exp \left( -\frac{A^2}{\langle A^2 \rangle} \right), \quad (2)$$

and the fluctuations of the intensity  $I \sim A^2$  obey a distribution  $W(I) = w(A)|dA/dI|$  in the form

$$W(I) = \frac{1}{\langle I \rangle} \exp \left( -\frac{I}{\langle I \rangle} \right). \quad (3)$$

The average duration  $\tau_{fl}$  of the fluctuation intensity is determined by the width  $\Delta\nu$  of the frequency band occupied by the axial modes of the laser ( $\Delta\nu = cm/2L$ ,  $c$ —velocity of light,  $L$ —length of resonator):

$$\tau_{fl} \approx 1/\Delta\nu. \quad (4)$$

An intensity fluctuation distribution of the form (3) obviously admits of fluctuation intensity spikes with amplitude greatly exceeding the average fluctuation amplitude. The probability  $P(\beta)$  of a fluctuation spike with amplitude  $\beta$  times larger than the average fluctuation amplitude can be approximately<sup>2)</sup> estimated as follows:

$$P(\beta) = \int_{\beta\langle I \rangle}^{\infty} W(I) dI = e^{-\beta}. \quad (5)$$

The repetition frequency of the intensity fluctuations of any amplitude is of the order of  $\Delta\nu$ . Consequently, a fluctuation spike with amplitude  $\beta\langle I \rangle$  occurs on the average within a time interval  $\tau(\beta)$  given by the expression

$$\tau(\beta) \approx \frac{1}{\Delta\nu} e^\beta = \frac{T}{m} e^\beta, \quad (6)$$

1) This process of gradual transformation of the distribution function of intensity fluctuations was recently observed experimentally<sup>[10]</sup>.

2) A more exact estimate can be obtained by taking into account the spectral distribution of the modes<sup>[13]</sup>.

where  $T = 2L/c$  is the time of passage through the resonator, and  $m$  is the number of excited axial modes.

If the axial modes are strictly equidistant, then the random intensity  $I(t)$  is a periodic function with period  $T$ . The scatter of the frequency interval between the modes and the drift of the phase of the field in the modes violate the rigorous periodicity of the intensity fluctuations. An appreciable change in the random function  $I(t)$  in the time interval  $T$  occurs after a time  $\tau' \approx 1/\delta\nu$ , where  $\delta\nu$  is the average scatter of the difference between the field frequencies of the neighboring axial modes. It is clear that the condition for the occurrence of periodically repeating fluctuation spikes with amplitude  $\beta(I)$  is

$$\tau(\beta) \leq T. \quad (7)$$

Then the maximum excess  $\beta$  of the amplitude of the fluctuation spikes over the average value is, in accordance with (6) and (7),

$$\beta \approx \ln m. \quad (8)$$

In solid-state lasers, the number of excited axial modes can reach  $\sim 10^4 - 10^5$ . Consequently, fluctuation spikes are possible in them with an amplitude exceeding by one order of magnitude the average amplitude of the fluctuations. Naturally, such spikes can be registered as single ultrashort pulses of light. However, actually these are strong spikes against the background of a large number of weaker ultrashort fluctuation spikes. The main radiation energy is concentrated not in the spikes with maximum amplitude, but in the background of the average spikes. This follows from the fact that the number of all the spikes in the interval  $T$  is of the order of  $m$ , i.e., it is much larger than  $\beta$ . This indeed is the major difference between the fluctuation picosecond pulses and the pulses in a laser with a saturable absorber<sup>[1]</sup>. In the latter, the nonlinear absorber ensures predominant development of one or, in principle, several most intense fluctuation spikes.

3. The considered mechanism of spontaneous formation of ultrashort pulses agrees with the experiments of<sup>[2,3]</sup>. Thus, for example in a Q-switched laser the pulse duration should be

$$\tau \approx (\alpha_0 \tau_d / T)^{1/2} / \Delta\nu_0, \quad (9)$$

where  $\Delta\nu_0$  is the luminescence line width in Hz,  $\alpha_0$  is the gain per pass, and  $\tau_d$  is the radiation-pulse delay time relative to the instant of Q switching. In (9) it is assumed that the only mechanism of decreasing the number of excited modes is their natural selection by the active medium. In the experiment of<sup>[2]</sup>,  $\alpha_0 \approx 0.2$ ,  $T = 3$  nsec,  $\tau_d \approx 10^{-7}$  sec, and in accordance with (9) the observed pulse should have a duration of the order of 1 psec for a neodymium laser and 10 psec for a ruby laser. These values agree well with experiment<sup>[2]</sup>. The number of excited modes is  $m \approx 3 \times 10^3$  for a neodymium and  $m \approx 3 \times 10^2$  for a ruby laser. The maximum excess of the amplitude of the fluctuation spikes, defined by expression (8) is  $\beta \approx 8$  for a neodymium laser and  $\beta \approx 6$  for a ruby laser. Under the experimental conditions of<sup>[3]</sup>, during the time between spikes ( $\sim 10^{-6}$  sec), the natural selection narrows down the mode spectrum by another factor of 3, and consequently the pulse duration should be three times larger than in the experiment

of<sup>[2]</sup>. This also agrees with the obtained experimental data. The maximum excess of the amplitude of the fluctuation spikes over the average background can amount to  $\beta = 6-8$ . These spikes are registered by the collision method<sup>[4]</sup>, as single picosecond pulses. This is the main feature of the collision method.

4. An experimental verification of the fluctuation mechanism of the spontaneous formation of ultrashort pulses can be realized in several methods. First, by using measurements performed by some other method it is possible to investigate the excess  $\beta$  of the fluctuation-spike amplitude over the average background. This can be done, for example, by measuring the absolute peak power of the spike and the average background power. Second, it is possible to observe fluctuation spikes in the radiation near the threshold in intensified spontaneous noise, when the role of the nonlinear effects is negligibly small. Finally, it is possible to observe fluctuation spikes in the non-equilibrium radiation of a medium with inverted population, for example in a giant superluminescence pulse<sup>[14]</sup>. According to the presented concepts, superluminescence radiation in the diffraction solid angle should have a pico-second structure.

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