INFLUENCE OF SECOND SOUND IN FERROMAGNETS ON THE DAMPING OF ELASTIC WAVES

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Account is taken of the interaction of elastic waves with second sound in the magnon gas in ferromagnets. This interaction leads to an increase in the velocity of the elastic waves with increasing temperature, and to the appearance of two maxima on the plot of the relative damping decrement Γ/ω against the frequency.

1. The problem of the interaction of elastic waves (first sound) with temperature waves (second sound) in dielectrics has been recently considered in a number of papers[1-4]. A study of such coupled elastic-temperature waves makes it possible to relate the microscopic processes of interaction of elementary excitations of the lattice with the fundamental macroscopic characteristic of the elastic and thermal properties of the medium—the complex elastic-modulus tensor, which takes into account the spatial dispersion.

Unlike dielectrics, in ferrodielectrics there can exist, besides the second sound in the gas of Debye phonons, also second sound in the magnon gas. The conditions for the existence of second sound in a magnon gas were investigated by Gurzhi[5] and by Gulyaev[6]. In the region of sufficiently low temperatures, only long-wave phonons and magnons with frequencies far from magnetoacoustic resonance are excited, and therefore their interaction can be neglected. Under these conditions, the second sound in the phonon and magnon gases can be considered independently[1)]. It is shown in[4,5] that the velocity of second sound in a phonon gas is smaller than the velocity of first sound, so that the unperturbed frequencies of the first and second sound are not equal. To the contrary, in a magnon gas, these frequencies can become equal, since the velocity of second sound depends strongly on the temperature, and a temperature T^* may exist such that the unperturbed first-sound velocity s_{01} becomes equal to the unperturbed second-sound velocity s_{02}. At this temperature it is necessary, of course, to satisfy the inequality $\tau_U^{-1} < \tau_N^{-1}$ (τ_U^{-1} is the frequency of magnon collisions with momentum loss, and τ_N^{-1} is the frequency of the normal collisions); this inequality is required for the existence of second sound. Allowance for the interaction between the first and second sounds leads to a change in their velocities, which is maximal when $s_{01} \approx s_{02}$ and is determined by the quantity $s_{12} \sim (T^* c_V/\rho)^{1/2}$ (c_V is the specific heat of the magnons and ρ is the crystal density). Observation of such a change in velocities is possible if the total damping decrement of such oscillations, Γ, due to all the dissipative processes, satisfies the inequality

$$\Gamma(T^*) < qs_{12},$$

(q is the wave vector of the sound). It will be shown below that the contribution made to Γ by the interaction with the magnons (under the considered conditions) is of the order of $\tau_U^{-1}(T^*) + \omega^2\tau_N(T^*)$. It is necessary to take into account in Γ also the contribution $\sim\omega^2\tau'$ from the scattering by thermal phonons (τ'—relaxation time of the thermal phonons). Thus, the last inequality can be rewritten in the form

$$\frac{qs_{12}}{\tau_U^{-1}(T^*) + \omega^2[\tau_N(T^*) + \tau'(T^*)]} > 1.$$

This inequality may not be satisfied, but if $\tau_U^{-1} < \tau_N^{-1}$, then the existence of second sound in the magnon gas should lead, according to our calculations, to an increase (and not to a decrease, as usual) in the velocity of first sound when T approaches T^*.

For an experimental observation of the effects due to second sound in the magnon gas, it is more convenient, in our opinion, to investigate the temperature and frequency dependences of the relative damping decrement Γ_1/ω of first sound. We shall show subsequently that if the second sound in the magnon gas exists, then the function Γ_1/ω has two maxima, the first near the point $\omega \sim \tau_U^{-1}$ and the second near $\omega \sim \tau_N^{-1}$. We present below a derivation and an analysis of the formulas for the frequencies and damping decrements of the waves of first and second sound.

2. To describe the interaction of the sound with the magnons it is necessary to have an expression for the magnetoelastic-energy density. We confine ourselves to the continuous-medium approximation, which is convenient for the description of long-wave excitations. The magnetoelastic energy density of a medium with arbitrary symmetry can then be represented in the form[7]

$$W_{\text{me}} = g_{\alpha\beta\gamma\delta}M_\alpha M_\beta u_{\gamma\delta} + f_{\alpha\beta\gamma\delta}\frac{\partial M_\nu}{\partial x_\alpha}\frac{\partial M_\nu}{\partial x_\beta}u_{\gamma\delta}$$
$$+ G_{\alpha\beta\gamma\delta\mu\nu}M_\alpha M_\beta u_{\gamma\delta}u_{\mu\nu} + F_{\alpha\beta\gamma\delta\mu\nu}\frac{\partial M_\varepsilon}{\partial x_\alpha}\frac{\partial M_\varepsilon}{\partial x_\beta}u_{\gamma\delta}u_{\mu\nu}, \qquad (1)$$

where $\mathbf{M}(\mathbf{r})$ is the magnetization vector, $u_{\alpha\beta}$ is the deformation tensor, and g, f, G, and F are the tensors of the magnetoelastic constants and depend on the symmetry of the medium.

1) For the same reason, we shall henceforth take into account only processes of scattering of long–wave first–sound by magnons and its interaction with second sound in a magnon gas, far from magnetoacoustic resonance.

As usual, when spin waves are introduced, we assume the fluctuations of $\mathbf{M}(\mathbf{r})$ about the equilibrium value M_0 to be small, meaning that

$$\mathbf{M}(\mathbf{r}) = \mathbf{M}_0 + \mathbf{M}'(\mathbf{r}), \quad |\mathbf{M}_0| \gg |\mathbf{M}'|, \quad \mathbf{M}_0 \| 0z;$$

In addition we stipulate $|\mathbf{M}| = |\mathbf{M}_0|$. It follows therefore that $M'_z = -(M'^2_y + M'^2_x)/2M_0$. We substitute this representation of $\mathbf{M}(\mathbf{r})$ in (1) and retain only the terms quadratic in M'_x and M'_y. In the formula obtained in this manner for W_{me}, we expand $\mathbf{M}'$ in a Fourier series

$$\mathbf{M}'(\mathbf{r}) = \sqrt{\frac{2\mu_0 M_0}{V}} \sum_{\mathbf{k}} \mathbf{e}_{\mathbf{k}} (b_{\mathbf{k}}(t) e^{i\mathbf{k}\mathbf{r}} + b_{\mathbf{k}}^+(t) e^{-i\mathbf{k}\mathbf{r}}),$$

$\mathbf{e}_{\mathbf{k}}$ is a unit polarization vector, μ_0 is the Bohr magneton, and V is the volume of the system.

After statistical averaging, $\langle W_{me} \rangle$ can be expressed in terms of the single-magnon distribution function

$$f(\mathbf{k}, \mathbf{r}, t) = \sum_{\mathbf{k}'} \langle b_{\mathbf{k}'}^+ b_{\mathbf{k}} \rangle e^{-i(\mathbf{k}'-\mathbf{k})\mathbf{r}},$$

which characterizes the density of the number of magnons with wave number $\mathbf{k}$ at the point $\mathbf{r}$ and at the instant of time t. In the quantum case, when the $b_{\mathbf{k}}$ are replaced by the second-quantization operators, $f(\mathbf{k}, \mathbf{r}, t)$ is a single-particle density matrix of the magnons in the Wigner representation. As a result we obtain for the case of weak spatial inhomogeneities, $|\mathbf{k}' - \mathbf{k}|/k \ll 1$, $u|\mathbf{k}' - \mathbf{k}| \sim |\partial \mathbf{u}/\partial \mathbf{r}|$, the formula

$$\langle W_{me} \rangle = \sum_{\mathbf{k}} [c_{\alpha\beta}(\mathbf{k}) u_{\alpha\beta}(\mathbf{r}) + d_{\alpha\beta\gamma\delta}(\mathbf{k}) u_{\alpha\beta} u_{\gamma\delta}] f(\mathbf{k}, \mathbf{r}, t). \tag{2}$$

The expression in the square brackets in (2) is none other than the additions to the unperturbed magnon energy, due to the propagating acoustic waves. The tensors $c_{\alpha\beta}$ and $d_{\alpha\beta\gamma\delta}$ entering in (2) are determined completely by formula (1). In order of magnitude we have

$$c_{\alpha\beta}(\mathbf{k}) \sim d_{\alpha\beta\gamma\delta}(\mathbf{k}) \sim \mu_0 M + (ak)^2 \Theta_C,$$

where Θ_C is the Curie temperature and a is the lattice constant.

3. The magnon distribution function satisfies the Boltzmann equation

$$\frac{\partial f}{\partial t} + (\mathbf{v}\nabla) f - \frac{1}{\hbar}\left(\frac{\partial \varepsilon}{\partial \mathbf{r}} \frac{\partial}{\partial \mathbf{k}}\right) f = I^N[f] + I^U[f]. \tag{3}$$

Here

$$\begin{aligned} \varepsilon(\mathbf{k}, \mathbf{r}, t) &= \varepsilon(\mathbf{k}) + c_{\alpha\beta} u_{\alpha\beta}(\mathbf{r}, t) + \mu_0 \mathbf{n}_0 \mathbf{h}(\mathbf{r}, t), \\ \varepsilon(\mathbf{k}) &= 2\mu_0 H_0 + J(ak)^2, \quad \mathbf{n}_0 = \mathbf{H}_0 / H_0, \end{aligned} \tag{4}$$

H_0 is the constant magnetic field (the sum of the external field and of the anisotropy field), $J \sim \Theta_C$, and $\mathbf{h}(\mathbf{r}, t)$ is the self-consistent magnetic field of the magnons. The solution of Eq. (3) should satisfy the local energy and momentum conservation laws. Such solutions can be constructed, following[4], by introducing a local-equilibrium magnon distribution function that depends on the local values of the energy $\epsilon(\mathbf{k}, \mathbf{r}, t)$, the temperature $T(\mathbf{r}, t) = T_0(1 + \vartheta(\mathbf{r}, t))$, and velocity $\lambda(\mathbf{r}, t)$. It will be assumed henceforth that the relaxation to the local equilibrium is due principally to normal collisions, i.e., $I^N[f] \gg I^U[f]$ or $\tau_N^{-1} > \tau_U^{-1}$. The functions $\vartheta(\mathbf{r}, t)$ and $\lambda(\mathbf{r}, t)$ are determined in this case by the local energy and momentum conservation laws that follow from (3):

$$\int (I^N[f] + I^U[f]) \varepsilon(\mathbf{k}) d\mathbf{k} = 0, \tag{5}$$

$$\int \left\{ \frac{\partial f}{\partial t} + (\mathbf{v}\nabla) f - \frac{1}{\hbar}\left(\frac{\partial \varepsilon}{\partial \mathbf{r}} \frac{\partial}{\partial \mathbf{k}}\right) f \right\} \mathbf{k} d\mathbf{k} = \int I^U[f] \mathbf{k} d\mathbf{k}. \tag{6}$$

We can integrate (3) only in the relaxation-time approximation, by putting

$$I^N[f] = \left\{ f_0\left(\frac{\varepsilon(\mathbf{k}, \mathbf{r}, t) - \hbar\mathbf{k}\lambda - \varepsilon(\mathbf{k})\vartheta}{T_0}\right) - f \right\} \Big/ \tau_N,$$

$$I^U[f] = \left\{ f_0\left(\frac{\varepsilon(\mathbf{k}, \mathbf{r}, t) - \varepsilon(\mathbf{k})\vartheta}{T_0}\right) - f \right\} \Big/ \tau_U.$$

We then get from (3), in the approximation linear in $u_{\alpha\beta}$, λ, ϑ, and $\mathbf{h} \sim \exp[i(\omega t - \mathbf{q}\cdot\mathbf{r})]$, the following formula for the Fourier components of f:

$$\begin{aligned} f(\mathbf{k}, \mathbf{q}, \omega) &= f_0(\varepsilon(\mathbf{k})/T_0) + \frac{\partial f_0/\partial \varepsilon}{\omega - \mathbf{q}\mathbf{v} + i\tau^{-1}} \\ &\times \{[c_{\alpha\beta} u_{\alpha\beta} + \mu_0(\mathbf{n}_0\mathbf{h})](-\mathbf{q}\mathbf{v} + i\tau^{-1}) - i\tau_N^{-1}\hbar(\mathbf{k}\lambda) - i\tau^{-1}\varepsilon(\mathbf{k})\vartheta\}, \end{aligned} \tag{7}$$

where

$$\tau^{-1} = \tau_N^{-1} + \tau_U^{-1}, \quad \mathbf{v} = \hbar^{-1}\partial\varepsilon / \partial\mathbf{k}.$$

4. Our problem involves the quantities $\mathbf{u}$, λ, ϑ, and $\mathbf{h}$. The vector $\mathbf{u}$ is determined by the elasticity-theory equation; when account is taken of the magneto-elastic energy, this equation is of the form

$$\rho \ddot{u}_\alpha - \overset{0}{\Lambda}_{\alpha\beta\gamma\delta} \frac{\partial^2 u_\delta}{\partial x_\beta \partial x_\gamma} = \frac{\partial}{\partial x_\beta} \frac{\delta}{\delta u_{\alpha\beta}} \langle W_{me} \rangle. \tag{8}$$

The vector $\mathbf{h}$ satisfies Maxwell's equations

$$\operatorname{rot}\operatorname{rot}\mathbf{h} + \frac{1}{c^2}\ddot{\mathbf{h}} = -\frac{4\pi}{c^2}\ddot{\mathbf{M}}, \tag{9}$$

where

$$\mathbf{M} = \frac{\mathbf{H}}{H}\left\{ M_0 - \frac{\mu_0}{(2\pi)^3}\int f d\mathbf{k} \right\}, \quad \mathbf{H} = \mathbf{H}_0 + \mathbf{h}.$$

The functions λ and ϑ satisfy Eqs. (5) and (6). Using formulas (2) and (6)—(9) for the Fourier components of $\mathbf{u}$, λ, ϑ, and $\mathbf{h}$, we obtain the following system of homogeneous linear equations:

$$\begin{aligned} \Lambda_{\alpha\beta} u_\beta + A_{\alpha\beta}\lambda_\beta + B_\alpha\vartheta + C_{\alpha\beta} h_\beta &= 0, \\ A_{\beta\alpha} u_\beta + \pi_{\alpha\beta}\lambda_\beta + P_\alpha\vartheta + Q_{\alpha\beta} h_\beta &= 0, \\ B_\beta u_\beta + P_\beta\lambda_\beta + \varkappa\vartheta + R_\beta h_\beta &= 0, \\ C_{\beta\alpha} u_\beta + Q_{\beta\alpha}\lambda_\beta + R_\alpha\vartheta + \chi_{\alpha\beta} h_\beta &= 0. \end{aligned} \tag{10}$$

Here

$$\Lambda_{\alpha\beta} = \rho\omega^2\delta_{\alpha\beta} - (\overset{0}{\Lambda}_{\alpha\gamma\delta\beta} + 2\langle(-\mathbf{q}\mathbf{v} + i\tau^{-1}) c_{\alpha\gamma} c_{\beta\delta}\rangle + 2\langle d_{\alpha\gamma\beta\delta}\rangle_0) q_\gamma q_\delta,$$

$$\begin{aligned} \pi_{\alpha\beta} &= i\omega^{-1}\hbar^2\langle\tau_N^{-1}(\omega - \mathbf{q}\mathbf{v} + i\tau_U^{-1}) k_\alpha k_\beta\rangle, \\ \varkappa &= i\omega^{-1}\langle\tau^{-1}(\omega - \mathbf{q}\mathbf{v})\varepsilon^2\rangle, \end{aligned}$$

$$\begin{aligned} \chi_{\alpha\beta} &= \frac{c^2}{4\pi\omega^2}\left\{ \delta_{\alpha\beta}\left[q^2 - \frac{\omega^2}{c^2}\left(1 + \frac{4\pi M_0}{H_0}\right)\right] - q_\alpha q_\beta \right. \\ &\left. + \frac{\omega^2}{c^2}\frac{4\pi M_0}{H_0} n_{0\alpha} n_{0\beta} \right\} + \mu_0^2 n_{0\alpha} n_{0\beta} \langle(-\mathbf{q}\mathbf{v} + i\tau^{-1})\rangle, \end{aligned}$$

$$\begin{aligned} A_{\alpha\beta} &= \hbar\langle\tau_N^{-1} c_{\alpha\gamma} q_\gamma k_\beta\rangle, \quad B_\alpha = \langle\tau^{-1} c_{\alpha\gamma} q_\gamma \varepsilon\rangle, \\ C_{\alpha\beta} &= i\langle(-\mathbf{q}\mathbf{v} + i\tau^{-1}) c_{\alpha\gamma} q_\gamma\rangle \mu_0 n_{0\beta}, \\ P_\alpha &= i\omega^{-1}\hbar\langle\tau_N^{-1}(\omega - \mathbf{q}\mathbf{v})\varepsilon k_\alpha\rangle, \\ Q_{\alpha\beta} &= -i\hbar\langle\tau_N^{-1} k_\alpha\rangle \mu_0 n_{0\beta}, \quad R_\alpha = -i\langle\tau^{-1}\varepsilon\rangle \mu_0 n_{0\alpha}, \end{aligned} \tag{11}$$

where

$$\langle\ldots\rangle = \frac{1}{(2\pi)^3}\int d\mathbf{k}(\ldots)\frac{\partial f_0/\partial\varepsilon}{\omega - \mathbf{q}\mathbf{v} + i\tau^{-1}}$$

$$\langle\ldots\rangle_0 = \frac{1}{(2\pi)^3}\int d\mathbf{k}(\ldots) f_0.$$

Equating the determinant of this system to zero, we obtain the dispersion equation for the frequencies and for the damping decrement. It is interesting to note that the determinant of this system is symmetrical. This is not surprising, since the Onsager symmetry principle holds for the four fluxes under consideration.

If we omit the nondiagonal terms of $A_{\alpha\beta}$, B_α, $C_{\alpha\beta}$, $Q_{\alpha\beta}$, and R_α, then the dispersion equation breaks up into equations for the elastic waves

$$|\Lambda_{\alpha\beta}| = 0,$$

the temperature waves

$$\begin{vmatrix} \pi_{\alpha\beta} & P_\alpha \\ P_\beta & \varkappa \end{vmatrix} = 0$$

and the electromagnetic waves

$$|\chi_{\alpha\beta}| = 0.$$

A general analysis of the dispersion equation is a rather complicated matter, and we confine ourselves to a study of the particular case when $q \parallel H_0$, with $\tau\omega < 1$ and $\tau_U^{-1} < \tau_N^{-1}$. Neglecting terms of order $s^2/c^2 \ll 1$, we obtain from (10)

$$(\omega^2 - s_1^2q^2 + 2i\Gamma_1\omega)(\omega^2 - s_2^2q^2 + 2i\Gamma_2\omega) - s_{12}^2q^2(\omega^2 + 2i\Gamma_3\omega) = 0. \tag{12}$$

Here (when $\mu_0H_0 < T$) we have

$$s_1^2 = s_{01}^2 - aTc_v/\rho, \quad a \sim 1, \tag{13}$$

$$s_2^2 = \frac{\langle kv\varepsilon\rangle_{00}^2}{3\langle k^2\rangle_{00}\langle\varepsilon^2\rangle_{00}}\frac{\omega^2\tau_U^2}{1+\omega^2\tau_U^2} = s_{02}^2\frac{\omega^2\tau_U^2}{1+\omega^2\tau_U^2}, \quad s_{02}^2 \sim \frac{a^2\Theta_C T}{\hbar^2}, \tag{14}$$

$$s_{12}^2 = -\frac{2\langle\varepsilon c_{33}\rangle_{00}^2}{\rho\langle\varepsilon^2\rangle_{00}} \sim \frac{Tc_v}{\rho}, \tag{15}$$

$$\Gamma_1 = \frac{Tc_v}{\rho}q^2\tau, \tag{16}$$

$$\Gamma_2 = s_2^2q^2\frac{\tau_U + \alpha'\omega^2\tau\tau_U^2}{1+\omega^2\tau_U^2} + \alpha'\omega^2\tau, \quad \alpha' \sim 1, \tag{17}$$

$$\Gamma_3 = 2\omega^2\tau, \tag{18}$$

where

$$\langle\ldots\rangle_{00} = (2\pi)^{-2}\int k^2\partial f/\partial\varepsilon\, dk(\ldots).$$

In the case when $\tau_U\omega > 1$, the quantities s_2 and Γ_2 represent the velocity and the damping decrements of the second-sound waves in the magnon gas[5,6].

For the phase velocity and the damping decrement of coupled elastic-temperature oscillations we get from (12)

$$(\omega/q)_{1,2}^2 = \tfrac{1}{2}(s_1^2 + s_2^2 + s_{12}^2) \pm \tfrac{1}{2}[(s_1^2 - s_2^2)^2 + 2(s_1^2 + s_2^2)s_{12}^2 + s_{12}^4]^{1/2}, \tag{19}$$

$$\Gamma = \frac{(\omega^2 - s_2^2q^2)\Gamma_1 + (\omega^2 - s_1^2q^2)\Gamma_2 - s_{12}^2q^2\Gamma_3}{\omega^2 - s_2^2q^2 + \omega^2 - s_1^2q^2 - s_{12}^2q^2}. \tag{20}$$

We shall consider below only the case $s_{12}/s_1 \ll 1$. It is seen from (19) and (20) that the interaction is most significant when $s_1 = s_2$. Such a case is possible in principle, since $s_2 \sim 10^{-5}$ cm/sec when $T \sim 10\,^\circ$K and $\Theta_C \sim 10^{3\circ}$K. The oscillation frequency should satisfy in this case, of course, the condition $\omega\tau_U > 1$.

When $|s_1 - s_2| \ll s_{12}$, the phase velocity of the oscillations is described by the formulas

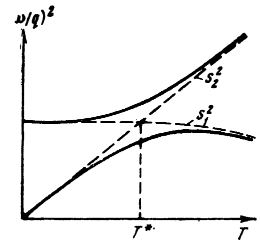
$$(\omega/q)_1 = s_1(1 + s_{12}/s_1), \quad (\omega/q)_2 = s_2(1 - s_{12}/s_2), \tag{21}$$

and the damping decrement is given by

$$\Gamma_1' = \Gamma_2' \approx \Gamma_1 + \Gamma_2 \approx \Gamma_1 + \tfrac{1}{2}\tau_U^{-1} + a\omega^2\tau, \quad a \sim 1. \tag{22}$$

It follows from (21) that the interaction between the first and second sounds leads, in a narrow temperature interval $\Delta T \sim T^*s_{12}/s$, to an increase of the velocities of both the first and second sounds as T approaches T^*, whereas allowance for the anharmonicity in solids leads to a decrease of the velocity of first sound[4]. From a comparison of these two changes of the sound velocity it follows that in ferromagnets the first cause is the more significant one, and leads to an increase in the velocity of first sound with increasing temperature. The approximate temperature dependence of ω/q is shown in Fig. 1.

FIG. 1.

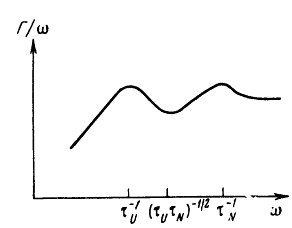


As already noted above, the experimental difference in the velocities of the first and second sounds at $T = T^*$ can be observed only if $\Gamma < s_{12}q$. Formula (22) gives an expression for the damping decrement under the condition $\tau_U^{-1} < \omega < \tau_U^{-1}$. From (10) we can also calculate the sound damping decrement in other frequency intervals. In the region of low frequencies, satisfying the inequality $\omega^2\tau_N\tau_U < 1$, we can obtain the formula

$$\Gamma \approx \Gamma_1 + \frac{s_{02}^2q\tau_U}{1+\omega^2\tau_U^2} \tag{23}$$

When $\omega \gtrsim \tau_U^{-1}$ this formula gives a result that agrees with (20). From (23) it follows that when $\omega \sim \tau_U^{-1}$ the function Γ_1/ω has a maximum. On the other hand, according to (22), when $\omega \approx (\tau_U\tau_N)^{-1/2}$ the quantity Γ_1/ω has a minimum. Near the frequency $\omega \sim \tau_N^{-1}$ (as can be verified with the equations (10)), Γ_1/ω reaches a second maximum, and at $\omega \gg \tau_N^{-1}$ the value of Γ_1/ω does not depend on τ_N and becomes a constant. The variation of Γ_1/ω described above is shown schematically in Fig. 2.

FIG. 2.



We recall that our equations (10) are exact only in the limit when τ_U and τ_N tend either to zero or to infinity (i.e., $\omega\tau \gg 1$ and $\omega\tau \ll 1$). In all the intermediate frequency intervals these formulas are of the interpolation type and in the best case can claim only a qualitative description of the phenomenon.

In conclusion we note that the broader the interval $(\tau_U^{-1} < \tau_N^{-1})$ and the closer T is to T^*, the more pronounced are the maxima on Fig. 2. Experimental observation of such a dependence would make it possible not only to confirm the existence of second sound in a magnon gas, but also to measure directly (which is more important) the value of τ_U, which determines the magnon part of the thermal conductivity, and also τ_N. Unfortunately, we cannot present reliable estimates for τ_N, and particularly for τ_U.

According to theoretical estimates, the main contribution to τ_N is made by magnon-magnon collisions. The order of magnitude of τ_N near $T \sim 10°K$, is 10^{-9} sec. As to τ_U, it is very difficult to estimate it, because at present there is no experimental method for separating the magnon part of the thermal conductivity of ferromagnets. We can therefore only point out the optimal conditions for a search of the discussed effect, namely maximally pure ferromagnets (to make τ_U^{-1} small), a temperature on the order of T^* (to satisfy the condition $s_1 \sim s_2$), and frequencies such that $\omega < \tau_N^{-1} \sim 10^{-9}\ \mathrm{sec}^{-1}$, but $\omega > \tau_U^{-1}$.

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