THEORY OF SUPERCONDUCTIVITY OF QUASI ONE-DIMENSIONAL STRUCTURES

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The effect of the finite probability of jumps between filaments on the nature of the superconducting transition is examined. It is shown that in a broad range of transition temperatures $\beta\epsilon_0 \lesssim T_c \lesssim \beta^{1/3}\epsilon_0$ (β is the jump probability and ϵ_0 is the Fermi energy) the superconducting state retains characteristic "one-dimensional" properties.[1,3]

IN a previous paper[1] the authors showed that in quasi one-dimensional (filamentary) structures a transition to the superconducting state is possible in principle whose properties coincide with those of the "one-dimensional" superconducting state considered by Little,[2] and Bychkov, Gor'kov and one of the authors.[3] At the same time only one of the factors preventing the transition to the superconducting state in the purely one-dimensional case was taken into account—the electron density fluctuations. There is, however, another factor which destroys superconductivity—peculiar fluctuations of the phase of the wave function of a Cooper pair at finite temperature. Their existence was first pointed out by Vaks, Galitskiĭ, and Larkin[4]; the effect of these fluctuations on the superconductivity in the one-dimensional case was first considered by Rice.[5]

The presence of phase fluctuations in filamentary media necessitates an account of the finite probability of a jump between filaments β ($\beta \ll 1$). This in turn leads to the fact that the superconducting transition temperature turns out to be bounded from above:

$$T_c \lesssim \beta^{1/3}\varepsilon_0. \tag{1}$$

For $T \gg \beta^{1/3}\epsilon_0$ the transition is in general impossible on account of the destructive action of the phase fluctuations. For $T_c \ll \beta\epsilon_0$ the superconducting state has the usual three-dimensional character, and finally for

$$\beta\varepsilon_0 \lesssim T_c \lesssim \beta^{1/3}\varepsilon_0 \tag{2}$$

a transition to the previously considered[1,3] "quasi one-dimensional" superconducting state is possible.

Inequality (1) can be derived as follows. In the purely one-dimensional case Hohenberg[6] derived the inequality

$$\int \frac{T\Delta^2}{q^2}dq < \infty,$$

where Δ is the gap. In the quasi one-dimensional case the inequality is replaced by

$$\int \frac{T\Delta^2}{q^2+\beta^2k^2}dq\,d^2k < \infty, \tag{3}$$

where k is the momentum of the transverse motion, or

$$T\Delta^2/\beta < \infty,$$

whence (1) follows for $\Delta \sim T \sim T_c$. The appearance of the quantity β^2 in the denominator of (3) is due to the fact that the coefficient of k^2 is according to the method of deriving Hohenberg's inequalities proportional to

$$\langle [j,\rho]\rangle \infty j_{mn}(\rho_{mm}-\rho_{nn}) \infty j_{mn}^2,$$

where j is the transverse current, ρ is the density, and the matrix elements are taken over states localized on different filaments.

If at zero temperature the gap $\Delta \ge \beta\epsilon_0$, then there exists a lower limit for the transition temperature

$$T_c \gtrsim \beta\varepsilon_0. \tag{4}$$

In order to derive this inequality, let us determine the temperature at which the above-mentioned phase fluctuations begin to affect the size of the gap appreciably.

Vaks, Galitskiĭ, and Larkin[4] have shown in the three-dimensional case rigorously that the correlation function of Cooper pairs

$$P(1,2) = -\langle T\psi_1^+\psi_1^+\psi_2\psi_2\rangle$$

has a peculiar singularity at $\omega_n = 0$ ($\omega_n = 2\pi nT$) and finite T, namely

$$P(\mathbf{K}) \infty \Delta^2/K^2,$$

K is the three-dimensional momentum. Unfortunately, one cannot derive an analogous formula rigorously in the quasi one-dimensional case[1,3] because in[1,3] all considerations are carried out in the logarithmic approximation. In order to obtain such a formula, one must proceed to the next approximation which meets with so far insurmountable difficulties.

We have made the natural assumption that in our case

$$P \infty \frac{\Delta^2}{q^2+\beta^2k^2}, \tag{5}$$

where q is the longitudinal and k the transverse momentum. Formula (5) can be obtained rigorously if one neglects the effect of the doubling of the lattice period.[3] Then $\beta \sim \alpha/\epsilon_0$ where α is the energy of the transverse motion from[1]. We note that (5) is in agreement with the Hohenberg inequality.

On the same basis one can use for the function Z connected with the doubling of the period the expression

$$Z \infty \frac{\varkappa^2}{(q\pm 2p_0)^2+\beta^2k^2} \tag{6}$$

where κ is the dielectric gap.[3]

The contribution of fluctuations of P and Z to the expression for the Green's functions is given by diagrams of the type shown in the Figure where the wavy line de-

notes P and the dashed line—Z.

The contribution of the first diagram is of the form

$$T\varkappa^2 \int \frac{dq\, d^2k}{q^2 + \beta^2 k^2} F(p+q).$$

In the region $T \lesssim \beta\epsilon_0 \lesssim \Delta \sim \kappa$ the important region of integration is $q \sim \beta p_0$. In it one can replace F by $1/\Delta$ which gives for the first diagram a contribution $T\Delta/\beta\epsilon_0$. Hence it follows that

$$T \lesssim \beta\varepsilon_0. \qquad (7)$$

The second diagram imposes a weaker limitation on T. The same region of integration $q \sim \beta p_0$ is important here. Replacing in it F by $1/\Delta$ and G by T/Δ^2, we find the contribution of this diagram to be $T^4/\beta^2\epsilon_0^2\Delta$ whence it follows that $T \lesssim (\beta\epsilon_0\Delta)^{1/2}$. One can convince oneself that the remaining diagrams do not impose on T a limitation which is stronger than (7).

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