THE BEHAVIOR OF A SUPERCONDUCTOR IN A VARIABLE FIELD

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The dynamical equations previously found for the model of a gapless superconductor are used for the calculation of some nonlinear high-frequency effects. The high-frequency region is primarily considered here, the parameter Δ mainly being determined by the time average of the square of the field intensity. Actually, the results do not depend on the choice of model. In each individual case, the boundary of the high frequency region should be specified. For type I superconductors, the behavior of Δ near the boundary is calculated for the presence of a large-amplitude high-frequency field. The case of the simultaneous effect of a constant and a varying field is also considered. Special interest attaches to the effect of a high frequency field on the hysteresis pattern calculated for a stationary Ginzburg field. Formulas are derived which describe the destruction of the superconductivity of films of various thicknesses in a variable field. For type II superconductors, the destruction of surface superconductivity by a high-frequency field is calculated. In particular, the dependence of the field strength H_{c3} on the variable field amplitude is found, and equations are obtained for the description of the dynamics of the surface layer.

1. INTRODUCTION

THE properties of superconductors in a nonstationary electromagnetic field have thus far been inadequately studied. On the one hand, experimental difficulties connected with heating in a strong variable field do not permit us to outline a clear picture of the phenomena, and on the other hand, theoretical results based on modern microscopic representations are virtually nonexistent. A general approach to the problem of nonstationary properties of a superconductor was proposed previously by the authors in[1], but it was noted at the same time that the general arrangement of the problem, independent of the experimental conditions, is possible in the region in which there are sufficient normal carriers to guarantee heat elimination, and the superconductivity is a relatively small effect. This is, above all, the region near the critical temperature or, for example, the case considered in detail in[1] of gapless superconductivity in alloys with paramagnetic impurities. For the latter problem, the generalization of the Ginzburg-Landau equation in a varying field was written in[1]:

$$\dot{\Delta}+\frac{\tau_s}{3}\left\{\pi^2(T^2-T_c^2)+\frac{|\Delta|^2}{2}\right\}\Delta+D\left(-i\nabla-\frac{2e}{c}\mathbf{A}\right)^2\Delta=0, \qquad (1)$$
$$\mathbf{j}=\sigma\mathbf{E}-\frac{2\sigma\tau_s}{c}|\Delta|^2(\mathbf{A}-\nabla\varphi).$$

Equations (1) are valid in the region of frequencies comparable with the characteristic scale frequency $\omega_0 = 2\tau_S|\Delta_0|^2$ (Δ_0 is the equilibrium value of the gap). These equations, obtained in[1] for the special case of paramagnetic alloys, nevertheless allows us to understand the qualitative picture of the phenomena taking place in superconductors in a variable field. In this connection, we note that the parameter $\varkappa$ of the Ginzburg-Landau theory in this system

$$\varkappa^2=c^2/48\pi\sigma D \qquad (2)$$

can be either small or large, depending on which initial pure metal was chosen for the preparation of the alloy. In other words, we shall regard below the system (1) as a reasonable model, which allows us to study the properties of both Pippard ($\varkappa \ll 1$) and London ($\varkappa \gg 1$) superconductors.

We transform to the dimensionless variables used in the Ginzburg-Landau theory:[2]

$$H'=\frac{H}{\sqrt{2}H_{cm}},\quad A'=\frac{A}{\sqrt{2}H_{cm}\delta_L},\quad \Delta'=\frac{\Delta}{\Delta_0}, \qquad (3)$$

and to the nondimensional time parameter

$$t'=\omega_0 t. \qquad (3')$$

(Here, we shall omit the primes for the nondimensional quantities. The transition to the ordinary variables is everywhere indicated in the corresponding places.) In what follows, we shall limit ourselves to the consideration of one-dimensional situations (films, half-space), where one can assume Δ to be real. In terms of the new variables for all quantities, which depend only on a single coordinate, the set of equations for Δ and the field A has the form

$$12\dot{\Delta}+(\Delta^2-1)\Delta-\frac{1}{\varkappa^2}\frac{\partial^2\Delta}{\partial z^2}+A^2\Delta=0,$$
$$\frac{\partial^2 A}{\partial z^2}=\dot{A}+\Delta^2 A. \qquad (1')$$

2. PIPPARD SUPERCONDUCTORS ($\varkappa \ll 1$)

For slow changes in the field, up to the point at which the amplitudes of the magnetic field is no longer smaller than the critical value, the change in Δ is small. If the fields exceed critical, then a first-order transition takes place in thick samples, as a result of which a moving separation boundary appears between the normal and superconducting phases. The kinetics of such a transition was considered by I. Lifshitz.[3] We shall therefore not linger over the details of the situation. We only note that the criteria of slowness of change in the field depends on the degree of perfection of the sample and of its surface, inasmuch as precisely these circumstances determine the formation of the boundary. We shall therefore study below the case of high frequencies ($\omega \gg 1$) where the formation and motion of the separation boundary are impossible.

For proof of the latter assertion, we recall that, as is well known, the structure of the boundary separating the two phases is characterized by two lengths: the London penetration depth and the scale of change in Δ (in dimensionless units, these are 1 and $1/\kappa$, respectively). Thus, for $\kappa \ll 1$, the location of the boundary is fixed with an accuracy to within a length ~ 1. At the same time, it is seen from the second of Eqs. (1′) that, during the period of change of the field, the boundary cannot go out from the surface a distance greater than $\omega^{-1/2} \ll 1$.

If $\omega \gg 1$, then $\Delta(t) = \Delta + \Delta_1(t)$, where the time average $\overline{\Delta_1(t)} = 0$ and the value of Δ_1 is small (see below). Therefore, the equation for the static part of the gap has the usual form of Ginzburg-Landau theory:

$$(\Delta^2 - 1)\Delta - \frac{1}{\kappa^2}\frac{\partial^2 \Delta}{\partial z^2} + \overline{A^2}\Delta = 0, \qquad (4)$$

where, however, the variable part of A, A_1 (before time averaging) is determined from the usual equations of the normal skin effect:

$$\partial^2 A_1 / \partial z^2 = \dot{A}_1. \qquad (4')$$

Assuming that the skin depth is less than all the characteristic dimensions of the problem, we find for A_1, in the case of a monochromatic field,

$$A_1 = \frac{h_1}{\sqrt{2}}\delta_s \exp\left(-\frac{z}{\delta_s}\right)\cos\left(\omega t - \frac{z}{\delta_s} - \frac{\pi}{4}\right),$$
$$\overline{A_1^2} = \frac{h_1^2 \delta_s^2}{4}\exp\left(-\frac{2z}{\delta_s}\right),$$

where h_1 is the amplitude of the alternating field, $\delta_S = \sqrt{2/\omega}$.

If a constant field h_0 also figures in the problem, then its penetration depth is Δ^{-1}.

A. Boundary of a Bulk Superconductor

Let us consider the superconducting half-space $z > 0$ and assume that the field is sufficiently small so that one can determine the change of Δ by perturbation theory: $\Delta = 1 + \Delta_1$ ($\Delta_1 \ll 1$). Then

$$2\Delta_1 - \frac{1}{\kappa^2}\frac{\partial^2 \Delta_1}{\partial z^2} + \frac{h_1^2}{4}\delta_s^2 \exp\left(-\frac{2z}{\delta_s}\right) + h_0^2 e^{-2z} = 0.$$

The solution of this equation with the boundary condition $(\partial\Delta/\partial z)_{z=0} = 0$ is

$$\Delta_1(z) = \frac{h_1^2 \delta_s^4 \kappa^2}{8(2-\kappa^2\delta_s^2)}\left[\exp\left(-\frac{2z}{\delta_s}\right) - \frac{\sqrt{2}}{\kappa\delta_s}\exp(-\kappa\sqrt{2}z)\right] \qquad (5)$$
$$+ \frac{h_0^2\kappa^2}{2(2-\kappa^2)}\left[e^{-2z} - \frac{\sqrt{2}}{\kappa}\exp(-\kappa\sqrt{2}z)\right]$$

or, for $\kappa \ll 1$ and $z = 0$,

$$\Delta(0) = 1 - \left(\frac{\kappa h_1^2 \delta_s^3}{8\sqrt{2}} + \frac{\kappa h_0^2}{2\sqrt{2}}\right). \qquad (5')$$

Solving the first of Eqs. (1′) for the variable part $\Delta_1(t)$, we find that the component $\Delta_{1,2\omega}$ is equal to ($\kappa \ll 1$)

$$\Delta_{1,2\omega}(z) = \frac{h_1^2\delta_s^4\kappa^2}{16}\left[\exp\left(-\frac{2z}{\delta_s}\right) - \frac{e^{i\pi/4}}{2\sqrt{3}\kappa}e^{-kz}\right],$$

where $k = 4\sqrt{3}(\kappa/\delta_S)e^{-i\pi/4}$. In other words, in the absence of a constant component, $\Delta_{12\omega}$ has a diffusion "tail" at comparatively small distances $z \sim \delta_S/\kappa$ and its value

$$\Delta_{1,2\omega}(z) \approx -\frac{h_1^2\delta_s^4\kappa}{32\sqrt{3}}e^{i\pi/4}e^{-kz}$$

is smaller by the factor $\delta_S \ll 1$ than the constant component $\Delta_1(z)$ of (5′). Therefore, we see that in the region where $h_1^2\delta_S^3\kappa \lesssim 1$, the stationary part of Δ is actually large in comparison with $\Delta_1(t)$, the rapidly oscillating part of the gap, which justifies our assumption in the derivation of the averaged equations (4).

Equations (5) and (5′) are naturally valid only so long as $h_1^2\delta_S^2\kappa \ll 1$ and $h_0^2\kappa \ll 1$. Continuation of the solution into the region of stronger fields when $\kappa \lesssim 1$ generally requires numerical calculations similar to those which were carried out by Ginzburg[4] in the determination of the supercooling field. In the region $\kappa \ll 1$, however, an analytical approach is possible, which was not noted in[1]. Having it in mind to study the qualitative picture, we now make use of it. The essential fact is that the field arises in the superconductor at a distance from the surface $\sim \delta_0$ for the variable and $1/\Delta(0)$ for the constant component of the magnetic field. If these distances are small in comparison with $1/\kappa$, then there is no field at great distances from the surface of the sample and the solution for $\Delta(z)$ is[2]

$$\Delta = \text{th}[\kappa(z + C)/\sqrt{2}]. \qquad (6)$$

The problem consists of connecting the solution in the region of change of the field with the solution (6). Taking it into account that in the region of the field itself, the function Δ and its derivative

$$\frac{\partial\Delta}{\partial z} = \kappa^2\left\{(\Delta^2(0) - 1)\Delta(0)z + \Delta(0)\int_0^z \bar{A}^2 dz\right\} \qquad (7)$$

are slowly changing, we can "join" (7) and (6) together at $z \ll 1/\kappa$. This is possible if

$$h_1^2\delta_s^3 \gg 1, \qquad h_0^2/\Delta^2(0) \gg 1. \qquad (8)$$

The conditions (8) overlap for $\kappa \ll 1$ with the region of applicability of (5) and (5′). Thus the "joining" conditions are

$$\Delta(0) = \text{th}(\kappa C/\sqrt{2}),$$
$$\kappa\Delta(0)\int_0^\infty \overline{A^2}dz = \frac{1}{\sqrt{2}\,\text{ch}^2(\kappa C/\sqrt{2})} = \frac{1}{\sqrt{2}}(1 - \Delta(0)^2). \qquad (9)$$

Comparing the first of the conditions (9), as $C \to \infty$,

$$\Delta(0) = 1 - 2e^{-\kappa\sqrt{2}C}$$

with the expression (5′), we see that C is logarithmically large for small $\kappa h_1^2\delta_S^3$ and $h_0^2\kappa$.

We now undertake a detailed study of the general case of (8). The second of Eqs. (9), in which C is contained, can be written in the form

$$h_0^2 = \frac{\sqrt{2}}{\kappa}\Delta(0)^2(1 - \Delta(0)^2) - \frac{h_1^2\delta_s^3\Delta(0)^3}{4}. \qquad (10)$$

If there is no constant field ($h_0 = 0$), it then follows from (10) that

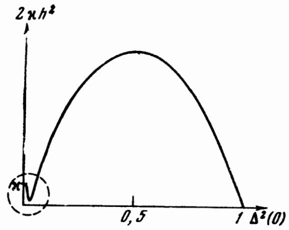

FIG. 1

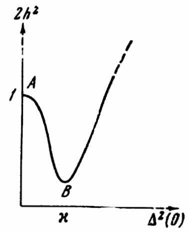

FIG. 2

$$\Delta(0) = -\frac{\varkappa h_1^2 \delta_s^3}{8\sqrt{2}} + \left(1 + \frac{\varkappa^2 h_1^4 \delta_s^6}{128}\right)^{1/2}. \qquad (11)$$

For small fields (11) coincides with (5′); in the limit of large field amplitudes,

$$\Delta(0) \approx 64 / \varkappa h_1^2 \delta_s^3 \qquad (11')$$

asymptotically approaches zero. Therefore, in the superconductor for $\Delta(x)$ in the limit of extremely strong fields, we obtain essentially the dependence

$$\Delta(z) = \operatorname{th}(\varkappa z / \sqrt{2}).$$

Thus, for the infinite half-space, the superconductivity is not destroyed in a high-frequency field. There is only one stationary regime. Once again, we recall that we assume a small Joule heating of the boundary, for which it is necessary to remove heat from the surface of the same.

Before concerning ourselves with the case of the complete Eq. (10), let us first make more precise the results obtained in[4], which pertain to the superheating field. Let $h_1 = 0$; then $h_0^2 = \sqrt{2}\,\kappa^{-1}\Delta(0)^2(1 - \Delta(0)^2)$. This is a curve with a maximum, i.e., $\Delta(0)$ as a function of h_0 is not well defined. The location of the maximum determines the superheating field H_s for $\kappa \ll 1$:

$$H_s = \frac{H_{cm}}{2^{1/4}\sqrt{\varkappa}} \approx \frac{0.84}{\sqrt{\varkappa}} H_{cm},$$

which differs somewhat from the result obtained in[4] by numerical calculation. Another circumstance is much more interesting. In Fig. 5 of[4] is given the curve $\psi(0)$ (i.e., $\Delta(0)$) of the field h_0. When inverted as a plot of h_0 vs. $\Delta(0)$, this curve has a single maximum, while the important fact is that this curve must reach the point (in our variables) $\Delta(0) = 0$, $h_0 = 1/\sqrt{2}$, which corresponding to a first order phase transition (formation of a boundary between the normal and superconducting phases). The region $\Delta(0) \to 0$ cannot be considered by our method, inasmuch as, along with the condition $h_0^2/\Delta^2(0) \gg 1$, the condition of the localization of the field near the boundary $1 \ll \Delta(0)/\kappa$ must be satisfied. However, upon satisfaction of the latter condition as $\kappa \to 0$ Eq. (10) ($h_1 = 0$) is still valid in the region $h_0 \ll 1$, $\Delta(0) \ll \sqrt{\kappa}$, i.e., curve (10) enters into the region of fields smaller than critical.

In Fig. 1 a graph is constructed of the dependence of $2\kappa h_0^2$ on $\Delta^2(0)$, from which it is seen that in the region indicated by the dashed line, $\Delta(0)$ has four values as a function of h_0. In the usual scale of critical fields, this region is shown schematically in Fig. 2. It is evident that the portion of the curve AB will also be metastable. It is physically probable that this portion corresponds to a certain surface current which screens the external field.

Our discussion refers to the limiting case $\kappa \to 0$. Now, assuming h_0 to be given in Eq. (10), we shall consider the destruction of the superconductivity by an alternating field in the presence of a constant field less than critical ($h_0 < 1/\sqrt{2}$). The dependence of $\Delta(0)$ on h_1, in contrast with (11) becomes non-single valued. The maximum of h_1^2 corresponds to the value

$$\Delta_{cr}^2 = \frac{3\sqrt{2}\,\varkappa h_0^2}{1 + [1 + 6\sqrt{2}\,\varkappa h_0^2]^{1/2}} \approx \frac{3}{\sqrt{2}} \varkappa h_0^2,$$

which generally corresponds to much larger amplitudes of the variable field

$$\frac{h_1^2 \delta_s^3}{4} = \frac{2h_0^2}{\Delta_{cr}^3} \sim \frac{1}{h_0 \varkappa^{3/2}}.$$

The value of Δ_{cr}^2 still satisfies our criterion of the localizability of the field

$$1 \ll \Delta_{cr} / \varkappa \sim h_0 / \sqrt{\varkappa}.$$

The existence of ambiguity of $\Delta(0)$ as a function of h_1 means that in the region $\Delta^2(0)$ there exists still another branch, similar to the case in a constant field discussed above, which is stable in the sense that $\Delta(0)$ falls off with increase in amplitude of the alternating fields. We did not succeed in analyzing this branch analytically. However, it can be established that in the presence of a constant magnetic field, an increase in the amplitude of the alternating field can produce a "collapse" of the regime, described by Eqs. (5), (5′) and (11). This confirmation generally refers only to sufficiently small κ ($\sqrt{\kappa} \ll 1$).

B. Superconducting Plates

As before, let $\kappa \ll 1$. We consider the case of a film with thickness $2d \ll 1/\kappa$. Then, as is known, we can neglect the dependence of Δ on the coordinate, as a result of which, we can omit in (1) terms with second derivative in z, and the set (1) is considerably simplified. In[1] the destruction of superconductivity by an alternating field was studied in detail for the case in which the dimensions of the layer are small in comparison with the penetration depth ($d \ll 1$). Such films in a magnetic field experience a second-order phase transition. In particular, it was discovered in[1] that in the case of a high-frequency field, the destruction of the superconductivity is determined by the mean-square field. Here we would wish only to add several observations, which touch on the properties of such a film in a slowly changing, strong field.

It was shown in[1] that generally, in a slow field, the change of $\Delta(t)$ follows the field change adiabatically. However, if we want to obtain a steady solution (one not depending on the initial conditions), in which a periodic behavior of Δ takes place in a periodically changing magnetic field then, in accord with[1], the condition $1 - h^2 > 0$ is always necessary (notation from[1]). In other words, the regime, even in a suitably slow field, in which the plate undergoes periodic transition from the superconducting state to the normal, cannot always be established and must "collapse" for sufficiently large amplitude. The new regime is determined by the value of the random fluctuations which arise during the time when the field is less than critical and the normal phase of the film is made absolutely unstable. Such a regime will certainly be aperiodic. The discovery of a collapse of the periodic regime and the study of the new regime is of considerable interest to us. We have based our arguments above on the formulas of[1], but it is clear that a similar "turbulent" regime will exist for all films with $2d < \sqrt{5}$, for which the phase transition in the magnetic field is of second order.[2] We are dealing here with variable fields with amplitude of the order of the value of the critical field H_{cm}.

We now proceed to the case of high frequencies $\omega \gg 1$ ($\delta_s \ll 1$) and $1 \ll d \ll 1/\kappa$. We shall assume that a constant magnetic field h_0 is applied to the film from both sides. So far as the variable field with amplitude h_1 is concerned, let it be applied only from one side. Inasmuch as $\delta \ll 1$, the variable skin field penetrates only very slightly into the layer. Averaging the field (4) over the z coordinate (from $-d$ to $+d$), we obtain by a direct solution of the Ginzburg-Landau equations[2] the relation

$$\frac{[\Delta^2-(1-\alpha_1)]\cdot 2\Delta^2\operatorname{ch}^2\Delta d}{1-(\operatorname{sh}2\Delta d)/2\Delta d}=h_0^2, \tag{12}$$

where

$$\alpha_1=\frac{1}{2d}\int_{-d}^{+d}\overline{A_1^2}\,dz=\frac{h_1^2}{16d}\delta_s^3.$$

By the substitution of variables

$$\Delta'=\frac{\Delta}{\sqrt{1-\alpha_1}},\quad d'=d\sqrt{1-\alpha_1},\quad h_0'=\frac{h_0}{1-\alpha_1} \tag{13}$$

we reduce it to an equation similar to that studied for films in[2,4].

If $h_0^2 = 0$, then $\Delta^2 = 1-\alpha_1$ falls off monotonically to zero with increase in the power of the variable field. We see that (comparable with[1]) the superconductivity for films with $d \ll 1/\kappa$ exists only if the amplitude of the field does not exceed some limit

$$h_1^2 < 4\sqrt{2}d\omega^{3/2}. \tag{14}$$

Let a certain level of power of the variable field be given. Then the phase transition of the film in the constant magnetic field is significantly changed. For $d' < \sqrt{5/2}$, this transition is of second order. In other words, according to (13), in the presence of radiation a second order phase transition takes place in a constant field for larger thicknesses:

$$d_{cr}=\sqrt{5}/2\sqrt{1-\alpha_1}. \tag{15}$$

The dependence of the critical field on the thickness is determined now by the relation

$$h_{c2}=\sqrt{3}d^{-1}\sqrt{1-\alpha_1}. \tag{16}$$

For $d > d_{cr}$, from (15), the phase transition is a first order transition. Equation (16) has the meaning of a supercooling field. The superheating field is determined by the maximum of the curve (12). For $d' \gg 1$, the field h' was determined in[4]: $h_s' = 0.6\sqrt{d'}$. Therefore, from (13),

$$h_s=0.6\sqrt{d}(1-\alpha_1)^{5/4}. \tag{17}$$

The relations (16) and (17) determine the region of hysteresis (the region of ambiguity of the dependence of Δ on h_0). It is evident that this region contracts. However, in the presence of an alternating field, we cannot establish the criterion which corresponds to the thermodynamic criterion determining that field h_{0cr} in which the first order phase transition will be actually established. Similar experiments on the displacements of the critical field can easily be conducted.

In turn, by fixing the constant field h_0 in the limits between (16) and (17), we conclude, inasmuch as h_0' in this range has associated with it two values of Δ, that the ambiguity of Δ exists also with respect to the dependence on the power of the variable field for fixed h_0. It is seen from (16) that this ambiguity, i.e., the possibility of collapse of the superconducting steady state, arises for

$$\alpha_1=1-\frac{h_0^2d^2}{3}\quad(d<d_{cr}).$$

The new regime obviously corresponds to the skin effect in the normal phase.

We have shown above that in the semi-infinite sample, the superconductivity does not vanish in a rapidly alternating field up to very high powers. On the contrary, according to (14), there is always a limiting value of the power in a thin film, above which the superconducting state cannot exist. We shall show that for thin films $\kappa d \sim 1$, a transition takes place from the first situation to the second.

For this case, we consider a thick film ($\kappa d \sim 1$) in a rapidly changing field ($\delta_s \ll 1$), applied from one side ($z = 0$). Then, as also for the case of an infinite half-space, there is no field in the greater part of the film thickness. However, the gap is not equal to unity on the second wall (at $z = 2d$). We shall denote its value $\Delta(2d) \equiv \Delta_1$; then the first integral of Eq. (4) without a field can be written in the form

$$\frac{d\Delta}{dz}=\varkappa\left[(\Delta_1^2-\Delta^2)\left(1-\frac{\Delta_1^2+\Delta^2}{2}\right)\right]^{1/2},$$

where we used the fact that $(\partial\Delta/\partial z)_{z=2d} = 0$. Assuming as in Sec. A that

$$\int_0^\infty\overline{A^2}\,dz=\alpha\gg\delta_s,$$

and joining the results, we obtain two conditions

$$\int_{\Delta_0}^{\Delta_1}d\Delta\left[(\Delta_1^2-\Delta^2)\left(1-\frac{\Delta_1^2+\Delta^2}{2}\right)\right]^{-1/2}=2\varkappa d,$$

$$\left[(\Delta_1^2-\Delta_0^2)\left(1-\frac{\Delta_1^2+\Delta_0^2}{2}\right)\right]^{1/2}=\Delta_0\varkappa\alpha$$

(where $\Delta_0 = \Delta(0)$, $\alpha = h_1^2\delta_s^3/8$). Introducing the variable $\Delta/\Delta_1 = \cos z$, $0 < x < \pi/2$, we transform both conditions to the much more convenient form

$$\int_0^u\frac{dx}{\sqrt{k^2+\sin^2x}}=\frac{2\varkappa d}{\sqrt{k^2+2}}, \tag{18}$$

$$\sqrt{k^2+\sin^2u}=\alpha\varkappa\sqrt{k^2+2}\operatorname{ctg}u, \tag{19}$$

while

$$u=\arccos(\Delta_0/\Delta_1),\qquad \Delta_1=\sqrt{2/(k^2+2)}. \tag{20}$$

Equation (19) can be solved relative to u:

$$\cos^2 u = 1 + {}^1/_2\{k^2 + \alpha\varkappa - [(k^2+\alpha\varkappa)^2 + 4\alpha\varkappa]^{1/2}\}.$$

After this, the problem of the determination of Δ_0 and Δ reduces to the numerical solution of Eq. (18). However, we shall not deal with this, bearing in mind only to show that there exists some critical thickness, above which no increase in the power can destroy the superconductivity over the entire volume of the film. For this purpose, we note that the destruction of superconductivity means $k \to \infty$ ($\Delta_1 \to 0$). It then follows from (18) that $u = 2\kappa d$, while

$$\varkappa d < \pi/4. \tag{21}$$

Here the threshold power that destroys superconductivity is determined by

$$1 = a_{cr}\varkappa \operatorname{ctg} 2\varkappa d. \tag{22}$$

For $\kappa d \ll 1$, we again get (13). For $\kappa d \sim 1$, we see that for destruction of the superconductivity in the film by the alternating field alone, high powers are necessary ($\kappa \ll 1$):

$$h_1^2\varkappa\omega^{-3/2} \sim 1.$$

Summing up what has been said about type I superconductors, we note only that although the majority of the formulas were obtained by us in the limiting case $\kappa \ll 1$, the phenomena described above undoubtedly are also valid for $\kappa \sim 1$. In this case, the situation deals with entirely realistic powers of the alternating field.

3. LONDON SUPERCONDUCTORS (ALLOYS WITH $\kappa \gg 1$)

The case of low frequencies of the alternating field for alloys is even more complicated in comparison with type I superconductors. If the amplitude of the variable field exceeds the critical value of the field H_{c1}, for which vortices are formed, then the penetration of the field into the superconductor is connected with the motion of the quantized vortices. This effect has always had a hysteresis character, both because of the presence of the surface barrier and because of the imperfection of the sample. Therefore, it seems to us that one should consider the general picture of a similar nature rather on a phenomenological basis rather than on the basis of a microtheory constructed for the homogeneous alloy. In this connection, we limit ourselves to the remark that the "motion" of the boundary of the field, for the reasons given, takes place with a velocity v that is less than follows from the second of Eqs. (1'): $v < \sqrt{\omega}$, and in what follows, we shall study the case of high frequencies $\delta_s \ll 1/\kappa$, where the motion of the vortices is practically non-existent.[1)]

In weak fields we have (5') for the correction to the gap as before ($\kappa \gg 1$, but $\kappa\delta_s \ll 1$). For strong fields, we can apply the method of "joining" the two solutions described by Eqs. (7), (9) of the previous section. The joining together of the two solutions (7) and (6) takes place at distances $z \sim \delta \ll 1/\kappa$. Therefore, the criterion of applicability of the method is

$$h_1^2\delta_s^3 \gg 1 \tag{23}$$

and Eq. (5') ($\kappa\delta_0 \ll 1$) can be obtained from (11) as the limiting case.

Thus, in this limiting high-frequency case, all the results of the previous section are valid. These were obtained for the half-space and film (if $h_0 = 0$). Once again, however, we note the fact that for films with $d > 1/\kappa$ for type II superconductors, there is the possibility of a new solution with the formation of vortices inside the film.

We now proceed to the inverse situation, where $\delta_s \gg 1/\kappa$.

$$\Delta^2(z) = 1 - \overline{A^2}(z). \tag{24}$$

The applicability of the quasiclassical solution (24) is limited to the vicinity of x_0, where $\Delta(z_0) = 0$. If the field is large enough,

$$h_1^2\delta_s^2/4 \gg 1, \tag{25}$$

and such a point exists, one must connect the solution (24) with the solution of the equation ($x = z - z_0$)

$$\frac{1}{\varkappa^2}\Delta'' + \frac{4x}{\delta_s}\Delta - \Delta^3 = 0 \tag{26}$$

with the conditions for the latter

$$\partial\Delta/\partial x \to 0, \quad x \to -\infty, \quad \Delta \propto x^{1/2}, \quad x \to +\infty.$$

By the substitution

$$x = (\delta_s/4\varkappa^2)^{1/3}\xi, \qquad \Delta = \bar\Delta(4/\delta_s\varkappa)^{1/3}$$

we reduce (26) to the form

$$\bar\Delta'' + \xi\bar\Delta - \bar\Delta^3 = 0. \tag{26'}$$

Equation (26') is the transcendental equation of Painleve (see [7]), relative to which it is shown that this solution is always a single-valued function of the complex variable ξ. Thus, even in this case, the superconductivity in a strong variable field for the half-space changes monotonically with increase in the power. However, in contrast with the type I superconductors (see [6]), to the left of the point z_0 there arises a region of normal phase. The very point z_0 moves to the right with increase in the field. Without solving (26) and (26'), we can see that, to the left of z_0, Δ decays according to the law

$$\Delta \propto \exp\left\{-\frac{4}{3}\frac{\varkappa}{\sqrt{\delta_s}}|z - z_0|^{3/2}\right\}. \tag{27}$$

The two latter problems, considered below in this section, pertain to the surface superconducting layer in fields $H_{c2} < H < H_{c3}$, the existence of which was shown first by Saint-James and de Gennes[8]. We shall first show that a sufficiently large varying field destroys the surface layer. According to[8], we ought to study the existence of an infinitely small localized solution of Eq. (1) for Δ in a constant field h_0 and an alternating field $h_1(t)$. Let $\delta_s \gg 1/\kappa$ and the solution $\Delta(z, y)$ is sought in the form $\Delta(x, y) = \Delta(z)e^{iky}$. In correspondence with[8],

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1) In experiments in a variable field, the amplitude of which is greater than H_{cl}, it is naturally always possible to have the existence of a finite vortex density in the sample which is oriented in a different way from these, so that the mean field connected with them was equal to zero. The formation of vortices is always possible, since this is one of the mechanisms of Joule loss. The regime in the variable field in the present of vortices, which are formed in fluctuating fashion, will necessarily be aperiodic (it is possible that a similar picture was observed in [5,6]; however, theoretical investigation of it is too complicated, and we shall in what follows digress from the possibility of a similar mechanism of collapse of the established regime.

we get for $\omega \gg 1$ Eq. (4) in the form

$$-\frac{1}{\varkappa^2}\frac{\partial^2\Delta_0}{\partial z^2} + \left[h_0^2(z-z_0)^2 + \frac{h_1^2}{4}\delta_s^2\exp\left(-\frac{2z}{\delta_s}\right)\right]\Delta_0 = \Delta_0. \quad (28)$$

The region of change of $\Delta_0(z)$ is of the order of $z \sim z_0 \sim 1/\kappa$ (see below). Therefore the exponent above can be replaced by unity. By the change of variables

$$z' = z\left(1 - \frac{h_1^2\delta_s^2}{4}\right)^{1/2}, \quad h'^2 = h_0^2\left(1 - \frac{h_1^2\delta_s^2}{4}\right)^{-3/2} \quad (29)$$

we transform (28) to the form studied in[8]. This equation has its first solution for $h' = 1.7\kappa$. Then

$$h_{c3} = 1.7\varkappa\left(1 - \frac{h_1^2\delta_s^2}{4}\right)^{3/4}.$$

As soon as h_{c3} becomes equal to κ ($H = H_{c2}$), the superconducting state becomes absolutely unstable in the bulk of the sample. Thus the condition for the vanishing of the surface layer is

$$1{,}7\left(1 - {}^1\!/_4 h_1^2\delta_s^2\right)^{3/4} = 1. \quad (30)$$

In conclusion, we consider a problem of unquestioned methodological interest—the derivation of equations which control the superconductivity in the surface layer for a varying field. We shall consider the case of small subcriticalities:

$$H(t) - H_{c3} \ll H_{c3} \quad (31)$$

and low frequencies. Thus, let $h(t) = h_{c3} + h_1(t)$. Then, assuming that $\Delta = c(t)\Delta_0(z)$, where $\Delta_0(z)$ is the solution of (28) for $h_1 = 0$, and multiplying the first of Eqs. (1) by this expression, integrating over z, we get

$$12c\dot{c}\overline{\Delta_0^2} + c^4\overline{\Delta_0^4} - c^2\overline{\Delta_0^2} - \frac{c^2}{\varkappa^2}\overline{\left(\Delta_0\frac{\partial^2\Delta_0}{\partial z^2}\right)} + \overline{(A' + h_{c3}(z-z_0))^2\Delta_0^2 c^2} = 0. \quad (32)$$

Here integration over z is indicated by the bar, A' is the field composed of the external field h_1 and the field of the superconducting currents. By virtue of the smallness of h_1 we can expand (31) in Eq. (32) in terms of A':

$$12c\dot{c}\overline{\Delta_0^2} + c^4\overline{\Delta_0^4} + 2h_{c3}\overline{A'(z)(z-z_0)\Delta_0^2(z)}c^2 = 0. \quad (33)$$

The field A' satisfies the equation

$$\partial^2 A'/\partial z^2 = \dot{A}' + c^2\Delta_0^2(z)\{h_{c3}(z-z_0) + A'(z)\}. \quad (34)$$

Equation (34) must be solved by the method of successive approximations, taking it into account that c is small. For this purpose, we introduce the notation

$$A'(z) = A_1'(z) + A_S(z),$$

where

$$\partial^2 A_S/\partial z^2 = c^2 h_{c3}\Delta_0^2(z)(z-z_0).$$

Then, following[9], we integrate the last term in (33) by parts:

$$2\overline{A'(z)h_{c3}(z-z_0)\Delta_0^2(z)c^2} = -2\overline{h'(z)h_S(z)}, \quad (35)$$

where $h' = \partial A_1'/\partial z + \partial A_S/\partial z$ and we have used the property noted in[8] that the total current in the superconducting layer is equal to zero:

$$h_{S,\,z=0} = h_{c3}c^2\int_0^\infty \Delta_0^2(z)(z-z_0)\,dz = 0. \quad (36)$$

For A_1', we have, according to (34), the equation

$$\frac{\partial^2 A_1'}{\partial z^2} = \dot{A}_1' + \dot{A}_S + c^2\Delta_0^2(z)[A_S + A_1']. \quad (34')$$

In Eq. (33), the term with the time derivative is substantial in the region $\omega \sim c^2$. We shall be guided by this scale of frequencies. It is then seen from (36) that the screening currents are small and the field h' in the first approximation can be set equal to $h' = h_1(t) + h_S$. Thus (35) is transformed to the form

$$-2h_1(t)\overline{h_S(z)} - 2\overline{h_S^2(z)}, \quad (37)$$

where

$$h_S(z) = -c^2 h_S\int_z^\infty \Delta_0^2(z)(z-z_0)\,dz.$$

The calculation of all the integrals entering into (33) and (37) is generally possible only numerically. However, in the limit $\kappa \gg 1$, the calculation can be greatly simplified. In particular, it is possible to omit the term $\overline{h_S^2}$ in (37). Actually, this term makes a contribution to the coefficient before c^4 in (33). Let $\overline{\Delta_0^2} = 1$. Then, inasmuch as the characteristic length dimension is $1/\kappa$, while the field is $h_{c3} \sim \kappa$, we have

$$\overline{\Delta_0^4} \sim \varkappa, \quad h_S(z) \sim c^2 F(\varkappa z),$$
$$\overline{h_S(z)} \sim c^2/\varkappa \sim c^2/h_{c3}, \quad \overline{h_S^2(z)} \sim c^4/\varkappa.$$

Final formulas are conveniently obtained by using the variational method of determination of $\Delta_0(z)$ proposed by Kittel (see[9]). Choosing $\Delta_0(z)$ in the form e^{-rz^2}, we find

$$r = \frac{\varkappa h_{c3}}{2}\sqrt{1 - \frac{2}{\pi}}, \quad z_0 = \frac{1}{\sqrt{2\pi r}}, \quad h_{c3} \approx 1.66.$$

Using the latter expression, after simple calculations, we get

$$12\dot{c} + \frac{c^3}{\sqrt{2}} + \frac{h_1(t)}{h_{c3}}c = 0. \quad (38)$$

The general solution of an equation of the type (38) is given in[1]. By means of it one can, from Eqs. (34'), find the corrections, for example, to the impedance in a metal, arising from the surface layer.

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