NONLINEAR LOW-FREQUENCY OSCILLATION SPECTRA OF AN ANISOTROPIC PLASMA

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The effect of high-frequency Langmuir turbulence on the nonlinear low-frequency oscillation spectra of an anisotropic plasma is investigated. It is shown that under certain conditions aperiodic instabilities appear in such a plasma and a slight temperature anisotropy of the plasma may remain.

GREAT interest has been shown recently in the problem of nonlinear interaction of different collective degrees of freedom in a plasma. Vedenov et al.[1] and Gaĭlitis[2], using ion-acoustic oscillations as an example, have already demonstrated the strong influence excerted on the dispersion properties of plasma at low frequencies (LF) by the presence of high-frequency (HF) and specially Langmuir turbulence. This fact was fully confirmed in subsequent investigations[3,4] of the excitation of spontaneous magnetic fields by HF turbulence and nonlinear-drift instabilities, and a general method of investigating similar problems was proposed

It is known that an aperiodic LF instability is produced in a plasma having temperature anisotropy, say anisotropy of the electron temperature, in the region $\omega \ll kv_{Te}$ (the symbols are defined below). As the result of the development of this instability, as shown in[5], the plasma relaxes to an isotropic state. On the other hand, it is known that a rather considerable temperature anisotropy is observed in the interplanetary plasma. The purpose of the present paper is to clarify the influence of the Langmuir HF turbulence on the dispersion properties of the anisotropy plasma in the frequency region $\omega \ll kv_{Te}$.

It was noted above that a method of obtaining the nonlinear dielectric constant ϵ_{ij} under rather general assumptions was developed earlier[3,4], so that we shall present here only the final results for ϵ_{ij}:

$$\varepsilon_{ij}^{(e)} = \delta_{ij} + \frac{4\pi e^2}{\omega m_e}\int \frac{v_i[\delta_{lj}(1-\mathbf{kv}/\omega) + k_l v_j/\omega]}{\omega - \mathbf{kv} + i\delta}\frac{\partial f_0}{\partial v_l}d\mathbf{v}$$
$$+ \frac{4\pi e^2}{m_e\omega}\int\frac{v_i d\mathbf{v}}{\omega - \mathbf{kv} + i\delta}\frac{\partial}{\partial v_l}(\omega - \mathbf{kv})\frac{\partial f_0}{\partial v_s}\left\{d_{ls}\int\frac{d\mathbf{v}'}{\omega - \mathbf{kv}' + i\delta}\right.$$
$$\times\left[\left(\delta_{mj}\left(1 - \frac{\mathbf{kv}'}{\omega}\right) + \frac{k_m v_j'}{\omega}\right)\frac{\partial f_0(\mathbf{v}')}{\partial v_m'} + d_{mnj}\frac{\partial}{\partial v_m'}(\omega - \mathbf{kv}')\frac{\partial}{\partial v_n'}f_0(\mathbf{v}')\right]$$
$$\left.\times\left[1 - d_{rp}\int\frac{d\mathbf{v}''}{\omega - \mathbf{kv}'' + i\delta}\frac{\partial}{\partial v_r''}(\omega - \mathbf{kv}'')\frac{\partial f_0(\mathbf{v}'')}{\partial v_p''}\right]^{-1} + d_{lsj}\right\} \quad (1)$$

We have introduced here the following symbols: ω, $\mathbf{k}$–frequency and wave vector of the LF oscillations; ω_1, $\mathbf{k}_1$–frequency and wave vector of HF pulsations; f_0–distribution function of the fundamental (turbulent) state. The coefficients d_{ij} and d_{ij} are determined from the formulas

$$d_{ij} = -\frac{\pi e^2}{n_0\omega_{0e}m_e^2}\int d\mathbf{k}_1\frac{k_{1i}k_{1j}}{k_1^2}\frac{(\mathbf{k}\partial/\partial\mathbf{k}_1)W_{\mathbf{k}_1}}{\omega - \mathbf{kv}_{g1} + i\delta}, \quad (2)$$

$$d_{ijl} = \frac{6\pi e^2}{m_e\omega_{0e}^3}\int d\mathbf{k}_1\frac{k_{1i}k_{1j}k_l}{k_1^2}\frac{W_{\mathbf{k}_1}}{\omega - \mathbf{kv}_{g1} + i\delta} + \frac{\pi e^2}{m_e^2\omega\omega_{0e}^2}$$
$$\times\int d\mathbf{k}_1\frac{k_{1i}k_{1j}}{k_1^2}k^2\frac{(k_{1l} - (\mathbf{kk}_1)k_l/k^2)(\mathbf{k}\partial/\partial\mathbf{k}_1)W_{\mathbf{k}_1}}{\omega - \mathbf{kv}_{g1} + i\delta}, \quad (3)$$

where $W_{\mathbf{k}_1} = |e_{\mathbf{k}_1}|^2/4\pi$ is the spectral energy density of the HF oscillations. $\mathbf{v}_{g_1} = (d\omega_{\mathbf{k}_1}/d\mathbf{k}_1)$ is their group velocity, and $\omega_{\mathbf{k}_1}$ is the solution of their linear dispersion equation¹⁾. The coefficients (2) and (3) were written out accurate to $|e_{\mathbf{k}_1}|^2$, higher-order terms have been discarded²⁾, $W = \int W_{\mathbf{k}_1}d\mathbf{k}_1$, and T_e is the electron temperature.

We proceed to investigate the dispersion equation. It is advantageous to choose a coordinate system such that $\mathbf{k} = \{k, 0, 0\}$, $\mathbf{E} = \{0, 0, E\}$, and the distribution function is

$$f_0 = \frac{n_0}{2\pi v_\| v_\perp}\exp\left\{-\left(\frac{v_z^2}{2v_\perp^2} + \frac{v_x^2}{2v_\|^2}\right)\right\}, \quad v_\| = \sqrt{\frac{T_\|}{m_e}}, \quad v_\perp = \sqrt{\frac{T_\perp}{m_e}}, \quad (4)$$

then the dispersion equation of the collective motions of the plasma, leading in the linear approximation to an anisotropic instability, is[6]

$$k^2c^2/\omega^2 - \varepsilon_{33} = 0. \quad (5)$$

Let us find the tensor component ϵ_{33}. To this end, we calculate d_{ij} and d_{ij3}, assuming that the HF pulsations are of the Langmuir type and are isotropic:

$$d_{ij3} = \frac{\pi e^2}{\omega_{0e}^2\omega m_e^2}\int d\mathbf{k}_1\frac{k_{1i}k_{1j}}{k_1^2}k^2k_{1z}\frac{k\partial W_{k_1}/\partial k_{1x}}{\omega - kv_{g1,x} + i\delta}, \quad (6)$$

$$d_{ij} = \frac{\pi e^2}{\omega_{0e}n_0m_e^2}\int d\mathbf{k}_1\frac{k_{1i}k_{1j}}{k_1^2}\frac{k\partial W_{k_1}/\partial k_{1x}}{\omega - kv_{g1,x} + i\delta}. \quad (7)$$

Calculating the integral with respect to $d\mathbf{v}$ in (1), we can easily show that (for isotropic HF pulsations) only the coefficient d_{133} contributes to ϵ_{33}; this coefficient is given by

$$d_{133} = \frac{\pi e^2}{\omega_{0e}^2\omega m_e^2}\int d\mathbf{k}_1\frac{k_{1x}^2k_{1z}^2k^3}{k_1^5}\frac{\partial W_{k_1}/\partial k_{1x}}{\omega - 3kk_{1x}v_{Te}^2/\omega_{0e} + i\delta}. \quad (8)$$

We shall consider (8) in two regions: 1) $\omega \gg kv_{g1}$ and 2) $\omega \ll kv_{g1}$. In the first frequency region we have

¹⁾We can confine ourselves here, for $\omega_{\mathbf{k}_1}$, to the solution of the linear dispersion equation. Indeed, even for Langmuir waves whose group velocity is quite small, the frequency corrections connected with their nonlinear interaction makes no contribution to $\mathbf{v}_{g1}$ for an isotropic turbulence (see [7,8]). However, in a sufficiently dense plasma, where it is important to take the collisions into account, this result may change [7].

²⁾Inasmuch as $W/n_0T_e \ll 1$. The next higher-order term is estimated below.

$$d_{133} = -\frac{\pi k^3}{9n_0 m_e \omega^2}\int_0^\infty W_{k_1} dk_1, \quad (9)$$

with

$$\varepsilon_{33} = 1 + \frac{\omega_{0e}^2}{\omega^2}\left(\frac{T_\perp}{T_\parallel} - 1\right) + \frac{T_\perp}{T_\parallel}\frac{\omega_{0e}^2}{\omega^2} i \sqrt{\frac{\pi}{2}}\frac{\omega}{kv_\parallel}\left[1 + \frac{\pi}{9}\left(\frac{kv_\parallel}{\omega}\right)^2 \frac{k^2}{n_0 T_\perp}\int_0 W_{k_1} dk_1\right]. \quad (10)$$

We substitute (10) in (5) and solve the resultant quadratic equation:

$$\frac{\omega}{kv_\parallel} = \left\{-i\left[k^2c^2 + \omega_{0e}^2\left(1 - \frac{T_\perp}{T_\parallel}\right)\right] \pm i\left(\left[k^2c^2 + \omega_{0e}^2\left(1 - \frac{T_\perp}{T_\parallel}\right)\right]^2 + 4\left(\frac{T_\perp}{T_\parallel}\right)^2 \omega_{0e}^4 \frac{\pi^2}{18}\frac{k^2}{n_0 T_\perp}\int_0^\infty W_{k_1} dk_1\right)^{1/2}\right\} \times \left(2\frac{T_\perp}{T_\parallel}\omega_{0e}^2\sqrt{\frac{\pi}{2}}\right)^{-1}. \quad (11)$$

It follows from this expression that the instability connected with the anisotropy of the temperatures is insignificant if

$$2\left(\frac{T_\perp}{T_\parallel}\right)^2 \frac{\pi^2}{9} k^2 \left(\frac{1}{n_0 T_e}\int W_{k_1} dk_1\right) \gg \left|\frac{k^2c^2}{\omega_{0e}^2} - \frac{T_\parallel - T_\perp}{T_\parallel}\right|^2 = \alpha^2. \quad (12)$$

In this case spontaneous magnetic fields are generated by the Langmuir turbulence (in analogy with[3]), therefore a change should take place in the plasmon distribution function as a result of the relaxation of such an instability.

In the region $\omega \ll kv_{g1}$, to calculate the integral in (8) it is necessary to know the form of the stationary spectrum W_{k_1} of the high frequency turbulence. If it is assumed that the HF pulsations have a Gaussian distribution, then[3)]

$$d_{133} = \frac{1}{12}\frac{kW\omega_{0e}^2}{n_0 T_\parallel}\left[-1 - \frac{3}{4}\sqrt{\frac{\pi}{2}} i\beta (C_1 + \ln\beta)\right], \quad (13)$$

where

$$\beta = \frac{\omega}{kv_\parallel}\frac{\omega_{0e}}{k_{10}v_\parallel 3\sqrt{2}} \ll 1$$

by virtue of $\omega \ll kv_{g1}$. Retaining in (13) the highest-order term and solving (5), we get

$$\frac{k^2c^2}{\omega_{0e}^2} + 1 - \frac{T_\perp}{T_\parallel} = i\frac{T_\perp}{T_\parallel}\sqrt{\frac{\pi}{2}}\frac{\omega}{kv_\parallel} \times \left[1 - \frac{1}{108}\frac{k^2}{k_{10}^2}\left(\frac{v_{\mathrm{ph}1}}{v_\parallel}\right)^2 \frac{W}{n_0 T_\perp}\right].$$

$$v_{\mathrm{ph}\,1} = \omega_{0e}/k_1. \quad (14)$$

It is seen from this expression that the influence of the HF turbulence becomes appreciable when

$$\frac{1}{108}\frac{k^2}{k_{10}^2}\left(\frac{v_{\mathrm{ph}1}}{v_\parallel}\right)^2 \frac{W}{n_0 T_\perp} > 1, \quad (15)$$

with a change occurring in the sign of the increment, and with the instability developing at the expense of the energy of the HF turbulence. Therefore the temperature anisotropy may be conserved. If (15) is violated, the usual anisotropic instability takes place.

The foregoing investigation allows us to conclude that an insignificant temperature anisotropy (on the order of several per cent) may be conserved in a turbulent plasma, but the quantitative result depends strongly on the form of the stationary spectrum of the HF oscillations.

In conclusion, let us estimate the discarded terms $\sim |e_{k_1}|^4$. Using the results of[4], we can show that the condition for the applicability of (11) is

$$10\frac{v_{Te}^5}{v_{Ti}^2 v_{\Phi 1}^3}\frac{k^4}{k_1^4} \gg \alpha^2$$

(k_1—fundamental scale of the HF turbulence), and that of formula (14) is

$$3\frac{v_{Te}^4}{v_{Ti}^2 v_{\mathrm{ph}\,1}^2}\frac{k^4}{k_1^4} \gg 1$$

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3) In the derivation of (13) we used the series expansion of the functions Ei(x) with small values of x, accurate to the linear terms $\mathrm{Ei}(-x) \approx C + \ln x - x$. Here C is order of constant, and C_1 in formula (13) is equal to C + 1.