CONCERNING THE VIBRATIONAL SPECTRUM OF VORTEX LINES

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The vibrational spectrum of isolated vortex lines is investigated for two microscopic models: 1) the model of a charged Bose gas, and 2) charged superconductors containing impurities of paramagnetic atoms. It is shown that in case 1) the function $\omega(k)$ has three branches, two of which have a gap and the third—the gapless branch—is associated with the conductivity of normal electrons; in case 2) the frequency has only an imaginary part which corresponds to only a damping of the vibrations. The obtained results indicate that the hydrodynamical treatment is not valid.

It is well known[1,2] that in superfluid helium vortex lines exist which are able to undergo vibrational motion; in this connection the spectrum of these vibrations has been established.[3] Vortex lines also exist in superconductors of the second kind. However, the problem of the vibrational spectrum of superconducting vortices in a microscopic theory has, as yet, not been investigated.

Determination of the vibrational spectrum in superconductors is associated with the solution of an essentially nonstationary problem which, in principle, may be solved by starting from the Gor'kov equations. However, in view of the complexity of these equations, the problem is difficult to solve. Another possibility is the establishment and investigation of a nonstationary generalization of the Ginzburg-Landau equations. Unfortunately, in the general case such equations have not been obtained.

The possibility of oscillations of the Abrikosov lines has been studied in a number of articles[4-7] on the basis of phenomenological equations to zero order in the ratio of the coherence length r_0 to the penetration depth λ. In article [6] it is shown that in an uncharged superconductor the vibrational spectrum of a vortex line coincides with the vibrational spectrum in an uncharged ideal liquid, in the same way as this occurs in the case of a nonideal, uncharged Bose gas. The spectrum for a charged superconductor was investigated in articles [4,5,7]. The obtained results indicate the presence in the spectrum of low-frequency oscillations of a gapless type.

In the present article, in order to understand the influence of charge, the vibrational spectrum of a single vortex is investigated in two microscopic models: 1) the model of a charged Bose gas, and 2) charged superconductors containing paramagnetic impurities. In both of these cases time-dependent equations of the Ginzburg-Landau type exist which permit one to comparatively easily investigate the spectrum of the low-frequency vibrations.

1. CHARGED BOSE GAS

If a charged Bose gas is placed in a constant magnetic field, then after the field reaches a certain critical value vortex lines appear in the gas; these lines are completely analogous to the Abrikosov vortex lines in superconductors of the second kind. In this Section we shall dwell on a study of the vibrational spectrum of an isolated line.

A Bose gas may be described by a system of equations, one of which is the equation of motion and the second is Maxwell's equation. This system (in units $\hbar = 1$) is written in the following form:

$$i\frac{\partial\Phi}{\partial t} = -\frac{1}{2m}\left(\nabla - i\frac{e}{c}\mathbf{A}\right)^2\Phi + g|\Phi|^2\Phi - gn_0\Phi,$$

$$\operatorname{rot}\operatorname{rot}\mathbf{A} = \frac{4\pi}{c}\frac{ie}{2m}\left\{\Phi\left(\nabla + i\frac{e}{c}\mathbf{A}\right)\Phi^* - \Phi^*\left(\nabla - i\frac{e}{c}\mathbf{A}\right)\Phi\right\} - \frac{4\pi}{c^2}\sigma_n\frac{\partial\mathbf{A}}{\partial t} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2}, \quad (1)$$

where $\Phi(\mathbf{r}, t)$ is the wave function of the condensate, $\mathbf{A}(\mathbf{r}, t)$ is the vector potential of the field, g is a constant characterizing the strength of the pair interaction, n_0 is the density of particles in the condensate at an infinite distance from the vortex line, and σ_n is the normal conductivity. The term proportional to σ_n takes into account the presence of normal electrons, leading to electrical neutrality of the sample.

In the stationary case an isolated vortex line corresponds to the solution

$$\Phi(\mathbf{r}) = \sqrt{n_0}e^{i\theta}a_0(r), \quad (2)$$

where r is the distance to the center of the vortex line, and θ is the polar angle. With Eq. (2) taken into account, the system (1) can be written in the form of the well-known equations of article:[8]

$$-\frac{1}{r}\frac{d}{dr}r\frac{d}{dr}a_0 + \frac{1}{r_0^2}Q^2a_0 + 2mgn_0(a_0^2 - 1)a_0 = 0,$$
$$\frac{d}{dr}\frac{1}{r}\frac{d}{dr}rQ = \frac{1}{\lambda^2}a_0^2Q, \quad (3)$$

where

$$\lambda^2 = \frac{mc^2}{4\pi n_0e^2}, \quad Q(r) = \frac{er_0}{c}A^0(r) - \frac{r_0}{r}, \quad (4)$$

and r_0 is a quantity having the dimensions of a length and having the meaning of the radius of the vortex line.

The boundary conditions for the system of equations (3) are the following requirements: $a_0 \to 1$, $Q \to 0$ as $r \to \infty$ and $a_0 \to 0$, $|Q| \to r_0/r$ as $r \to 0$. The solutions of the obtained system are known, and we shall not write them down; we only note that we are interested in the case $\kappa = \lambda/r_0 \gg 1$.

In order to determine the vibrational spectrum of a vortex line it is necessary to linearize Eq. (1) close to the stationary solution (2). Let us assume

$$\Phi(\mathbf{r},t)=\sqrt{n_0}\,\{a_0(r)e^{i\theta}+\psi(r)e^{-i\omega t+ikz}\}=\sqrt{n_0}\,\{\varphi_0+\psi\},$$

$$\Phi^*(\mathbf{r},t)=\sqrt{n_0}\{a_0(r)e^{-i\theta}+\psi^*(r)e^{-2i\theta-i\omega t+ikz}\}=\sqrt{n_0}\{\varphi_0^*+\psi^*\}, \quad (5)$$

$$\mathbf{A}(\mathbf{r},t)=\mathbf{A}^0(r)+\mathbf{A}_1(r)e^{-i\theta-i\omega t+ikz}=\mathbf{A}^0+\mathbf{A}_1,$$

where a_0, $\mathbf{A}^0$ are the solutions of the system (3), and ψ, ψ^*, and $\mathbf{A}_1$ are small corrections. Choice of the angular dependence is determined by the fact that in this case the variables are separable. We note that in the gauge chosen by us the scalar potential is equal to zero.

In addition, let us divide the potential $\mathbf{A}_1$ into solenoidal and potential parts so that

$$\mathbf{A}_1=\mathbf{A}_1{}^s+\nabla\chi,$$

where χ is an unknown function and div $\mathbf{A}_1^S = 0$.

Thus, we have six unknown quantities ψ, ψ^*, $\mathbf{A}_1^S$, and χ. It is convenient to supplement the obtained system of five scalar equations by one more equation which is obtained by taking the operation div on the linearized electrodynamical equation . However, as a consequence of the fact that the solutions of the system (3) do not depend on the coordinate z, and also div $\mathbf{A}_1^S = 0$, the equation for the component A_1^{Sz}, which is contained in only one of the equations of the system, is split off from all remaining equations, and one does not need to consider this equation in order to determine the spectrum. Finally we obtain

$$\begin{aligned}
&\left(\omega-\frac{k^2}{2m}\right)\psi+\frac{ie}{2mc}k^2\varphi_0\chi=-\frac{1}{2m}\left(\nabla-i\frac{e}{c}\mathbf{A}^0\right)^2\psi+gn_0(2a_0^2-1)\psi\\
&\quad+gn_0a_0^2e^{2i\theta}\psi^*+\frac{ie}{2mc}\left(2\nabla\varphi_0-2i\frac{e}{c}\mathbf{A}^0\varphi_0\right)\mathbf{A}_1{}^s\\
&\quad+\frac{ie}{2mc}\left(\varphi_0\nabla^2+2\nabla\varphi_0\cdot\nabla-2i\frac{e}{c}\mathbf{A}^0\varphi_0\nabla\right)\chi,\\
&\left(\omega+\frac{k^2}{2m}\right)\psi^*+\frac{ie}{2mc}k^2\varphi_0^*\chi=\frac{1}{2m}\left(\nabla+i\frac{e}{c}\mathbf{A}^0\right)^2\psi^*-gn_0(2a_0^2-1)\psi^*\\
&\quad-gn_0a_0^2e^{-2i\theta}\psi+\frac{ie}{2mc}\left(2\nabla\varphi_0^*+2i\frac{e}{c}\mathbf{A}^0\varphi_0^*\right)\mathbf{A}_1{}^s\\
&\quad+\frac{ie}{2mc}\left(\varphi_0^*\nabla^2+2\nabla\varphi_0^*\cdot\nabla+2i\frac{e}{c}\mathbf{A}^0\varphi_0^*\nabla\right)\chi,\\
&-\left(\frac{4\pi i}{c^2}\sigma_n\omega+\frac{\omega^2}{c^2}\right)(\mathbf{A}_1{}^s+\nabla\chi)+k^2\mathbf{A}_1{}^s\\
&\quad=-4\pi i\frac{en_0}{2mc}\left(\varphi_0^*\nabla-\nabla\varphi_0^*-2i\frac{e}{c}\mathbf{A}^0\varphi_0^*\right)\psi\\
&+4\pi i\frac{en_0}{2mc}\left(\varphi_0\nabla-\nabla\varphi_0+2i\frac{e}{c}\mathbf{A}^0\varphi_0\right)\psi^*-\operatorname{rot}\operatorname{rot}\mathbf{A}_1{}^s-\frac{1}{\lambda^2}a_0^2\mathbf{A}_1{}^s-\frac{1}{\lambda^2}a_0^2\nabla\chi,\\
&-\left(\frac{4\pi i}{c^2}\sigma_n\omega+\frac{\omega^2}{c^2}\right)\nabla^2\chi-4\pi i\frac{en_0}{2mc}k^2(\varphi_0^*\psi-\varphi_0\psi^*)-\frac{k^2}{\lambda^2}a_0^2\chi\\
&\quad=-4\pi i\frac{en_0}{2mc}\left(\varphi_0^*\nabla^2-\nabla^2\varphi_0^*-2i\frac{e}{c}\mathbf{A}^0\varphi_0^*\nabla-2i\frac{e}{c}\mathbf{A}^0\nabla\varphi_0^*\right)\psi\\
&\quad+4\pi i\frac{en_0}{2mc}\left(\varphi_0\nabla^2-\nabla^2\varphi_0+2i\frac{e}{c}\mathbf{A}^0\varphi_0\nabla+2i\frac{e}{c}\mathbf{A}^0\nabla\varphi_0\right)\psi^*\\
&\quad-\frac{1}{\lambda^2}\mathbf{A}_1{}^s\nabla a_0^2-\frac{1}{\lambda^2}\nabla a_0^2\cdot\nabla\chi-\frac{1}{\lambda^2}a_0^2\nabla^2\chi.
\end{aligned} \quad (6)$$

The boundary conditions are the requirements that all solutions be regular as r → 0 and that all solutions vanish as r → ∞.

We shall solve the system (6) by the method of perturbation theory, assuming ω and k to be small. For this purpose, let us write Eq. (6) in vector form:

$$\hat{L}\mathbf{f}=(\hat{L}_0+\hat{L}_1)\mathbf{f}=0, \quad (7)$$

where $\hat{L}_0$ does not contain the quantities ω and k, and $\mathbf{f} = (\psi, \psi^*, \mathbf{A}_1^S, \chi)$ is the vector of the solutions.

One can construct a solution of the matrix equation

$$\hat{L}_0\mathbf{f}_0=0 \quad (8)$$

by starting from the properties of the system (1). Equations (1) are gauge invariant and, in addition, their form does not change during a displacement of all coordinates by a constant vector. Therefore, if $\mathbf{f}(\mathbf{r})$ is a vector of solutions, then $\mathbf{f}(\mathbf{r}+\mathbf{b})$ will also be a solution of (1); similarly if $\mathbf{f}(\mathbf{r})$ is a solution then the vector $\tilde{\mathbf{f}}(\mathbf{r},\chi)$ which is transformed according to the gauge transformation $\psi\to\psi e^{ie\chi/c}$, $\mathbf{A}\to\mathbf{A}+\nabla\chi$ will also be a solution of (1) for any arbitrary time-independent function $\chi(\mathbf{r})$. Linearizing $\mathbf{f}(\mathbf{r}+\mathbf{b})$ and $\tilde{\mathbf{f}}(\mathbf{r},\chi)$ with respect to $\mathbf{b}$ and χ, we obtain solutions of the system (6) with $\omega = 0$.

Thus, we have

$$\mathbf{f}_0=\begin{pmatrix}\psi\\ \psi^*\\ A_1^{c\varphi}\\ A_1^{c\theta}\\ \chi\end{pmatrix}=\begin{pmatrix}\alpha\left(\dfrac{da_0}{dr}+\dfrac{a_0}{r}\right)+ia_0\chi\\ \alpha\left(\dfrac{da_0}{dr}-\dfrac{a_0}{r}\right)e^{-2i\theta}-ia_0\chi e^{-2i\theta}\\ \alpha i\dfrac{A^0}{r}e^{-i\theta}\\ \alpha\dfrac{dA^0}{dr}e^{-i\theta}\\ \dfrac{c}{e}\chi e^{-i\theta}\end{pmatrix}. \quad (9)$$

Here α is an arbitrary constant, and χ is an arbitrary function of r.

Further, let us construct the operator $\hat{K}_0$ conjugate to the operator L_0 and the function $\mathbf{F}_0$ conjugate to $\mathbf{f}_0$; these are determined in the following way. Let there be a vector function $\hat{L}_0\mathbf{f}_0$, let us multiply it from the left by $\mathbf{F}_0$, take the integral and integrate by parts, then the expression $(\mathbf{f}_0, \hat{K}_0, \mathbf{F}_0)$ is obtained. Let us define the function $\mathbf{F}_0$ so that the integral vanishes for an arbitrary function $\mathbf{f}_0$; then we call $\mathbf{F}_0$ the function conjugate to $\mathbf{f}_0$, and the operator $\hat{K}_0$ arising in this connection is called the conjugate to the operator $\hat{L}_0$.

$\mathbf{F}_0$, just like $\mathbf{f}_0$, consists of two parts each of which corresponds to two kinds of invariance of Eqs. (1). We denote the part containing the arbitrary function $\tilde{\chi}$ by $\mathbf{P}$, the second part is denoted by $\mathbf{M}$. Multiplying the system (6) in turn by $\mathbf{P}$ and $\mathbf{M}$, integrating it and taking into consideration that $\hat{K}_0\mathbf{F}_0 = 0$, we obtain the following two equations:

$$\begin{aligned}
&\alpha i\omega\int_0^\infty\tilde{\chi}\frac{da_0^2}{dr}r\,dr\\
&+\left(\frac{4\pi i}{c^2}\sigma_n\omega+\frac{\omega^2}{c^2}\right)\frac{c^2}{4\pi n_0e^2}\int_0^\infty\tilde{\chi}\left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-\frac{1}{r^2}\right)\chi\,r\,dr=0,
\end{aligned} \quad (10)$$

$$\begin{aligned}
&i\omega\int_0^\infty\chi\frac{da_0^2}{dr}r\,dr+2\alpha\omega\\
&+\left(\frac{4\pi i}{c^2}\sigma_n\omega+\frac{\omega^2}{c^2}\right)\frac{\alpha}{4\pi n_0}\int_0^\infty\left[\left(\frac{A^0}{r}\right)^2+\left(\frac{dA^0}{dr}\right)^2\right]r\,dr\\
&-\alpha\frac{k^2}{m}\int_0^\infty\left[\left(\frac{a_0}{r}\right)^2+\left(\frac{da_0}{dr}\right)^2+\frac{m}{4\pi n_0}\left\{\left(\frac{A^0}{r}\right)^2+\left(\frac{dA^0}{dr}\right)^2\right\}\right]r\,dr=0.
\end{aligned} \quad (11)$$

It should be noted that the vector $\mathbf{P}$ is a solution of the system (6) for $\omega = 0$ and arbitrary $k \neq 0$, where **M** satisfies (6) for $\omega = 0$ and $k = 0$. Therefore, the terms proportional to k^2 are not present in Eq. (10), but they are present in Eq. (11).

As a consequence of the arbitrariness of $\tilde{\chi}(r)$, Eq. (10) is a differential equation determining the unknown in the zero-order approximation function $\chi(r)$ which, being substituted into Eq. (11), determines the desired dispersion equation. The requirements that the function $\chi(r)$ be bounded as $r \to 0$ and $\chi(r) = 0$ as $r \to \infty$ serve as the boundary conditions for the differential equation following from (10). Integrating this equation we obtain

$$\chi(r) = i\alpha \frac{mc^2}{\lambda^2} \frac{1}{(\omega + 4\pi i\sigma_n)} \left(\frac{r}{2} - \frac{1}{r} \int_0^r a_0^2(t)\, t\, dt \right). \quad (12)$$

Substituting (12) into (11), we find the dispersion equation

$$-\frac{mc^2}{\varkappa^2} \frac{\omega}{(\omega + 4\pi i\sigma_n)} + 2\omega + \frac{1}{2mc^2} \omega(\omega + 4\pi i\sigma_n) - \frac{k^2}{m} \ln \frac{1}{kr_0} = 0 \quad (13)$$

From the obtained Eq. (13) it follows that $\omega(k)$ has three roots. Two of the branches of $\omega(k)$ have a gap whose magnitude can be expressed in the following way:

$$\omega_0 = -(2mc^2 + 4\pi i\sigma_n) \pm 2mc^2(1 + 1/2\varkappa^2)^{1/2}. \quad (14)$$

From the obtained expression it follows that one gap is associated with charge, but the other does not depend on it. The latter is not small, and therefore we shall not consider it. Thus, the quantity of interest to us is

$$\omega_0 = -4\pi i\sigma_n + mc^2/2\varkappa^2. \quad (15)$$

Hence it is immediately evident that this gap vanishes as the charge tends to zero ($e \to 0$, $\kappa \to \infty$).

For $\sigma_n = 0$ we obtain

$$\omega = -2mc^2 \pm 2mc^2 \left[1 + \frac{1}{2\varkappa^2} + \frac{k^2}{2m^2c^2} \ln \frac{1}{kr_0} \right]^{1/2}. \quad (16)$$

Once again let us consider the smallest root, having the form

$$\omega = \frac{mc^2}{2\varkappa^2} + \frac{k^2}{2m} \ln \frac{1}{kr_0}. \quad (17)$$

As already noted, the gap vanishes as $e \to 0$, i.e., in case the spectrum has a gapless form, which agrees with the spectrum of hydrodynamical oscillations of an uncharged vortex.[3]

In addition, there is one more gapless branch ($\sigma_n \neq 0$, $e \neq 0$); however it is related to the value of σ_n and as $\sigma_n \to 0$ it goes over into $\omega = 0$.

Let us estimate the magnitude of the gap. According to Eqs. (15) and (17)

$$\omega_0 \sim mc^2/\varkappa^2 = mr_0^2\omega_p^2, \quad (18)$$

where $\omega_p^2 = 4\pi n_0 e^2/m$ is the plasma frequency. In order to estimate the quantity mr_0^2 we note that we did not take into account the Coulomb terms in Eq. (1), corresponding to a violation of electrical neutrality near the axis of the vortex line. At distances of the order of r_0 where the charge density is changing in a substantial way, we have (in order of magnitude)

$$-\frac{1}{2m}\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} \right)\psi - e\varepsilon\psi = 0,$$

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} \right)\varepsilon = 4\pi n_0 e(|\psi|^2 - |\varphi_0|^2),$$

where ϵ is the scalar potential.

From the equations written down it follows that

$$-\frac{1}{2mr_0^2}\psi - 4\pi n_0 e^2 r_0^2(|\psi|^2 - |\varphi_0|^2)\psi = 0,$$

or $1/mr_0^2 \sim \omega_p$.

From the obtained results it is clear that in the absence of conductivity due to normal electrons, the vibrational spectrum of a charged vortex starts with a gap. This gap vanishes in the limit $\kappa = \infty$, in accordance with the results of a hydrodynamical treatment.[4,5] However, for actual values of κ (even $\kappa = 100$) this gap has a magnitude of the order of ω_p which actually makes the results of a hydrodynamical treatment inapplicable.

2. SUPERCONDUCTOR CONTAINING PARAMAGNETIC IMPURITIES

Let us consider a superconductor containing paramagnetic impurity atoms placed in a field. In this case, as established in article [9], one can obtain a generalization of the Ginzburg-Landau equations to the nonstationary case. These equations have the form

$$\frac{\partial\Delta}{\partial t} - D\left(\nabla - 2i\frac{e}{c}\mathbf{A} \right)^2 \Delta + \frac{\tau_s}{3}\pi^2(T_c^2 - T^2)(|\Delta|^2 - 1)\Delta = 0,$$

$$\operatorname{rot}\operatorname{rot}\mathbf{A} = ie\frac{4\pi n\tau_1\tau_s}{2mc} 2\pi^2(T_c^2 - T^2)\left\{ \Delta\left(\nabla + 2i\frac{e}{c}\mathbf{A} \right)\Delta^* - \Delta^*\left(\nabla - 2i\frac{e}{c}\mathbf{A} \right)\Delta \right\} - \frac{4\pi\sigma_n}{c^2}\frac{\partial\mathbf{A}}{\partial t}, \quad (19)$$

where $\Delta(\mathbf{r}, t)$ is the order parameter, $2/\tau_s = (1/\tau_1) - (1/\tau_2)$ is the relaxation time associated with reversal of the spin, τ_1 and τ_2 are certain characteristic times between collisions, $D = v^2\tau_1/3$ is the coefficient of diffusion.

In the stationary case Eqs. (19) have solutions of the vortex type $a_0(r)e^{i\theta}$, $\mathbf{A}^0(r)$. In order to determine the vibrational spectrum, once again we linearize the system (19). Then, just as in Section 1, equations appear relative to the corrections ψ, ψ^*, $\mathbf{A}_1^S$ and χ. We shall solve the linearized system according to perturbation theory in exactly the same way that this was done in the case of a charged Bose gas.

The operator $\hat{\mathbf{L}}_0$ arising in this connection is completely analogous to the expression for the case of a Bose gas. However, the operator $\hat{\mathbf{L}}_1$ differs significantly from the previous case. The difference consists in the fact that in the first two lines of this operator the imaginary unit i appears in front of the quantity ω; this is associated with diffusion-type equations of motion. In addition the terms proportional to ω^2 are not present in $\hat{\mathbf{L}}_1$.

Once again the solution of the equation $\hat{\mathbf{L}}_0\mathbf{f}_0 = 0$ is constructed by starting from the invariant properties of Eqs. (10), and as before it has the form (9). The solution of the conjugate equation is constructed in analogous fashion. Then, using the method indicated in Sec. 1, we obtain two equations:

$$\frac{4\pi\sigma_n D}{c^2} \frac{mc^2}{4\pi^3 n(2e)^2\tau_1\tau_s(T_c^2 - T^2)}\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2} \right)\chi - a_0^2\chi = -i\alpha\frac{a_0^2}{r}, \quad (20)$$

$$\omega \int_0^\infty \chi a_0^2\, dr - i\omega\alpha \int_0^\infty \left[\left(\frac{a_0}{r}\right)^2 + \left(\frac{da_0}{dr}\right)^2 + \frac{4\pi\sigma_n D}{c^2} \right.$$

$$\left. \cdot \frac{m\,\{(A^0/r)^2 + (dA^0/dr)^2\}}{4\pi^3 n\tau_1\tau_s(T_c^2 - T^2)} \right] r\,dr + \alpha k^2 D \int_0^\infty \left[\left(\frac{a_0}{r}\right)^2 + \left(\frac{da_0}{dr}\right)^2 \right.$$

$$\left. + \frac{m}{4\pi^3 n\tau_1\tau_s(T_c^2 - T^2)} \left\{ \left(\frac{A^0}{r}\right)^2 + \left(\frac{dA^0}{dr}\right)^2 \right\} \right] r\,dr = 0. \quad (21)$$

where a_0 and A^0 are solutions of the steady-state Ginzburg–Landau equation.

One can approximately derive a solution of Eq. (20). In order to do this it is necessary to set $a_0^2 = 1$. Then, requiring boundedness of $\chi(0)$ and $\chi(r) = 0$ as $r \to \infty$, we obtain

$$\chi(r) = i\frac{\alpha}{r} - i\alpha\beta K_1(\beta r); \quad (22)$$

where $K_1(t)$ is a Bessel function of imaginary argument, and the constant β is defined as

$$\beta^2 = \frac{c^2}{4\pi\sigma_n D} \frac{12\pi n(2e)^2}{mc^2} \tau_1 \frac{\tau_s}{3} \pi^2 (T_c^2 - T^2).$$

Substituting (22) into (21) we obtain the dispersion equation

$$-i\omega\mathcal{R} + k^2 D \ln\frac{1}{kr_0} = 0, \quad (23)$$

$$\mathcal{R} = \beta \int_0^\infty a_0^2(r) K_1(\beta r)\, dr + \int_0^\infty \left[\left(\frac{da_0}{dr}\right)^2 + \frac{4\pi\sigma_n D}{c^2} \times \frac{m\{(A^0/r)^2 + (dA^0/dr)^2\}}{4\pi^3 n\tau_1\tau_s(T_c^2 - T^2)} \right] r\,dr$$

or

$$\omega = -i\frac{k^2 D}{\mathcal{R}} \ln\frac{1}{kr_0}. \quad (24)$$

The obtained result (24) indicates that the oscillations are strongly damped, i.e., free, undamped oscillations of vortex lines are impossible in a superconductor containing paramagnetic impurity atoms.

Thus, both of the considered models show that the results obtained with the aid of a hydrodynamical approach turn out to be invalid. This assertion apparently also holds for the case of an ordinary superconductor, although due to the complexity of the equations no one has yet been able to carry out the procedure stated above for obtaining the dispersion equation for the Gor'kov equations.

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