

## QUANTUM THEORY OF AN ELECTROMAGNETIC FIELD IN A GYROTROPIC MEDIUM

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It is shown that, despite spatial dispersion, the quantum field in a medium can be described by a second-order equation for the vector-potential. A covariant apparatus is developed and the causal Green function ("propagator") is calculated. Owing to the gyrotropic effect, the latter function has three poles and contains a part that is antisymmetric with respect to vector indices.

## 1. INTRODUCTION

THE phenomenological approach to the description of quantum electromagnetic processes in a medium was first used in the quantum theory of the Cerenkov effect<sup>[1,2]</sup>. Later, Ryazanov<sup>[3,4]</sup> developed a relativistic covariant formalism for an isotropic non-gyrotropic medium. Field quantization in a gyrotropic medium at rest was used by Karavaev to calculate first-order effects (emission of a single photon)<sup>[5]</sup>.

In this article the field is considered covariantly, and the main purpose is a consistent realization of the quantization and the development of a diagram technique, i.e., the determination of the causal Green's function ("propagator") for the electromagnetic field in a gyrotropic medium. This problem is unique in that the equation for the field potential in the gyrotropic medium contains higher derivatives. As shown in the paper, it is possible to get along with simpler second-order equations, but they admit only of complex solutions. Since, on the other hand, the electromagnetic field is neutral, it becomes possible to retain in the Fourier expansion of the potential only the positive-frequency part. During quantization, this leads to non-local commutation relations (the commutator does not vanish for the spacelike intervals). Nonetheless, the expressions for the operators of the observable quantities remain local, and the S-matrix satisfies all the necessary requirements.

From the point of view of the authors, greatest interest in the present paper lies in the theoretical-methodological aspect of the problem. At the same time, the formalism developed here makes it possible to calculate the probabilities of quantum electromagnetic processes in gyrotropic media.

## 2. CLASSICAL FIELD IN A GYROTROPIC MEDIUM

Maxwell's equations for a moving medium are of the form

$$e^{iklm} \frac{\partial F_{lk}}{\partial x^i} = 0, \quad (2.1)$$

$$\frac{\partial H^{ik}}{\partial x^k} = 4\pi j^i. \quad (2.2)$$

Here  $e^{iklm}$  is a completely antisymmetrical unit tensor of fourth rank,  $j^i$  is the current density,  $F_{lk}$  and  $H^{ik}$  are the field and induction tensors (see<sup>[6]</sup>), each of the indices runs through the values 0, 1, 2, 3. We use throughout the metric

$$-g^{00} = g^{11} = g^{22} = g^{33} = -1; \quad g^{ik} = 0, \quad i \neq k$$

and put  $f = c = 1$ .

The tensors  $F_{ik}$  and  $H^{ik}$  are connected by

$$H^{ik} = e^{iklm} F_{lm}. \quad (2.3)$$

(The dielectric tensor  $\epsilon^{iklm}$  is antisymmetrical in the indices (i, k) and (l, m).)

If we introduce a vector potential  $A_k$ , putting

$$F_{ik} = \frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k},$$

then Eq. (2.1) is satisfied identically, and Eq. (2.2) takes the form

$$e^{iklm} = \frac{\partial^2 A_m}{\partial x^k \partial x^l} = 2\pi j^i. \quad (2.4)$$

The dielectric constant of an optically inactive medium is invariant against the inversion of the spatial coordinate axis. Its components are numbers. On the other hand, in the case of a gyrotropic medium, the dielectric tensor changes upon inversion, and its components contain the gradient operator (see<sup>[6,7]</sup>).

The basis of the subsequent analysis will be the fact that for plane waves in a gyrotropic medium at rest the relation (2.3) reduces to the equations

$$D = \epsilon E - ifB, \quad B = \mu H, \quad (2.5)$$

where  $\epsilon$  is the dielectric constant,  $\mu$  the magnetic permeability,  $f$  the dimensionless gyrotropy constant defined in such a way that  $f\omega/2$  is the angle of rotation of the plane of polarization per unit length ( $\omega$ —cyclic frequency). The generalization of (2.3) to the case of plane waves in a moving homogeneous and isotropic medium can be readily effected by recognizing that  $\epsilon^{iklm}$  may contain only the following covariant quantities: the four-velocity of the medium  $u^i$ , the metric tensor  $g^{ik}$ , and the totally antisymmetric unit fourth-rank tensor  $e^{iklm}$  ( $e^{0123} = 1$ ). Then the only expression satisfying the symmetry requirement and going over into (2.5) for a medium at rest is given by

$$\begin{aligned} e^{iklm} = & \frac{1}{2\mu} [g^{il}g^{km} - g^{im}g^{kl} + \kappa(u^i u^l g^{km} \\ & + u^k u^m g^{il} - u^k u^l g^{im} - u^i u^m g^{kl})] \\ & - \frac{if}{2} (e^{inlm} u_n u^k - e^{knlm} u_n u^i). \end{aligned} \quad (2.6)$$

Here  $\kappa = \epsilon\mu - 1$ .

Substituting (2.6) in (2.4), we obtain an equation for the potential of the field in the gyrotropic medium. The following two circumstances must be noted here. The first consists in the fact that after going over to Fourier components of the field it is necessary to take into account the dependence of the parameters of the medium

on the frequency (in particular,  $f \approx \omega$ , where  $\omega$  is of the order of atomic dimensions). Second, expression (2.6) pertains to the description of the field by complex quantities, so that a physical meaning attaches to  $\operatorname{Re} F_{ik}$  and  $\operatorname{Re} H^{ik}$ .

The vector potential  $A_k$  can be subjected, as is well known, to an additional Lorentz condition, which in the rest system of the medium is given by

$$\operatorname{div} A + \epsilon\mu \frac{\partial A_0}{\partial t} = 0.$$

In covariant notation, this condition takes the form

$$\left( \frac{\partial}{\partial x_m} + \kappa u^m u \cdot \frac{\partial}{\partial x} \right) A_m = 0, \quad u \cdot \frac{\partial}{\partial x} = u^k \frac{\partial}{\partial x^k}. \quad (2.7)$$

Taking into account the additional condition (2.7), the equation for the potentials (2.4) can be written, after substituting the dielectric tensor (2.6), in the form

$$\begin{aligned} & \left\{ \frac{1}{\mu} \left[ \left( \frac{\partial}{\partial x} \right)^2 + \kappa \left( u \cdot \frac{\partial}{\partial x} \right)^2 \right] (g^{im} + \kappa u^i u^m) \right. \\ & \left. + i f e^{inlm} u_n \frac{\partial}{\partial x^l} u \cdot \frac{\partial}{\partial x} \right\} A_m = -4\pi j^i. \end{aligned} \quad (2.8)$$

We note that in the case  $j^i = 0$  this equation can be reduced to a somewhat simpler form. Multiplying (2.8) by the nonsingular matrix  $(g_{ik} - u_i u_k \kappa / (1 + \kappa))$ , we obtain

$$\left\{ \frac{1}{\mu} \left[ \left( \frac{\partial}{\partial x} \right)^2 + \kappa \left( u \cdot \frac{\partial}{\partial x} \right)^2 \right] g^{km} + i f e^{knlm} u_n \frac{\partial}{\partial x^l} u \cdot \frac{\partial}{\partial x} \right\} A_m = 0.$$

Equations (2.8) and (2.9) are not invariant to complex conjugation, and in general admit of no "charge conjugation" transformation, i.e., a substitution of the type

$$A_m(x) \rightarrow \sum_{m'} C_{mm'} A_{m'}^+(x),$$

where  $C$  is some unitary matrix. In this sense, the solutions of Eqs. (2.8) and (2.9) are essentially complex, whereas the field itself describes only one type of particle (photons and antiphotons are identical). These circumstances should be taken into account during the quantization.

### 3. LAGRANGIAN FORMALISM

Equation (2.8) can be obtained from a variational principle, by choosing the Lagrangian in the form

$$\mathcal{L} = - \left[ \frac{1}{\mu} (g^{im} + \kappa u^i u^m) (g^{kl} + \kappa u^k u^l) \right. \\ \left. + \frac{if}{2} (e^{inlm} u_n u^h + e^{inkm} u_n u^l) \right] \frac{\partial A_i^+}{\partial x^k} \frac{\partial A_m}{\partial x^l} - 4\pi j^i (A_i + A_i^+). \quad (3.1)$$

In the case of a free field it is possible to obtain from (3.1), with aid of a standard procedure, an expression for the energy and momentum tensor:

$$T^{ij} = \mathcal{L} g^{ij} + \left[ \frac{1}{\mu} (g^{km} + \kappa u^k u^m) (g^{il} + \kappa u^i u^l) \right. \\ \left. + \frac{if}{2} (e^{knlm} u^i u_n + e^{knml} u^i u_l) \right] \frac{\partial A_k^+}{\partial x^j} \frac{\partial A_l}{\partial x^i} + \text{h.c.}$$

The field energy density in the rest system of the medium is the contraction of the tensors  $T^{ij}$  and  $u_i u_j$ :

$$w = u_i u_j T^{ij} \\ = \frac{1}{\mu} (g^{km} + \kappa u^k u^m) [g^{il} - (\kappa + 2) u^i u^l] \frac{\partial A_k^+}{\partial x^j} \frac{\partial A_l}{\partial x^i}.$$

Accordingly, the total energy in the rest system is given by the integral

$$W = \int w d^3x = \int w \delta(ux) d^4x.$$

As noted above, the solutions of Eq. (2.8) or (2.9) are essentially complex even in the case of a free field. In this connection, it admits of a gauge transformation of the first kind. The Lagrangian (3.1) is invariant against such a transformation, leading to the appearance of a conserving "current-density" four-vector:

$$\begin{aligned} J^h = \text{const} \cdot & \left[ \frac{1}{\mu} (g^{im} + \kappa u^i u^m) (g^{kl} + \kappa u^k u^l) \right. \\ & \left. + \frac{if}{2} (e^{inlm} u^h u_n + e^{inkm} u_n u^l) \right] A_i^+ \frac{\partial}{\partial x^l} A_m. \end{aligned} \quad (3.2)$$

The "charge density" corresponding to (3.2) has in the rest system of the medium the form

$$\begin{aligned} \rho = u^k J_k = \text{const} \cdot & \left[ \frac{1}{\mu} (g^{im} + \kappa u^i u^m) (1 + \kappa) u^l \right. \\ & \left. + \frac{i}{2} f e^{inlm} u_n \right] A_i^+ \frac{\partial}{\partial x^l} A_m. \end{aligned}$$

Accordingly, the "total charge" of the field in a medium at rest is given by

$$Q = \int \rho d^3x = \int \rho \delta(ux) d^4x. \quad (3.3)$$

It will be made clear later on that when states with a definite circular polarization are considered, the "charge"  $Q$ , as well as the gauge transformation of the first kind itself, can be given a simple geometric meaning.

### 4. UNIT VECTORS OF THE CIRCULAR POLARIZATION AND THE MOMENTUM REPRESENTATION

Let us consider a free field and obtain the solution of Eq. (2.9) corresponding to a plane wave:

$$A(k, x) = \xi(k) e^{ikx}. \quad (4.1)$$

Substituting (4.1) in (2.9) and putting here and henceforth  $\mu = 1$  (which means only renormalization of  $\epsilon$  and  $f$ ), we obtain

$$[k^2 + \kappa(uk)^2] \xi^i(k) + if(uk) e^{inlm} u_n k_l \xi_m(k) = 0.$$

The complex amplitudes  $\xi(k)$  can be resolved in terms of any system of unit vectors. In this case it is natural to choose as these vectors the eigenvectors of the helicity operator  $\hat{S}$ . This matrix can be written in the form

$$\hat{S}^{km} = -ie^{kmnl} u_n v_l,$$

where

$$v_l = q_l / \sqrt{-q^2}, \quad q_l = k_l - (uk) u_l \quad (4.2)$$

(we note that in the rest system of the medium the vector  $v_l = (0, \mathbf{k}/|\mathbf{k}|)$ . The sought unit vectors should satisfy the equation

$$\hat{S}_{\xi_A}^* = N_{\Lambda \xi_A}, \quad (4.3)$$

which in expanded form are given by

$$(\hat{S}^{km} - N g^{km}) \xi_{Am} = 0. \quad (4.4)$$

In Eqs. (4.3) and (4.4), the index  $\Lambda$  numbers the four eigenvectors corresponding to the eigenvalues  $N_\Lambda$  (the latter are obtained from the condition that the determinant of Eq. (4.4) vanish). We obtain

$$N_\Lambda = 0, \pm 1.$$

The unit vectors corresponding to  $N_\Lambda = 0$  are assigned the index  $\Lambda = L, S$ . Then

$$\xi_L = iv, \quad \xi_S = u. \quad (4.5)$$

The unit vectors for which  $N_\Lambda = \pm 1$ , will be denoted by the indices  $\Lambda = \pm 1$ . The unit vectors  $\xi_{\pm 1}$  can be readily constructed by introducing a real unit four-vector  $\eta$  orthogonal to  $u$  and  $v$ , and arbitrary in all other respects:

$$\eta u = \eta v = 0, \quad \eta^2 = v^2 = -1.$$

It is easy to verify that  $\xi_{\pm 1}$  can be written in the form

$$\xi_{\pm 1}^k = \frac{i}{\sqrt{2}} (e^{knim} u_n v_m \mp i \eta^k). \quad (4.6)$$

The circular-polarization unit vectors  $\xi_\Lambda$  satisfy the following normalization relations:

$$\xi_\Lambda^+ \xi_{\Lambda'}^- = d_{\Lambda \delta_{\Lambda \Lambda'}} = d_{\pm 1} = d_L = -d_S = -1.$$

In addition, the following equalities hold

$$\xi_\Lambda^+(v) = \xi_\Lambda(-v) = d_\Lambda \xi_{-\Lambda}(v)$$

(the symbol  $-\Lambda$  denotes  $\pm 1$  if  $\Lambda = \pm 1$ ;  $-\Lambda = \Lambda$  if  $\Lambda = L, S$ ).

The general solution of (2.9) can now be written in the form of a superposition of plane waves  $A_\Lambda(k, x)$  with a given circular polarization:

$$A(x) = \sum_\Lambda A_\Lambda(x), \quad A_\Lambda(x) = \frac{1}{(2\pi)^{1/2}} \int c_\Lambda(k) A_\Lambda(k, x) dk. \quad (4.7)$$

Here

$$A_\Lambda(k, x) = \xi_\Lambda(k) e^{ikx}$$

and  $c_\Lambda(k)$  are scalar amplitudes.

We note that owing to the orthogonality of the vectors  $\xi_\Lambda$ , the Lagrangian (3.1) and the "charge"  $Q$  (formula (3.3)) can be written in the form

$$\mathcal{L}(x) = \sum_\Lambda \mathcal{L}_\Lambda(x), \quad Q = \sum_\Lambda Q_\Lambda, \quad (4.8)$$

where  $\mathcal{L}_\Lambda(x)$  and  $Q_\Lambda$  are obtained from (3.1) and (3.3) by substituting  $A_\Lambda(x)$  for  $A(x)$ .

We now turn to find the general solution. Substituting (4.7) in (2.9), we obtain

$$(n^2 - f N_\Lambda n - 1 - \kappa) c_\Lambda(k) = 0, \quad n = \sqrt{\frac{-q^2}{(uk)^2}} > 0. \quad (4.9)$$

It follows therefore that  $c_\Lambda \neq 0$  only if

$$n = n_\Lambda = -\frac{N_\Lambda f}{2} + \sqrt{1 + \kappa + \frac{f^2 N_\Lambda^2}{4}}.$$

The invariant  $n_\Lambda$  is the refractive index of light with circular polarization  $\Lambda$  in the rest system of the medium:

$$n_\Lambda = \frac{|k|}{\omega_\Lambda}, \quad \omega_\Lambda > 0, \quad (4.10)$$

where  $\omega_\Lambda$  is the frequency of the light at the specified  $k$  and  $\Lambda$ . Since according to (4.2) and (4.10) we have

$$k^2 = k_0^2 - k^2 = q^2 + (uk)^2 = (1 - n_\Lambda^2) \omega_\Lambda^2,$$

it follows that for the same value of  $k$  the frequencies  $k_0$  in an arbitrary reference frame are different for different  $\Lambda$ . The amplitudes  $c_\Lambda(k)$  satisfying (4.9) can now be written in the following form:

$$c_\Lambda(k) = \tilde{c}_\Lambda(k) \delta[q^2 + N_\Lambda f \omega \sqrt{-q^2} + (1 + \kappa) \omega^2]. \quad (4.11)$$

Here  $\omega$  is the frequency in the rest system of the medium

$$\omega = (uk). \quad (4.12)$$

In the rest system of the medium, the argument of the  $\delta$  function in (4.11) is given by

$$-|k|^2 + N_\Lambda f \omega |k| + (1 + \kappa) \omega^2. \quad (4.13)$$

We emphasize that (4.13) does not reduce to the difference  $n_\Lambda^2 \omega^2 - k^2$ , for at fixed  $k$  and  $\Lambda$  the solutions of (4.9) are  $\pm \omega_\Lambda$ , and not  $\pm \omega_\Lambda$ .

Substituting (4.11) in (4.7), and integrating with respect to  $k_0$ , we obtain, in the rest system of the medium, the following expression for the vector potential of the field satisfying Eq. (2.9) [sic!]:

$$A(x) = \frac{1}{(2\pi)^{1/2}} \sum_\Lambda \frac{1}{\sqrt{2\bar{n}_\Lambda}} \int \frac{d^3 k}{\sqrt{|k|}} (a_\Lambda(k) \xi_\Lambda e^{ik_\Lambda x} + b_\Lambda^+(k) \xi_{-\Lambda} e^{-ik_{-\Lambda} x}).$$

We have introduced here the notation

$$\bar{n}_\Lambda = \frac{n_\Lambda + n_{-\Lambda}}{2}, \quad k_\Lambda = (\omega_\Lambda, k),$$

$$a_\Lambda(k) = \frac{\tilde{c}_\Lambda(k_\Lambda)}{(2\bar{n}_\Lambda n_\Lambda \omega_\Lambda)^{1/2}}, \quad b_\Lambda^+(k) = \frac{d_\Lambda \tilde{c}_\Lambda(-k_{-\Lambda})}{(2\bar{n}_\Lambda n_\Lambda \omega_\Lambda)^{1/2}}.$$

## 5. QUANTIZATION

Expression (4.14) differs from the normal-mode expansions usually employed in field theory in two respects: first, in the dependence of the frequency on the state of the polarization and, second, in the fact that the exponentials in both terms are not complex conjugates ( $k_\Lambda \neq k_{-\Lambda}$ ). Inasmuch as the photons and antiphotons are identical, in the quantization of the electromagnetic field in vacuum or in a non-gyrotropic medium, one usually puts  $a_\Lambda = b_\Lambda$ . In our case this equality contradicts the translational invariants, and therefore leads to an incorrect expression for the field energy. In exactly the same manner, it is impossible to identify  $a_\Lambda$  with  $b_{-\Lambda}$ . In practice this leads to a dependence of the physically observable quantities on the arbitrary vector  $\eta$  that enters in  $\xi_{\pm 1}$  (formula (4.6)). From the purely theoretical side, the point is that  $\mathcal{L}_\Lambda(x)$  in (4.8) are invariant against independent gauge transformations of the first kind

$$A_\Lambda(x) \rightarrow \exp \{i N_\Lambda \varphi_\Lambda\} A_\Lambda(x), \quad (5.1)$$

where  $\varphi_\Lambda$  are arbitrary real parameters.

We note that the transformation (5.1) has a geometrical meaning—this is the rotation of the unit vector  $\xi_\Lambda(k)$  in the rest system of the medium around  $k$ . The generators and the transformation (5.1) are the "charge" operators  $Q_\Lambda$ :

$$\exp \left\{ -i \sum_\Lambda Q_\Lambda \varphi_\Lambda \right\} A_\Lambda \exp \left\{ i \sum_\Lambda Q_\Lambda \varphi_\Lambda \right\} = \exp \{i N_\Lambda \varphi_\Lambda\} A_\Lambda. \quad (5.2)$$

It follows from (5.2) that

$$[A_\Lambda, Q_\Lambda] = N_\Lambda \delta_{\Lambda \Lambda'} A_\Lambda. \quad (5.3)$$

Relation (5.3) contradicts the equality  $a_\Lambda = b_{-\Lambda}$  (at  $\Lambda = \pm 1$ ).

To avoid the appearance of excessive states (i.e., antiphotons), we assume

$$b_\Lambda = 0. \quad (5.4)$$

The inequality (5.4) does not contradict the quantization postulates connected with the homogeneity of the medium, relativistic invariants, and transformation (5.1). Here, however, the commutation relations turn out to be nonlocal.

$$[A_m(x), A_{m'}(x')] \neq 0 \text{ при } (x - x')^2 < 0.$$

The latter should not worry us, since the observable physical quantities are represented by Hermitian operators for which the operators are local (see Sec. 2). In particular,  $\text{Re } F_{ik} = (F_{ik} + F_{ik}^*)/2$  is expressed in terms of the operators

$$\mathcal{A}(x) = A(x) + A^+(x),$$

which satisfy commutation relations of the ordinary type (we note that  $\mathcal{A}(x)$  is the solution of a fourth-order equation obtained by squaring Eq. (2.9). Since it is precisely  $\mathcal{A}(x)$  which enters in the Hamiltonian of the interaction of the field with the charges, the S matrix will have the usual causal properties.

Thus, we assume

$$A(x) = \frac{1}{(2\pi)^{3/2}} \sum_{\Lambda} \frac{1}{\sqrt{2\bar{n}_\Lambda}} \int \frac{d^3k}{\gamma|\mathbf{k}|} a_\Lambda(\mathbf{k}) \xi_\Lambda(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}}. \quad (5.5)$$

The commutation relations for the operators  $a_\Lambda(\mathbf{k})$  are given by

$$[a_\Lambda(\mathbf{k}), a_{\Lambda'}(\mathbf{k}')] = 0, \quad [a_{\Lambda'}(\mathbf{k}), a_{\Lambda''}(\mathbf{k}'')] = i\delta_{\Lambda\Lambda'}\delta(\mathbf{k} - \mathbf{k}'), \quad (5.6)$$

where

$$\delta_{\Lambda L} = -(1 + \kappa)\delta_S = 1. \quad (5.7)$$

Relations (5.6) and (5.7) are not compatible with the additional condition (2.7), if it is stipulated that the latter be satisfied for the field operators. Just as in the case of field quantization in a vacuum, the way out of the situation is to require satisfaction of Eq. (2.7) in the mean for all the physical states of the free field. Under this condition, the longitudinal and scalar photons make no contribution to the field energy, the expression for which in the rest system of the medium is

$$W = \sum_{\mathbf{k}, \Lambda=\pm 1} \omega_\Lambda a_\Lambda^+(\mathbf{k}) a_\Lambda(\mathbf{k}).$$

The expression for the "charge" of the field is

$$Q_\Lambda = N_\Lambda \sum_{\mathbf{k}} a_{\Lambda'}^+(\mathbf{k}) a_\Lambda(\mathbf{k}).$$

Thus, conservation of the "charge" denotes conservation of the helicity of the field.

On the basis of (5.5)–(5.7), we can obtain the connection between the operators  $\mathcal{A}(x)$  in different space-world points  $x$  and  $x'$ , i.e., the causal Green's function

$$D_{ij}^c(x - x'): D_{ij}^c(x - x') = i\langle 0 | T(\mathcal{A}_i(x) \mathcal{A}_j(x')) | 0 \rangle,$$

$$D_{ij}^c(x) = \frac{1}{(2\pi)^4} \int D_{ij}^c(k) e^{i\mathbf{k}\cdot\mathbf{x}} d^3k,$$

$$D_{ij}^c(k) = -\sum_{\Lambda} \frac{n_\Lambda}{\bar{n}_\Lambda} \frac{\delta_{\Lambda i} \delta_{\Lambda j}}{k^2 - (1 - n_\Lambda^2) \omega^2 - i0} \quad (5.8)$$

(we recall that  $\omega$  is an invariant quantity defined by

formula (4.12) and equal to the frequency in the rest system of the medium).

Owing to the fact that  $n_\Lambda \neq n_{-\Lambda}$  for  $\Lambda = \pm 1$ , the Green's function (5.8) is not symmetrical in the indices  $i$  and  $j$ . We put

$$D_{ij}^c(k) = S_{ij}(k) + P_{ij}(k), \quad (5.9)$$

where

$$S_{ij}(k) = S_{ji}(k), \quad P_{ij}(k) = -P_{ji}(k). \quad (5.10)$$

The symmetrical and antisymmetrical parts of the Green's function can be written, with the aid of the explicit expressions for  $\xi_\Lambda$  (formulas (4.5) and (4.6)) in the following form:

$$S_{ij}(k) = S_1(k^2) g_{ij} + S_2(k^2) v_i v_j + S_3(k^2) u_i u_j,$$

$$P_{ij}(k) = P(k^2) e_{ijlm} u^l v^m, \quad (5.11)$$

where

$$S_1(k^2) = \frac{k^2 + \kappa\omega^2}{D_+(k^2) D_-(k^2)},$$

$$S_2(k^2) = \frac{f^2 \omega^2 (k^2 + \omega^2)}{D_+(k^2) D_-(k^2) D_0(k^2)},$$

$$S_3(k^2) = \frac{-\kappa k^4 + (f^2 - 2\kappa^2) k^2 \omega^2 - (f^2 + \kappa^2) \omega^4}{(1 + \kappa) D_+(k^2) D_-(k^2) D_0(k^2)},$$

$$P(k^2) = -\frac{if}{2\bar{n}_+} \frac{k^2 - (\kappa + 2) \omega^2}{D_+(k^2) D_-(k^2)},$$

$$D_\pm(k^2) = k^2 + (n_\pm^2 - 1) \omega^2 - i0,$$

$$D_0(k^2) = k^2 + \kappa\omega^2 - i0. \quad (5.13)$$

It is seen from (5.9)–(5.13) that the Green's function has three different poles; two of them correspond to transverse photons ( $\Lambda = \pm 1$ ) and one to longitudinal and scalar photons. In this case, as expected, the antisymmetrical part of  $P_{ij}(k)$  has poles corresponding only to transverse photons.

Formulas (5.9)–(5.13) make it possible to calculate the amplitudes of the different quantum processes in any order of perturbation theory. Of course, their use is limited to the region of applicability of the phenomenological description of the medium, i.e., to processes caused by sufficiently soft photons. We note that the gyrotropy effect increases the number of phenomena that can be of interest from the experimental point of view.

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