SELF-FOCUSING OF EMISSION FROM A CONTINUOUS-WAVE GAS LASER

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The results of theoretical and experimental investigation of a new effect are reported concerning thermal self-focusing of argon laser emission in absorbing crystals and glasses in which $dn/dT > 0$. It is found that in the case of a number of well-known glasses and crystals the critical power is fairly moderate and does not exceed a few hundreds of milliwatts. The minimum diameter of the self-focusing beam in the focal region is $\sim 50\,\mu$ in optical glasses.

1. The energy of optical radiation passing through real media is partially absorbed. The medium heated by a laser beam forms a thermal lens whose effect depends on the sign of the derivative dn/dT. In media with $dn/dT < 0$ the divergence of the transmitted beam increases (defocusing effect), and in media with $dn/dT > 0$ the beam divergence decreases manifesting a self-focusing effect.

The first observations of the thermal self-focusing effect in glasses and $LiNbO_3$ crystals were reported by us;[1-3] we then observed external self-focusing of an argon laser beam (the focal region was beyond the limits of the nonlinear medium). Recently Kelley and Karman succeeded in observing internal self-focusing in glasses (private communication).

This paper presents the results of further experiments with glasses in which self-focusing of a laser beam was obtained within the nonlinear specimen.[1)] (The results of this part of the work are presented in [4] and [5]). Below we also derive the theory of thermal self-focusing and, in particular, reveal the conditions of realization of internal self-focusing.

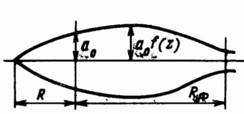

FIG. 1. Diagram of internal self-focusing. α_0 – radius of the beam; R – wave front curvature characterizing beam divergence; f(z) – dimensionless beam width; R_f – length of self-focusing; z – distance to the front face of the specimen.

2. We first consider the computation of power necessary to observe the internal thermal self-focusing (Fig. 1). Stationary thermal self-focusing[2)] at the axial part of the Gaussian beam (zero-aberration approximation) is described by an equation for a dimensionless beam width f(z)

$$\frac{d^2f}{dz^2} = -\frac{P_0\delta\exp(-\delta z)\,dn/dT}{\pi n a_0^2 \varkappa f} + \frac{1}{L_d^2 f^3}. \qquad (1)$$

The method of deriving (1) is similar to that we used earlier in the self-focusing theory[8] (see also [1] and [3]). In (1) P_0 is beam power, δ is the linear (single photon) attenuation decrement, a_0 is beam radius at the entrance to the medium, κ is the coefficient of thermal conductivity, and $L_d = \frac{1}{2}ka_0^2$ is the diffraction length of the beam. In the analysis of (1) it is convenient to introduce a parameter with a dimension of length:

$$R_{nl} = \frac{\pi n a_0^2 \varkappa}{P_0 dn/dT}.$$

In (1) the first term characterizes nonlinear refraction and the second, diffraction effects. The qualitative features of self-focusing follow from the general form of (1). The larger the losses δ the stronger nonlinear refraction; at the same time the nonlinear refraction effect develops over lengths that do not exceed $L_\delta = \delta^{-1}$ (exponent of $e^{-\delta z}$ (1)). Hence it follows in particular that the self-focusing length cannot be less than the value of R_{nl}. It is characteristic that the "strengths" of nonlinear refraction and diffraction in (1) depend differently on beam width f. In this connection, in contrast to the Kerr self-focusing (cf. [8-10]), the focal spot has a finite diameter even in the zero-aberration approximation.[3)] For comparison with experimental data the following parameters are of greatest interest: P_{cr} is the power at which nonlinear refraction and diffraction are completely compensated, and P_{th} is the power at which the self-focusing length R_f is equal to the length l of the specimen.

We integrate (1) by approximating the exponential law of attenuation $e^{-\delta z}$ with a weaker relationship $(1 + \delta z)^{-2}$ (it is readily apparent that the power absorbed by the medium as $\delta z \to \infty$ equals input power P_0 in either dissipation law). In this case the introduction of new variables $\tilde{f} = f/(1 + \delta z)$ and $\tilde{z} = z/(1 + \delta z)$ reduces (1) to a simpler equation

$$\frac{d^2\tilde{f}}{d\tilde{z}^2} = -\frac{\delta}{R_{nl}\tilde{f}} + \frac{1}{L_d^2\tilde{f}^3}, \qquad (2)$$

whose first integral is

$$\left(\frac{d\tilde{f}}{d\tilde{z}}\right)^2 = -\frac{\delta}{R_{nl}}\ln\tilde{f} - \frac{1}{L_d^2\tilde{f}^2} + \frac{1}{L_d^2} + \left(\frac{1}{R} - \delta\right)^2, \qquad (3)$$

[1)]We are thus concerned with CW self-focusing of a type that is observed in solid-state laser beams propagating in liquids with a large Kerr constant.

[2)]Nonstationary thermal self-focusing neglecting thermal conductivity was considered in [6,7].

[3)]Note that the finite spot diameter in Kerr self-focusing effect is due to the saturation of nonlinearity. In the case of thermal nonlinearity however the limiting factor is the escape of heat from the beam.

where R is the radius of curvature of the beam wave front at the entrance to the nonlinear medium.

We use (3) to analyze the behavior of the self-focusing beam, at first in the geometric optics approximation, assuming that $L_d \to \infty$. A simple derivation results in the following dependence of beam width f from the z coordinate:

$$-\left(\operatorname{sign}\frac{d\tilde{f}}{d\tilde{z}}\right)\Phi\left(\left[(\delta-R^{-1})^2\frac{R_{\mathrm{nl}}}{2\delta}+\ln\frac{1+\delta z}{f}\right]^{1/2}\right)$$
$$+\Phi\left((R^{-1}-\delta)\sqrt{\frac{R_{\mathrm{nl}}}{2\delta}}\right)=\frac{z}{1+\delta z}\sqrt{\frac{2\delta}{\pi R_{\mathrm{nl}}}}\exp\left\{-\frac{(R^{-1}-\delta)^2R_{\mathrm{nl}}}{2\delta}\right\}, \tag{4}$$

where Φ is the error integral

$$\frac{d\tilde{f}}{d\tilde{z}}=\frac{df}{dz}(1+\delta z)-\delta f.$$

Setting f = 0 and $df/dz < 0$ in (4) we find the distance $z = R_f$ required for the beam to self-focus,

$$\frac{R_{\mathrm{nl}}}{R_f}=\frac{\sqrt{2\pi^{-1}\delta R_{\mathrm{nl}}}\exp\{-(R_{\mathrm{nl}}R^{-1}-\delta R_{\mathrm{nl}})^2/2\delta R_{\mathrm{nl}}\}}{1+\Phi[(R_{\mathrm{nl}}R^{-1}-\delta R_{\mathrm{nl}})/\sqrt{2\delta R_{\mathrm{nl}}}]}-\delta R_{\mathrm{nl}}. \tag{5}$$

The family of curves of R_f as a function of δR_{nl} is shown in Fig. 2 for various initial beam divergences (various values of R/R_{nl}). We see that the length of self-focusing shortens with increasing attenuation. However the focal length of the "thermal lens" cannot be made less than R_{nl}. Therefore also the initial beam divergence should at least not exceed the value of a_0/R_{nl}, i.e., to obtain the thermal self-focusing effect we should have $R \geq R_{nl}$.

Analysis of (5) shows that two limiting cases of self-focusing should be distinguished depending on the absorption in the medium, i.e., on the parameter δR_{nl}. This distinction appears particularly clear in the case of the self-focusing of a parallel beam, i.e., when $R = \infty$.

1. When $\delta R_{nl} \ll 1$ we have a thick lens, the wave is practically unattenuated and the entire nonlinear medium participates in self-focusing ($0 < z < R_f$); the focal length of the thick lens is

$$R_f=\sqrt{\pi R_{\mathrm{nl}}/2\delta}. \tag{6a}$$

2. When $\delta R_{nl} \gg 1$ we have a thin lens and the strong absorption causes a nonlinear refraction of the beam practically only in the first layer $L_\delta = \delta^{-1}$ thick. Beyond this layer ($z > L\delta$) the medium can be considered linear and the beam is thus focused in the distance

$$R_f=R_{\mathrm{nl}}, \tag{6b}$$

characterizing the optical power of the thin lens.

Thus thermal self-focusing of the beam in the length $R_f = l$ ($l \ll L_d$) requires a threshold power of

$$P_{\mathrm{th}}=\frac{\pi n\varkappa a_0^2}{l\,dn/dT}\left(1+\frac{2}{\pi}\frac{L_\delta}{l}\right). \tag{7}$$

(This notation combines the cases corresponding to the thick (6a) and thin (6b) lenses). So far we have not taken diffraction effects into account, considering lengths $z \ll L_d$. However if we set $l = L_d$ in (7) (here $z_f \to \infty$), we can obtain a formula to determine critical power

$$P_{\mathrm{cr}}=\frac{\lambda_0\varkappa}{dn/dT}\left(1+\frac{2}{\pi}\frac{L_\delta}{L_d}\right). \tag{8}$$

We evaluate P_{th} and P_{cr} for conditions close to the experimental situation. For an argon laser beam ($\lambda_0 = 0.5\ \mu$, $a_0 = 0.5$ mm) and glass with $dn/dT = 10^{-5}\ \mathrm{deg}^{-1}$, $\kappa = 10^{-2}$ w/cm·deg at $L_\delta \ll L_d$ we have $P_{cr} = 0.1$ W. For $l = 10$ cm and $\delta l \approx 1$ we have $P_{th} = 1.5$ W. This means that the internal self-focusing effect is fully observable in beams of continuous-wave gas lasers.

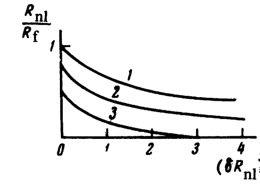

FIG. 2. Dependence of reverse length of self-focusing R_{nl}/R_f on the quantity δR_{nl} for various initial beam divergences. 1 – R = ∞, parallel beam; 2 – R = $4R_{nl}$; 3 – R = $2R_{nl}$. The limiting permissible divergence corresponds to R = R_{nl}.

It is of interest to compute the radius of the focal spot $a_f = f_f a_0$. The value of f_f can be found from (3), setting $z = R_f$ and $df_f/dz = 0$. For the case of the thick lens ($L_\delta \gg R_f$) we have

$$f_f\approx\left(\frac{P_0}{P_{\mathrm{cr}}}\ln\frac{P_0}{P_{\mathrm{cr}}}\right)^{-1}, \tag{9a}$$

and for the thin lens ($L_\delta \ll R_f$) as expected

$$f_f\approx R_{\mathrm{nl}}/L_d=P_{\mathrm{cr}}/P_0. \tag{9b}$$

Equations (9) allow us to compute the gain in brightness ($\mathrm{W/cm^2}$) due to thermal self-focusing. With optimal attenuation $\delta R_f \approx 1$ the gain is of the order of f_f^{-2}.

We note once again that (1)–(9) refer to zero-aberration self-focusing. Aberration can be readily taken into account in the thin-lens case. For a thick lens aberration can be computed analytically without considering diffraction. Such a computation is analogous to that performed by us in the analysis of defocusing.[3] The cumbersome derivation forced us to omit this analysis in this paper.

3. The experiment was performed with an argon laser generating the TEM_{00} mode at the wavelength of $\lambda_0 = 4880$ Å and power of up to 1 W in a beam with a radius $a_0 = 0.35$ mm. Thermal self-focusing was observed in colored $LiNbO_3$ crystals ($l = 0.4$ cm, $\delta l = 3.7$) and optical glasses TF = 105 ($l = 12$ cm, $\delta l = 1.3$). Internal self-focusing was obtained in glass; only external self-focusing was observed in the $LiNbO_3$ crystal. Figure 3 shows a photograph of an argon laser beam in glass for $P_0 > P_{th}$ illustrating the process of beam compression along its propagation path. In contrast to the Kerr self-focusing of solid-state laser beams, the gas laser beam is self-focused as a whole without decomposition into separate filaments.

A study of the spatial structure of the beam emerging from the above specimens showed that a complex ring

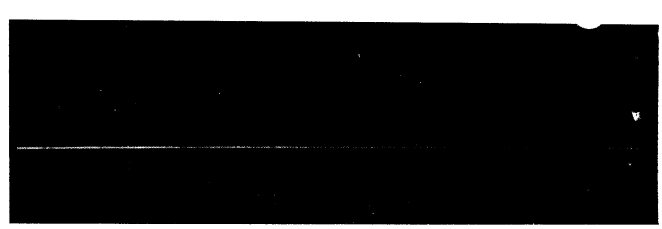

FIG. 3. Photograph of the lateral view of an argon laser beam in optical glass power $P_0 > P_{th}$. Beam compression along the path of propagation in glass is visible.

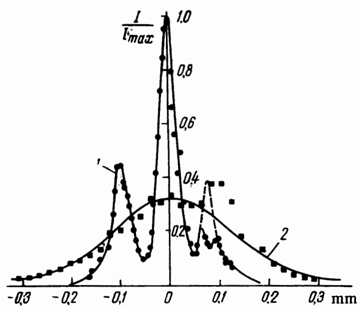

FIG. 4. Cross section intensity distribution of a self-focusing argon laser beam emerging from optical glass (curve 1) for $\delta l = 1.3$. Curve 2 shows intensity distribution for the beam that passed through a linear medium with the same value of δl.

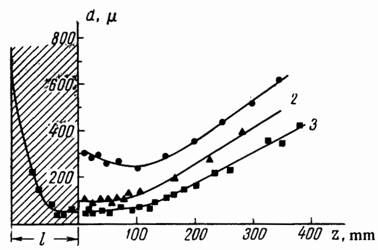

FIG. 5. Graphs of argon laser beam diameter as function of distance to front face of the specimen in the course of self-focusing in glass (we measured the diameter of the central spot of the aberration pattern containing about 50% of total energy). The graph parameter is total beam power. Shaded region denotes glass. Curves 1 – 100 mw; 2 – 180 mw; 3 – 400 mw.

structure with a bright central spot is formed in the cross section of the beam; the structure is due to the aberrations of the thermal lens.[1,3,8] The number of rings increases with distance from the absorbing medium. The results of photometry of the transverse structure of the beam at the output face of a glass specimen $l = R_f$ are given in Fig. 4. The minimum diameter of the central spot in the glass was $\sim 50\,\mu$.

Graphs in Fig. 3 characterize the dynamics of beams with varying power in the glass (transition from internal to external self-focusing). The characteristic power values for glass in our experiments were $P_{cr} = 0.04$ W and $P_{th} = 0.2$ W.

We also measured the diameter d_f of the focal spot as a function of input beam power (see Fig. 5). The experimental data for lithium niobate are determined by the relationship $d_f \sim P_0^{-1}$ (the thin-lens condition $L_\delta \ll R_f$ holds well in $LiNbO_3$); at maximum power $P_0 = 560$ mW; $d_f = 45\,\mu$. In glass the dependence of d_f on P_0 is slower but the limiting thick-lens case of $L_\delta \gg R_f$ is not yet realized here.

It is of interest to note that the behavior of the central spot of a beam that emerged from the absorbing medium cannot be described by the usual formulas for aperture diffraction; this is apparently due to variation in the phase-amplitude structure of the beam in the course of self-focusing (this condition was noted in [11] with respect to the Kerr self-focusing). We also note that the far field structure of the beam was analogous in internal and external self-focusing in our experiments.

4. Of interest is the problem whether it is possible to obtain thermal self-trapping of the laser beam. Under our experimental conditions it was impossible to obtain a waveguide with a length $L > L_\delta$. In this connection the system of thermal self-focusing of a biharmonic field investigated by Kelley and Karman (private communication) is of interest.

The thermal self-focusing effects are significant for a wide range of CW lasers. Special selection of materials and lengthwise distribution of absorbing centers can provide conditions necessary to maintain a sufficiently strong field over lengths of the order of several centimeters. This may be of importance to the technology of frequency multipliers, parametric generators, and observation of nonlinear threshold effects in a continuous regime.

We finally note that along with the thermal self-focusing due to single-photon absorption and analyzed above, thermal self-focusing due to two-photon absorption may be of definite interest (especially in the nonstationary case). The critical parameter here is power density rather than total power, which significantly alters the entire picture of the self-focusing effect. It is possible that precisely this type of self-focusing is the cause of the breakdown of optically transparent crystals.

The information on the self-focusing properties of optical glasses obtained in this work provides a basis for a realistic evaluation of the possibilities of nonstationary thermal self-focusing of solid-state laser pulses in absorbing media. In particular, it was found that it is possible to obtain self-focusing of the second harmonic from a pulsed neodymium laser in glasses of the type investigated in this paper.

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Note added in proof (May 27, 1969). After this paper went to press, the work of F. Dabbi and J. Whinnery (Appl. Phys. Lett. 13, 286 (1968) was published with similar experimental results.

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