STATIC SKIN EFFECT IN TUNGSTEN

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The transverse magnet resistance of thin single crystal plates of high purity tungsten is investigated at 4.2°K in fields up to 10 kOe. The electrical resistivity of thin plates (~0.1 mm) in a magnetic field parallel to the surface is significantly smaller than that in a perpendicular field. It is assumed that the observed anisotropy, which is related to the shape of the conductor, is due to forcing out of the direct electric current towards the surfaces of the plates. In very pure samples, the calculated diffusivity coefficient q is found to depend on the thickness d and is less than unity in thin crystals. The observed dependence of q on d is ascribed to the presence in the tungsten of small groups of carriers, which in principle are scattered more specularly at the surface than the majority carriers, and hence play an appreciable role in the phenomenon.

In strong transverse magnetic fields, under definite conditions, a constant electric current is forced to the surface of the specimen—a static skin effect arises, the theory of which has been developed in the works of Azbel' and Peschanskiĭ.[1-4] The concentration of the current at the surface takes place because the electrons moving near the boundary of the conductor are more mobile than in its interior. In a magnetic field parallel to the boundary, collisions with the surface are undergone by all electrons moving in a boundary layer of thickness of the order of the Larmor radius r; this leads to a break in the orbits and, under the conditions of specular reflection, to a forward motion of the carriers along the boundaries of the conductor.

Experimentally, the effect is most easily observed in thin conductors whose resistance in the bulk state increases without limit in the magnetic field.

It is well known that the asymptote of the resistance in strong fields is different for various metals and is determined by the electron structure of the conductor and by the direction of the magnetic field. Unbounded growth according to a square law is characteristic for metals with an equal number of electrons and holes $n_1 = n_2$, or for metals with $n_1 \neq n_2$ for directions in which there are open orbits. Under these conditions, the difference in the mobilities of the volume and surface carriers is seen to be small.

The effect appears especially simply in a magnetic field parallel to the plane of a thin plate. For such a geometry, conditions arise at the surface which guarantee an increase in the specific conductivity of the plate, the greater the smaller its thickness d. Rotation through a right angle leads to a decrease in the conductivity; an additional anisotropy of the resistance appears and is connected with the shape of the conductor.

Under specular reflection of the carriers, the effect of the surface is seen even in comparatively thick samples, where $d > l$, where l is the free path of the carriers.

As is shown in[4], the character of the interaction of the carriers with the surface determines essentially the law of change in the resistance of the plate in a magnetic field. Under the limiting conditions of specular (q = 0) and diffuse (q = 1) reflections, the resistance is a linear or quadratic function, respectively, of the magnetic field intensity (for metals with $n_1 = n_2$). For the intermediate cases ($0 < q < 1$), the linear dependence is maintained over a bounded interval of magnetic fields. Thus there is a possibility in principle of the experimental determination of the parameter q.

The first experimental investigation of the effect of the shape of the specimens on the magnetoresistance was carried out by Lazarev and Borovik in bismuth[5] even before the appearance of the theory. Analysis of recent measurements on purer samples[6] allows us, on the basis of the theory, to substantiate the current opinion on the practically specular character of the reflection of the carriers from the surface in this material.

The present research is devoted to the study of the magnetoresistance of thin plates of tungsten with $n_1 = n_2$ and with a much greater (in comparison with Bi) limiting Fermi momentum for the majority carriers and consequently with greater diffuseness, in principle, in their reflection from the surface.

EXPERIMENTAL PROCEDURE

The study of the anisotropy of the resistance of a single crystal plates of tungsten was carried out in a magnetic field up to 10 kOe at a temperature of 4.2°K. The samples had the shape of bars of length 20 mm and width 4 mm. In the study of the size effect, the thickness of the bars was systematically reduced from 1.6 to 0.1 mm by mechanical abrasion with subsequent treatment in an electropolishing solution. To prevent possible mechanical damage, samples of thickness less than 0.3 mm were treated only by an electrochemical method. The thickness of the plates was determined by direct measurement by means of a micrometer or was computed from the resistance at room temperature. The surface of the plates was in every case oriented parallel to the (110) faces with an accuracy no worse than ±2°; the current was passed in the direction of the cubic symmetry axis ⟨100⟩.

The distance between the clamping potential contacts of phosphor bronze was 8–9 mm. The current contacts were made of tantalum and were attached to the sample by spot welding. Special precautionary measures were taken which enabled us to avoid bending of the sample in mounting and in temperature changes. The sensitivity of the potentiometric apparatus was 5×10^{-8} V.

EXPERIMENT

The dependence of the resistance on the magnetic field is shown in logarithmic scale in Fig. 1 for a number of W samples with $\rho(300°K)/\rho(4.2°K) \sim 5 \times 10^5$. The curves 1 and 2 characterize the magnetoresistance of a plate of thickness 1.0 mm in magnetic fields that were respectively perpendicular and parallel to its surface. Curve 3 and 4 are the resistances of thinner plates (0.33 and 0.1 mm) in a parallel field. It is seen from the figure that the resistance of the plate in the perpendicular field is proportional to the square of its intensity; in the parallel field the dependence is much weaker in the region of small fields (<3 kOe) and almost the same in strong fields (<3 kOe).

Figure 2 shows the results of the measurement in polar diagrams of the resistance $\rho_{H=\text{const}}(\vartheta)$ in a constant magnetic field H = kOe for a given sample. For comparison, the data of[7] are plotted by a dashed curve with an arbitrary scale. These data characterize the magnetoresistance of a bulk specimen of W of the same crystallographic orientation.

For thin plates, a violation of the fourfold symmetry of the cubic axis of the crystal is observed. In a magnetic field, different resistivities ρ correspond to equivalent crystallographic directions. When the magnetic field coincides with the plane of the plate, its resistance is a minimum and decreases further with decrease in the thickness d. The resistance in the perpendicular field also depends on the thickness and falls off with decrease in d. Nevertheless, the anisotropy of the resistance continues to increase. We note that in the absence of a magnetic field the effect of the surface leads to an opposite result: the resistivity of bounded conductors increases with decrease in their thickness.

The general form of the polar plots (Fig. 2) remains stable to a decrease in the field strength down to ~3 kOe. Its subsequent decrease leads to a gradual disappearance of the resistance anisotropy associated with the shape of the conductivity. This is illustrated in Fig. 3, in which the dependences of the ratio $\rho_\perp/\rho_\parallel$ on the magnetic field strength is shown for the same range of thicknesses.

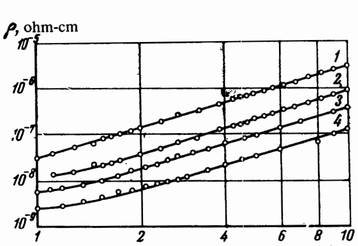


FIG. 1. Dependence of the resistance on the magnetic field for samples with the ratio $\rho(300°K)/\rho(4.2°K) \sim 5 \times 10^5$. Curves 1 and 2 for d = 1.0 mm are for perpendicular and parallel fields, respectively; curves 3 and 4 are for thicknesses 0.3 and 0.10 mm in a parallel field.

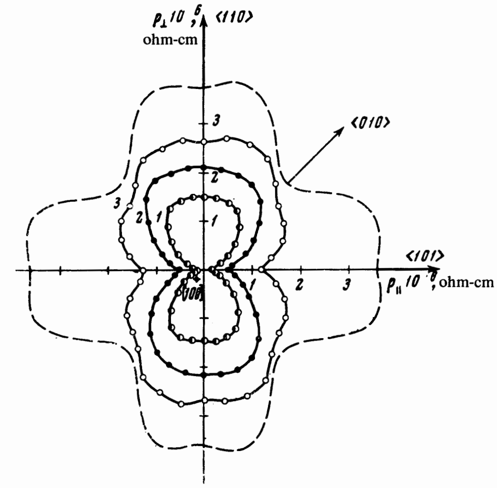


FIG. 2. Polar diagrams of the resistance $\rho_{H=8\,\text{kOe}}(\vartheta)$ for samples with $\rho(300°K)/\rho(4.2°K) \sim 5 \times 10^5$. Curves 1 for d = 1.6, 2 for 0.7, 3 for 0.2 mm. The dashed curve represents data from [7].

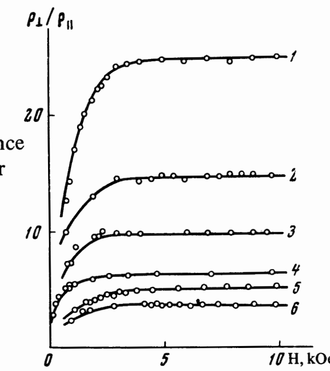


FIG. 3. Anisotropy of the resistance as a function of the magnetic field for samples with $\rho(300°K)/\rho(4.2°K) \sim 5 \times 10^5$. Curves 1 for d = 0.10, 2 for 0.20, 3 for 0.33, 4 for 0.40, 5 for 0.69, 6 for 1.00 mm.

DISCUSSION OF THE RESULTS

Account of boundary effects in the presence of a strong magnetic field leads to the following expression for the electrical conductivity of the plate in the parallel field:[4]

$$\sigma = \sigma_0 \frac{\gamma}{q+\gamma} \frac{r}{d} + \sigma_0 \gamma^2. \qquad (1)$$

Here σ_0 is the conductivity of the bulk metal in the absence of the field, $\gamma = r/l$. The surface conductivity (the first term on the right hand side of the expression) is the greater the higher the degree of specularity of the reflection of the electrons $q_1 = 1 - q$. In sufficiently high fields, almost all the current flowing in the conductor is concentrated in a bounded layer of thickness $\sim r$.

As is seen from Fig. 2, halving of the resistivity, which is already achieved for plates of thickness ~1 mm, means that $\sigma_{\text{surf}} \sim \sigma_{\text{bulk}}$. For thinner samples, $\sigma_{\text{surf}} > \sigma_{\text{bulk}}$. In this connection, the current density on the surface exceeds the density in the bulk of the conductor by more than an order of magnitude in fields ~10 kOe (if we assume that $r = 50\ \mu$ for $H = 10$ kOe).

We now make an estimate of the parameters q and l. For this purpose, we rewrite the expression (1) in the form

$$\frac{q}{l}+\frac{H_0}{l}\frac{1}{H}=F,\quad F^{-1}=\left(\frac{\sigma}{\sigma_\infty}-1\right)d, \qquad (2)$$

where $\gamma = r/l = H_0/H$ and σ_0 is the conductivity of the bulk metal in the magnetic field. The value of H_0 can be estimated directly from experiment by means of the condition that $H_0 = H$ when $\rho \sim \Delta\rho$.[8]

Evidently, in the coordinates F and 1/H, the expression (2) is the equation of a straight line which intercepts the quantity q/l on the ordinate and has a slope H_0/l.

Curve 1 of Fig. 4 shows the dependence F(1/H) for a plate of W with $d = 300\ \mu$ and $\rho(300°K)/\rho(4.2°K) \sim 6 \times 10^4$ and $H_0 \sim 0.5$ kOe. The quantities q and l, estimated from these data, are equal to ~1 and 0.4 mm, respectively. Then, the fact that $q \sim 1$ means that the scattering from the surface has a predominantly diffuse character. Account of the fact that $r = l$ when $H_0 = H$ allows us to estimate the mean momentum of the electron on the Fermi surface, $p = reH_0/c$, which is equal to 1 Å^{-1} in order of magnitude in the present case. This is in agreement with the data of[9]. In decrease in the thickness to 0.1 mm, the dependence F(1/H) remains the same.

Curves 2, 3 and 4 on the same drawing show the dependence of F(1/H) for much purer samples with $\rho(300°K)/\rho(4.2°K) \sim 5 \times 10^5$ for thicknesses of 1.6, 0.7, and 0.2 mm. In the calculation of F(1/H), the conductivity $\sigma_\perp$ of the thickest (d = 1.6 mm) plate investigated in this series in a perpendicular field was taken for σ_∞. One can assume that $\sigma_\perp$ does not differ much from σ_∞.

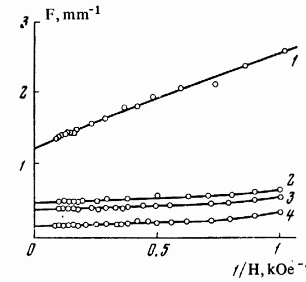
It is seen from the figure that the curves are not superimposed on one another and that a decrease in d leads to some change in the slope of the curves and their bending as a function of the field. The decrease of q/l with decrease in the thickness of the samples is characteristic.

We now discuss the reasons of the thickness for the effect on F(1/H) and, as a consequence, on q/l for pure samples.

The explanation that the decrease of q/l with d for this series is due only to the increase in l is not plausible, since it is natural to expect a decrease of l in thin samples. It is clear that, in the presence in the crystal of a spectrum of lengths l corresponding to different groups of electrons, the decrease in the thickness of the plate affects first the groups of carriers with large l and only then those with small l. Averaging over all groups, the length l ought only to fall off.

Thus the dependence of q/l on d can most likely be explained by a decrease in q (increase in the specularity of the reflection). The existence of this dependence in thin plates of high purity tungsten is evidently connected with the presence in the crystal of small groups of electrons that are scattered on the surface more specularly than the majority carriers. In tungsten, these can be small hole ellipsoids centered in the Brillouin zone at the point N.[9] According to the data

FIG. 4. Dependence of F on 1/H. Curve 1 for samples with $\rho(300°K)/\rho(4.2°K) \sim 6 \times 10^4$ for d = 0.3 mm, curves 2, 3, and 4 for samples with $\rho(300°K)/\rho(4.2°K) \sim 5 \times 10^4$ for d = 1.6, 0.7, 0.2 mm, respectively.



of[9] the concentration n′ in this group is ~12% of the concentration of majority carriers $n = n_1 + n_2$. It can be supposed that the corresponding conductivities σ_0' and σ_0 are in about the same relation.

We now consider in more detail the effect of this group on the electrical conductivity of the plate in the magnetic field. Equation (1), generalized to this case, has the following form

$$\sigma=\sigma_0'\frac{\gamma'}{q'+\gamma'}\frac{r'}{d}+\sigma_0\frac{\gamma}{q+\gamma}\frac{r}{d}+\sigma_0'\gamma'^2+\sigma_0\gamma^2. \qquad (3)$$

Here the first and third terms of the right hand side correspond to the surface and bulk conductivities associated with a small group of carriers.

In the limiting case of a strong magnetic field ($\gamma \ll q$) and completely specular reflection for carriers of this group ($q' = 0$), Eq. (3) transforms to the following:

$$\sigma=\sigma_0'\frac{r'}{d}+\sigma_0\gamma\frac{r}{d}+\sigma_0'\gamma'^2+\sigma_0\gamma^2. \qquad (4)$$

It then follows that in pure crystals and strong fields ($\gamma' \ll 1$) the contribution of the first term to the surface conductivity can be not too small in spite of the fact that $\sigma_0' r \ll \sigma_0 r$. Since virtually specular reflection from the surface is characteristic for carriers of this group, it follows that the integral parameter q, determined experimentally, is much less than unity and depends on the thickness. The latter is connected with the fact that the surface conductivity is masked by the bulk conductivity.

Thus the static skin effect, observed in tungsten in strong magnetic fields, is essentially determined by small groups of charge carriers, the participation of which has little effect on the conductivity of the crystal under ordinary conditions.

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