TRANSMISSION OF AN ELECTROMAGNETIC WAVE BY A FERROMAGNETIC METAL IN THE ANTIRESONANCE REGION

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Submitted March 12, 1970

Zh. Eksp. Teor. Fiz. 59, 424–433 (August, 1970)

Measurements of the position and profile of an antiresonance line of permalloy are reported for various values of the frequency, the temperature, and the thickness of a sample. The dependence of the intensity of a microwave signal transmitted by a ferromagnetic plate on an external magnetic field is calculated in the antiresonance region. A comparison is made between the observed and calculated line profiles in various specific cases. Optimum values of the sample thickness for the determination of the spin-spin relaxation time are found for ferromagnetic metals.

I. INTRODUCTION

THE skin depth in a ferromagnetic metal, governed by the high-frequency permeability $\mu = \mu' - i\mu''$, varies rapidly in the ferromagnetic resonance region. The reciprocal of the depth of penetration of an electromagnetic wave is

$$\operatorname{Re} k \propto [(\mu'^2 + \mu''^2)^{1/2} + \mu'']^{1/2}.$$

Figure 1 shows the dependence of μ' and μ'' on an external magnetic field H for $\omega/\gamma > 4\pi M$. At the ferromagnetic resonance point (FMR) the depth of penetration of an electromagnetic wave decreases strongly because of the large value of μ''. At the ferromagnetic antiresonance point (FMAR), we find that $\mu' \to 0$ and μ'' is quite small. Kittel,[1] Yager,[2] and Van Vleck[3] established that, at the ferromagnetic antiresonance point (which is defined by the condition $\omega/\gamma = B_{int}$), the skin depth should increase greatly and the transparency of a ferromagnetic metal should become greater.

Kaganov was the first to investigate this phenomenon theoretically[4] and he expanded his treatment in[5]. The intensity of an electromagnetic wave transmitted by a ferromagnetic film in the antiresonance region was calculated also by Tannenwald.[6] Lewis, Alexandrakis, and Carver[7] observed the resonance transmission of a signal through a 75 μ thick gadolinium foil (the skin depth for $\mu = 1$ was 6 μ) in the paramagnetic region near the Curie point. Hirst[8] demonstrated that the observed signal could be explained by an increase in the skin depth in the ferromagnetic antiresonance region. The necessary analysis was made by Van der Ven,[9] who obtained a good agreement with the results published by Hirst.[7]

Our own preliminary results were reported in[10]. We observed selective transmission, in the antiresonance region, of a 36 GHz electromagnetic wave incident on a ferromagnetic Permalloy plate, which was 14 μ thick. The present paper gives the results of calculations of the selective transmission by samples of various thicknesses, and of the spin-spin relaxation time. We shall also report the results of measurements of the position and profile of the ferromagnetic antiresonance line as a function of the frequency, the temperature, and the thickness of the Permalloy samples. In each case, a comparison will be made between the observed and calculated profiles.

II. THEORETICAL ANALYSIS

1. Results of Calculations

We shall consider a plane electromagnetic wave e, $\mathbf{h} \propto \exp(i\omega t - \mathbf{k}\cdot\mathbf{r})$, incident on an infinite ferromagnetic plate of thickness d. We shall use Maxwell's equations in the following form:

$$\operatorname{rot}\mathbf{h} = \frac{1}{c}4\pi\sigma\mathbf{e}, \quad \operatorname{rot}\mathbf{e} = -\frac{1}{c}\frac{\partial}{\partial t}(\mathbf{H} + 4\pi\mathbf{M}), \tag{1}$$

where the displacement currents are ignored, compared with the conduction currents, because the electrical conductivity σ is high. These equations yield expressions for the wave vector k, representing a wave whose damping in the antiresonance region decreases to a minimum:

$$k = \frac{k_1 + ik_2}{\delta_0}, \quad k_1 = \left(\frac{\sqrt{\mu_1^2 + \mu_2^2} + \mu_2}{2}\right)^{1/2}, \quad k_2 = \left(\frac{\sqrt{\mu_1^2 + \mu_2^2} - \mu_2}{2}\right)^{1/2}; \tag{2}$$

here, $\delta_0 = c/\sqrt{4\pi\sigma\omega}$ is the skin depth for $\mu = 1$; $\mu = \mu_1 - i\mu_2$ is the high-frequency permeability. The boundary conditions, representing the continuity of the tangential components of e and h at the boundaries of the sample, can be used in combination with Eq. (1) to find an expression for the amplitude of the power P of the transmitted wave represented by Eq. (2):

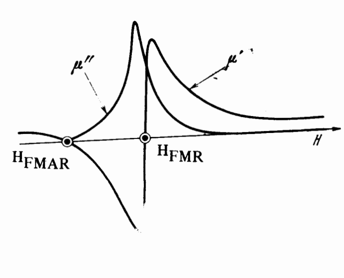

FIG. 1. Dependences of μ' and μ'' on the external magnetic field for $\omega/\gamma > 4\pi M$.

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$$\frac{P}{P_0} \sim (k_1{}^2 + k_2{}^2) \Big/ \left[\mathrm{sh}^2 \left(\frac{d}{\delta_0} k_1 \right) + \sin^2 \left(\frac{d}{\delta_0} k_2 \right) \right], \quad (3)$$

which is identical with the expression obtained by Kaganov.[5]

The high-frequency permeability μ is found from the equation of motion of the magnetization **M**, which has the Bloch-Bloembergen damping term. If the external magnetic field **H** is directed along the z axis, we find that

$$\left(\frac{d\mathbf{M}}{dt} \right)_{x,y} = \gamma [\mathbf{MH}_{\mathrm{int}}]_{x,y} - \frac{M_{x,y}}{T}. \quad (4)^*$$

Introducing dimensionless parameters

$$\eta = \frac{H}{4\pi M}, \quad \Omega = \frac{\omega}{4\pi M\gamma}, \quad \Omega' = \frac{1}{4\pi M\gamma T} \quad (5)$$

and solving Eq. (4), we obtain the following expressions for the parallel configuration (the external magnetic field parallel to the plane of the plate):

$$\mu_1 = \frac{(\eta+1)^2 + \Omega'^2 - \Omega^2}{\eta(\eta+1) + \Omega'^2 - \Omega^2}, \quad \mu_2 = \frac{2\Omega\Omega'(\eta+1)}{[\eta(\eta+1) + \Omega'^2 - \Omega^2]^2}, \quad (6a)$$

and the following expressions for the perpendicular case (the external magnetic field perpendicular to the plane of the plate):

$$\mu_1 = \frac{\eta - \Omega}{\eta - \Omega - 1}, \quad \mu_2 = \frac{\Omega'}{(\eta - \Omega - 1)^2}. \quad (6b)$$

Using Eqs. (2), (3), and (6) and specifying the values of the parameters Ω, Ω', and d/δ_0, we can find how the ratio P/P_0 depends on η. The results of computer calculations for $\Omega = 1.5$ will be given later.

2. Influence of the Thickness of a Sample on the Profile and Intensity of the Antiresonance Line

We now consider the profile of a transmitted microwave signal as a function of the external magnetic field. This will be done for samples of various thicknesses. Figure 2 shows the curves calculated for the case when the external field is parallel to the plane of the plate and the damping parameter is $\Omega' = 0.005$; in our case, this corresponds to an FMR line of width $\Delta H_{FMR} = 80$ Oe. The results obtained show that no appreciable change in the microwave power can be expected in the antiresonance region for $d/\delta_0 < 3$ ($\delta_0 = 1.4 \times 10^{-4}$ cm for Permalloy). When the thickness of the plate is increased, the dependence of the transmitted power on the magnetic field becomes a resonance curve with a maximum at the antiresonance point. Therefore, in the case of $d/\delta_0 > 6$, we may speak of the width of the FMAR line, which is measured at half the maximum transmitted power.

It seemed of interest to determine the variation in the maximum intensity of the transmitted signal in the antiresonance region for various thicknesses of a sample. The absolute value of this intensity can be found if we know the additional factor in Eq. (3) which is independent of the magnetic field and which represents the parameters of the microwave channel. Since our measurements established that $P/P_0 \sim 10^{-9}$ for $\Omega' = 0.005$ and $d/\delta_0 = 40$, we could determine this factor in Eq. (3) and then plot the curves which are shown in Fig. 3.

*[MH] ≡ M × H.

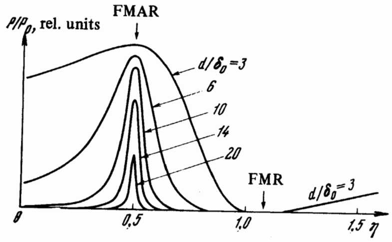

FIG. 2. Dependences of the transmitted microwave power on the external magnetic field, calculated for various thicknesses of a sample ($\Omega' = 0.005$, $\Omega = 1.5$). The magnetic field is assumed to be parallel to the plane of the plate.

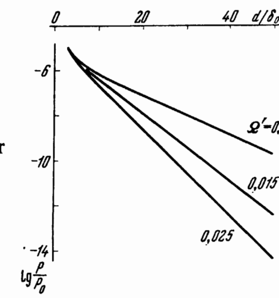

FIG. 3. Dependences of the maximum intensity of the microwave signal, transmitted at the antiresonance point, on the thickness of a sample for several values of the damping Ω' ($\Omega = 1.5$). The magnetic field is assumed to be parallel to the plane of the plate.

3. Influence of the Damping on the Width and Intensity of the Antiresonance Line

The influence of the damping Ω' on the width of the FMAR line for $d/\delta_0 = 6$ can be seen from Fig. 4a. The curves obtained do not differ greatly, although the damping differs by a factor of 9. The situation is different for $d/\delta_0 = 40$. We can see from Fig. 4b that the difference between the curves is now considerable. A comparison of the results presented in Figs. 4a and 4b shows that the influence of the damping on the antiresonance line width becomes greater when the thickness is increased.

An increase in the damping shifts the maximum of the transmitted power in the direction of weaker fields (relative to the antiresonance point).

The damping governs also the maximum intensity of the transmitted signal in the antiresonance region. Thus, for example, an increase in the damping by a factor of 5 for $d/\delta_0 = 25$ reduces the antiresonance signal by a factor of 1000 (Fig. 3).

III. SAMPLES AND MEASUREMENT METHOD

The frequency and temperature dependences were investigated using 14 μ thick circular Permalloy plates (82% Ni, 18% Fe), which were prepared as follows. A Permalloy strip of 53 μ initial thickness was rolled down to a thickness of 14 μ and cut into circular plates These plates were annealed in a hydrogen atmosphere at 1100°C for 3 h and were cooled to room temperature at a rate of 100 deg/h.

A circular Permalloy plate, 53 μ thick, and annealed by the method just described, was used in the measurements of the thickness dependences. After the FMR and FMAR lines had been recorded, the thickness of the plate was reduced by several microns by electrolytic polishing.[11] The thickness was measured with an optical comparator. The thinning operation

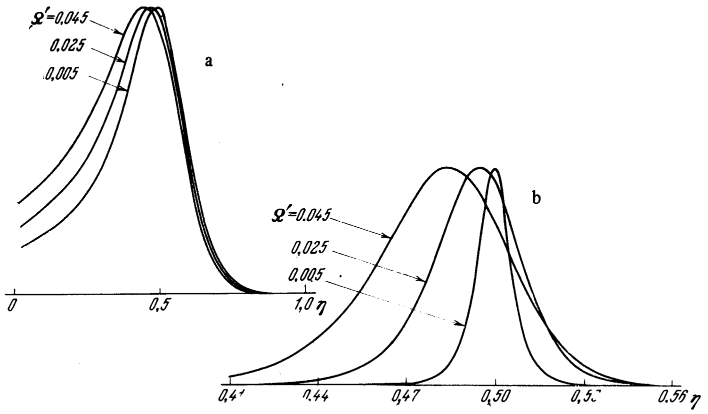

FIG. 4. Comparative curves showing the influence of the damping on the FMAR width: a) $d/\delta_0 = 6$; b) $d/\delta_0 = 40$. These curves are calculated for $\Omega = 1.5$ and an external magnetic field parallel to the plane of the sample.

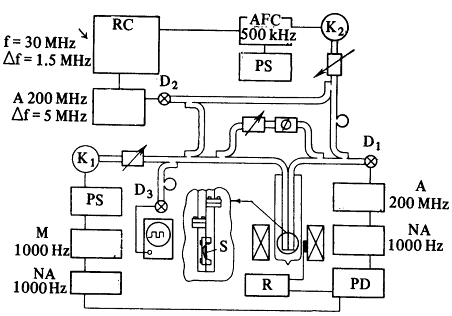

FIG. 5. Block diagram of the apparatus: S is a sample; PS is a power source; M is a modulator; NA is a narrow-band amplifier; A is an amplifier; PD is a phase sensitive detector; R is an X-Y automatic recorder; AFC is an automatic frequency control unit; RC is a resonance circuit; K_1 and K_2 are klystrons; and D_1, D_2, and D_3 are detectors.

was repeated several times until the final thickness of the sample was 16 μ. Annealing was not repeated after each polishing stage.

A block diagram of the appratus used in these measurements is shown in Fig. 5. A microwave signal ($\sim 10^{-2}$ W) was generated by a klystron K_1, modulated by square pulses of 100 Hz frequency, and applied to a resonator or a shorted waveguide (in investigations of the frequency dependences). The resonator (waveguide) wall had a coupling aperture of 3 mm diameter. This aperture was located in the region where the magnetic component of the microwave field had its maximum value. A conducting silver adhesive was used to bond the sample to the outside of this wall. The wall of the receiving waveguide, which had a similar coupling aperture, was pressed against the other side of the sample. The power reaching the receiving waveguide was applied directly to a mixer diode D_1, which also received a signal from a heterodyne klystron K_2. The signal produced by the diode was applied successively to amplifiers operating at 200 MHz and 1000 Hz. Next, the signal was passed through a phase detector to an automatic recorder. A compensation circuit, consisting of a precision attenuator and a phase shifter, eliminated strays in the microwave channel so that the intensity of the useful signal could be determined. The microwave signals produced by the klystrons K_1 and K_2 were applied also to a diode D_2, which was connected to an intermediate-frequency (200 MHz) stabilization circuit. The use of directional couplers of 10–30 dB attentuation made it possible to dispense

Table I

Sample, $d = 14\ \mu$	H_{FMR}, Oe	H_{FMAR}, Oe	ΔH_{FMR}, Oe	ΔH_{FMAR}, Oe	Ω'_{FMR}	Ω'_{FMAR}
Unannealed	8600	3810	520	1320	0,032	0,035
Annealed	8700	3900	260	1060	0,016	0,020

with isolators and bridges in the microwave channel and to tune easily the spectrometer frequency in the range between 30 and 43 GHz.

IV. EXPERIMENTAL RESULTS AND COMPARISON WITH THE THEORY

1. Position and Profile of the Ferromagnetic Antiresonance Line

The value of the conductivity σ was required before any comparisons could be made of the experimental and calculated results. This conductivity was measured and the following values were obtained:

T, °K:	293	77	4,2	2,0
σ, 10^{16} sec^{-1}:	1.49	1.87	1.89	1.90

The calculated and experimental dependences of the transmitted power on the magnetic field are given in Fig. 6 for the same sample in the perpendicular and parallel orientations. The position of the FMAR line is given by

$$\omega / \gamma = B_{\text{int}}$$

or

$$(\omega/\gamma)_\perp = H, \quad (\omega/\gamma)_\parallel = H + 4\pi M \tag{7}$$

for g = 2.17 and $4\pi M$ = 8070 Oe. These results are in agreement with the data on the ferromagnetic resonance given in Table I for the same values of the g factor and of the magnetization. H_{FMR} and H_{FMAR} in Table I represent the experimental values of the magnetic field at the absorption maximum in the case of the FMR and at the transmission maximum in the case of the FMAR. The width of the FMR line, ΔH_{FMR}, is defined as the distance between two extrema in the derivative of the absorption curve, and the damping Ω'_{FMR} is found from the condition

$$\Delta H_{FMR} = \frac{2}{\gamma T}, \quad \Omega' = \frac{1}{4\pi M \gamma T}. \tag{8}$$

The curves presented in Fig. 6 also show that the width of the antiresonance line depends on the direction of the external magnetic field. In the parallel field case, this width is 1060 Oe; in the perpendicular orien-

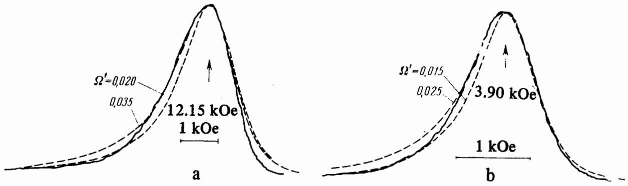

FIG. 6. Experimentally observed (f = 36.2 GHz, d = 14 μ) and calculated (dashed curves; $\Omega = 1.5$, $d/\delta_0 = 10$) antiresonance lines of a Permalloy plate: a) magnetic field perpendicular to the plane of the plate; b) magnetic field parallel to the plane of the plate.

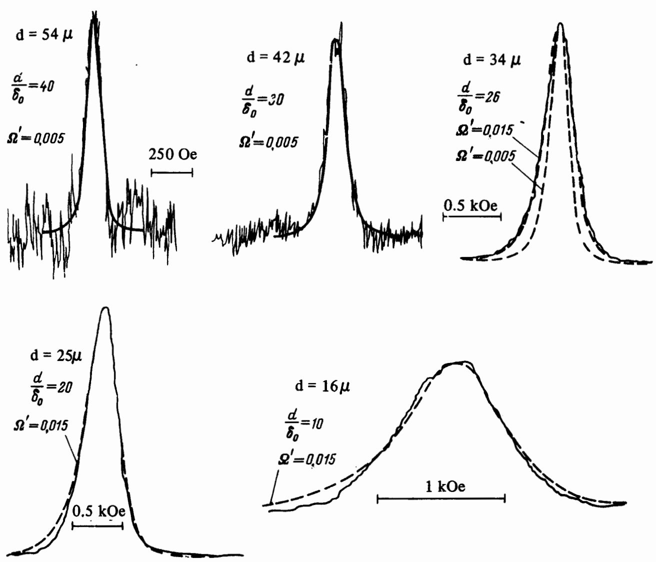

FIG. 7. Comparison of the observed (f = 36.6 GHz) and calculated ($\Omega = 1.5$) antiresonance line profiles obtained for various thicknesses of a sample. The magnetic field was parallel to the plane of the sample.

tation, it is 1900 Oe. The antiresonance line width, ΔH_{FMAR}, is defined as the distance between the points in the antiresonance curve, which correspond to half the maximum transmitted power. Although the antiresonance line width is in satisfactory agreement with the calculated width for both configurations and $\Omega' = 0.02$–0.035, the experimental and calculated profiles do not quite coincide. Therefore, the damping Ω'_{FMAR}, given in Table I, is found by equating the observed and calculated values of ΔH_{FMAR}.

The FMR and FMAR results were compared by making measurements on the same unannealed sample, 14 μ thick. Table I gives the results of these measurements at f = 36.6 GHz. The results for the annealed and unannealed samples indicate that the changes in the magnetization and damping are the same under the FMAR and FMR conditions.

2. Dependence of the Antiresonance Line Width on the Thickness of a Sample

It seemed of interest to investigate the dependence of the FMAR line width on the thickness of a sample. Figure 7 shows the recorded profiles of the transmitted signals for a magnetic field parallel to the plane of the sample, whose thickness was gradually reduced from 54 to 16 μ. The calculated antiresonance line

Table II

d, μ	ΔH_{FMR}, Oe	ΔH_{FMAR}, Oe	Ω'_{FMR}	Ω'_{FMAR}
54	410	110	0.025	0.005
42	320	160	0.020	0.005
34	210	290	0.013	0.015
25	210	370	0.013	0.015
16	210	960	0.013	0.015

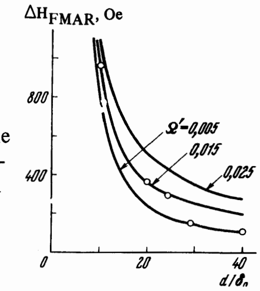

FIG. 8. Dependence of the FMAR line width (f = 36.6 GHz) on the thickness of a sample. The field **H** was parallel to the plane of the sample. The continuous curves represent the theoretical dependences calculated for various values of the damping and $\Omega = 1.5$.

profiles are represented by the dashed curves in Fig. 7. The agreement between the calculated and observed profiles is very good.

A comparison of the theoretical and experimental values of the FMAR line width is given in Fig. 8 for various thicknesses of a sample.

The FMR line was recorded for each value of the thickness of a sample. The results of the measurements of the FMAR and FMR are given in Table II. It is interesting to note that the damping deduced from the FMR and FMAR measurements is not the same for 54 and 42 μ. The observed difference may be explained as follows. Under the FMR conditions, the depth of penetration of the microwave field into the sample is much less than the thickness of the sample and, therefore, the measured damping is the value applicable to the surface layer. In the antiresonance measurements, an electromagnetic wave ($\mathbf{k} \rightarrow 0$) travels across the whole thickness of the sample and, therefore, the information obtained applies to the sample as a whole. Bearing in mind the technology used in the preparation (described earlier in the paper), we may conclude that the initial annealing failed to make the sample entirely homogeneous. When electrolytic polishing reduced the thickness of the sample by a factor of at least 1.5, the value of the damping deduced from the FMR and FMAR measurements were found to be the same. However, we should remember that the mechanical methods used in the determination of the thickness could have damaged the sample.

3. Frequency Dependences of the Position and Width of the Antiresonance Line

Figure 9 shows the frequency dependence of the FMAR line position, obtained for a Permalloy plate 14 μ thick. The external magnetic field was directed parallel to the plane of the plate and the results were described satisfactorily by the expression

$$(\omega/\gamma)_{\parallel} = H + 4\pi M, \tag{9}$$

where $4\pi M = 8070$ Oe, g = 2.17, in agreement with the ferromagnetic resonance data for the same sample. The width of the FMAR line, determined at half the

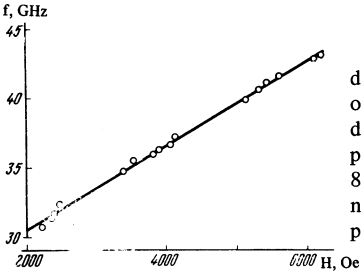

FIG. 9. Frequency dependence of the FMAR line position of Permalloy. The straight line drawn through the experimental points obeys Eq. (9) if $4\pi M$ = 8070 Oe and g = 2.17. The magnetic field was parallel to the plane of the sample.

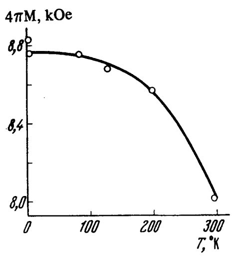

FIG. 10. Temperature dependence of the magnetization of Permalloy, calculated on the assumption that the formula $(\omega/\gamma)_{\|} = H + 4\pi M$ is valid.

maximum transmitted power, decreased from 1100 Oe at f = 30 GHz to 850 Oe at f = 43 GHz. When the frequency was increased, the skin depth δ_0 decreased from $\delta_{01} = 1.60 \times 10^{-4}$ cm at f = 30 GHz to $\delta_{02} = 1.35 \times 10^{-4}$ cm at f = 43 GHz. Thus, the relevant ratios were $d/\delta_{01} = 8.9$ and $d/\delta_{02} = 10.3$ It is evident from Fig. 8 that the frequency dependence of d/δ_0 was in agreement with the frequency dependence of the FMAR line width. The accuracy of the measurements of the damping was not high and, therefore, we could only say that the damping changed by a factor less than 2 in the investigated frequency range. This result was in agreement with that reported by Frait and Mitchell[12], who determined the relaxation time of Permalloy using the FMR method in a wide range of frequencies and found that the damping was independent of the frequency.

4. Temperature Dependences of the Position and Width of the FMAR Line

The temperature dependences of the position and width of the FMAR line were determined at f = 36.6 GHz in the temperature range from 293 to 2°K, using two orientations of the external magnetic field with respect to the plane of the sample. The measurements carried out in the perpendicular configuration, $(\omega/\gamma)_{\perp} = H$, showed that the line position was independent of the temperature. The FMAR line width was also constant (1900 Oe) throughout the temperature range 293–80°K. However, the line broadened between 80 and 2°K; at T = 4.2°K, the line width was 2600 Oe; at T = 2°K it was 3300 Oe.

In the parallel configuration, $(\omega/\gamma)_{\|} = H + 4\pi M$, the line position shifted when the temperature was lowered owing to the change in the magnetization of the sample. The temperature dependence of the magnetization was calculated using the formula given earlier for the antiresonance conditions in the parallel configuration; this dependence is presented in Fig. 10. The line width was found to be independent of the temperature throughout the investigated range and this was in agreement with measurements of the conductivity, carried out on the same sample (see Sec. IV.1).

The reported results indicate that the damping Ω' of Permalloy, deduced from the antiresonance line width, is independent of temperature between 293 and 80°K. The broadening of the antiresonance line below 80°K, observed when the external field is perpendicular to the plane of the sample, cannot be explained by a change in the skin depth since the measurements of the conductivity show that the depth is practically constant in this range of temperatures. We are not yet able to provide a satisfactory explanation of this line broadening.

V. CONCLUSIONS

The following conclusions can be drawn from our investigation. The observed transmission of microwaves in the antiresonance region is described satisfactorily by the solution of the equation of motion of the magnetization, in which the damping term is of the Bloch-Bloembergen type. The values obtained for the spin-spin relaxation time T_S and for the magnetization are mostly in agreement with the ferromagnetic resonance data. The characteristic relaxation time T_S of Permalloy is independent of the frequency and remains constant between 293 and 80°K.

Our investigation was carried out in order to establish the possibilities of the FMAR method for determining the relaxation time T_S of ferromagnetic metals. The low-temperature the value of T_S is difficult to determine by the FMR method for the following reasons. The skin depth in perfect samples decreases rapidly with falling temperature, and this reduces strongly the intensity of the ferromagnetic resonance signal. Since the value of k in the FMR region is large, the exchange interaction exerts a strong influence on the line width. Moreover, the appearance of the anomalous skin effect at fairly low temperatures complicates the interpretation and the line width measurements.

The optimum thickness of a sample in the measurements of the relaxation by the FMAR method is $d/\delta_0 = 20-30$ (Fig. 8). For these thicknesses, the signal intensity is still high and the accuracy of the measurements of the damping and the FMAR line position is close to its maximum value.

The authors are grateful to P. L. Kapitza for the opportunity to carry out this investigation, and to A. S. Borovik-Romanov for his valuable advice and discussions of the results obtained. The authors are also indebted to L. B. Luganskiĭ and Z. V. Gausman for the calculations they carried out on the computer.

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Translated by A. Tybulewicz
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