MAGNETIC-RESONANCE FREQUENCIES IN FILMS ON ANTIFERROMAGNETIC SUBSTRATES[1)]

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Owing to surface exchange interaction, the magnetization of a ferromagnet on the film-substrate interface is confined to a certain direction l lying in the plane of the film. When a static field **H** is applied in a direction opposite to l, the film magnetization is reversed, starting with a certain value $H = H_u$, by formation of magnetic structures of the interdomain-boundary type along its thickness. The magnetic resonance of such structures is investigated. It is shown that the dependence of the frequency ω on the magnetic field, which has a simple linear form at $H < H_u$ for all resonant modes, and becomes peculiar when $H > H_u$. When $H = H_u$, the $\omega(H)$ curves of all the resonant modes have an angle singularity.

1. INTRODUCTION. EQUATIONS OF MOTION

AMONG the different types of thin magnetic films investigated at the present time, considerable attention is being paid to ferromagnetic films on antiferromagnetic substrates[2-4]. In such a two-layer system, an exchange interaction between the ferromagnet magnetization **M** and the magnetization $\mathbf{M}_1$ of the sublattice closest to the surface is produced on the interface between the ferromagnet and the antiferromagnet. The real picture, naturally, is made more complicated by the inevitable partial mutual diffusion of the layers, but such an interaction can always be phenomenologically described by the term

$$-\alpha_s \mathbf{M}\mathbf{M}_1, \tag{1.1}$$

which enters in the surface Hamiltonian of the system. Then, neglecting the surface anisotropy, the boundary conditions on the surface of the contact will take for the ferromagnetic layer the form

$$\left[\mathbf{M}, \alpha\frac{\partial \mathbf{M}}{\partial n} - \alpha_s\mathbf{M}_1\right] = 0, \tag{1.2*}$$

and for the antiferromagnetic layer

$$\left[\mathbf{M}_1, \alpha_1\frac{\partial \mathbf{M}_1}{\partial n} + \alpha_{12}\frac{\partial \mathbf{M}_2}{\partial n} - \alpha_s\mathbf{M}\right] = 0. \tag{1.3}$$

Here α is the exchange constant of the ferromagnet, α_1 and α_2 are the exchange constants of the antiferromagnet, and α_s is the surface exchange constant.

Thus, the systems of equations describing the ferromagnet and the antiferromagnet are coupled by their boundary conditions and can be solved in the general case only simultaneously.

We shall consider below the widely prevalent situation, wherein the external magnetic field is much weaker than the effective antiferromagnetic-anisotropy field, and consequently the orientation of $\mathbf{M}_1$ remains unchanged and is assumed known. Then all the static and dynamic

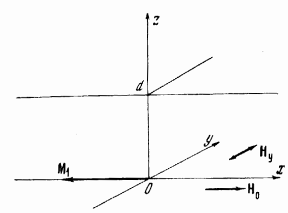

FIG. 1

properties of the two-layer system in the phenomenological approximation are described by the Landau-Lifshitz equation for the vector **M**:

$$\dot{\mathbf{M}} = g[\mathbf{M}\mathbf{H}^{(e)}] - \frac{\xi}{M}[\mathbf{M}\dot{\mathbf{M}}] \tag{1.4}$$

with boundary conditions (1.2).

We confine ourselves in this paper to the case $\alpha_s \to \infty$, when it follows from (1.2) that the vector **M** on the surface of the contact remains secured parallel to the immobile (in our approximation) vector $\mathbf{M}_1$. If certain methods are used to produce such two-layer systems, the vector $\mathbf{M}_1$ lies in the plane of the film.

Let us consider a plane of film d, whose plane is normal to the z axis (Fig. 1). The external static magnetic field $\mathbf{H}_0$ is directed along the x axis, and the alternating field along the y axis. The boundary conditions are

$$\mathbf{M} \parallel \mathbf{M}_1 \text{ for } z = 0, \qquad \partial\mathbf{M}/\partial n = 0 \text{ for } z = d \tag{1.5}$$

(we neglect the surface anisotropy on both surfaces of the film).

We write the effective magnetic field in the form

$$\mathbf{H}^{(e)} = \alpha\nabla^2\mathbf{M} + \mathbf{H}, \tag{1.6}$$

i.e., we neglect the magnetic anisotropy of the ferromagnetic film. Introducing normalized fields, magnetizations, and frequencies

$$\mathbf{h} = \mathbf{H}/M, \ \mathbf{m}' = \mathbf{M}/M, \ \sigma = \omega/gM, \tag{1.7}$$

projecting (1.4) on the coordinate axes and representing $\mathbf{m}'(z, t)$ in the form

$$\mathbf{m}'(z, t) = \mathbf{m}(z) + \boldsymbol{\mu}(z, t), \tag{1.8}$$

[1)] A brief content of this paper was published in [1].

*$[\mathbf{M}, \alpha\partial\mathbf{M}/\partial r - \alpha_s\mathbf{M}_1] \equiv \mathbf{M}\times(\alpha\partial\mathbf{M}/\partial r - \alpha_s\mathbf{M}_1)$.

we obtain a static system for m(z) in the form

$$\alpha\left(m_x\frac{\partial^2 m_y}{\partial z^2}-m_y\frac{\partial^2 m_x}{\partial z^2}\right)-m_y h=0,\quad m_x^2+m_y^2=1. \tag{1.9}$$

The dynamic linearized system for the projections of μ and h_y, which are proportional to $e^{i\omega t}$, under the condition

$$\left|\alpha\frac{\partial^2 m_x}{\partial z^2}\right| \ll |4\pi m_x+h| \tag{1.10}$$

(which is equivalent to the condition $h \ll 4\pi$), is

$$\alpha m_x\frac{\partial^2\mu_y}{\partial z^2}+\left[\frac{\sigma^2}{4\pi m_x+h}-h-i\xi\sigma m_x-\alpha\frac{\partial^2 m_x}{\partial z^2}\right]\mu_y$$
$$-\alpha m_y\frac{\partial^2\mu_x}{\partial z^2}+\left[i\xi\sigma m_y+\alpha\frac{\partial^2 m_y}{\partial z^2}\right]\mu_x=am_x,$$
$$4\pi m_y\mu_y=-(4\pi m_x+h)\mu_x. \tag{1.11}$$

Here $h = h_x$ is the normalized projection of the external constant magnetic field and a is the amplitude of the high-frequency field h_y.

If the vector M_1 is directed as shown in Fig. 1, then the boundary conditions for the static system are

$$m_x|_{z=0}=-1,\qquad \left.\frac{\partial m_x}{\partial z}\right|_{z=d}=0. \tag{1.12}$$

Such a problem was solved in[5]; the solution can be represented in the form

$$m_x=\begin{cases}-1, & h\leqslant h_u,\\ -1+2k^2\,\mathrm{sn}^2\left[\left(\frac{hd^2}{\alpha}\right)^{1/2}\frac{z}{d},k\right], & h\geqslant h_u,\end{cases} \tag{1.13}$$

where the dependence of the modulus k of the elliptic integral K on the magnetic field is given by

$$K^2(k)=hd^2/\alpha. \tag{1.14}$$

According to this solution, the film remains homogeneously magnetized along M_1 not only when the external field is directed along M_1, but also when h is oppositely directed, provided its value does not exceed

$$h_u=(\pi/2)^2\alpha/d^2, \tag{1.15}$$

which in this case plays the role of a sort of effective field of unidirectional anisotropy. When $h > h_u$, a smooth rotation of the magnetization begins, such that an inhomogeneous layer of the interdomain-boundary type is produced through the thickness of the film. Several succeeding distributions of the magnetization projection $m_u(z)$ with increasing h are shown in Fig. 2.

The magnetization process is reversible, i.e., the fact that M becomes fixed on the surface, just like unidirectional anisotropy of a uniformly mixed ferro-antiferromagnetic system[6], does not lead to hysteresis phenomena. When ordinary magnetic anisotropy is taken into account, we obtain a hysteresis loop that is shifted by an amount h_u as a result of the fixing of M on the surface.

The dynamic system (1.11) can be reduced to an equation for one projection of μ:

$$\frac{\partial^2\mu_y}{\partial\eta^2}+P(\eta)\frac{\partial\mu_y}{\partial\eta}+Q(\eta)\mu_y=R(\eta), \tag{1.16}$$

where $\eta = (hz^2/\alpha)^{1/2}$. The coefficients of this equation

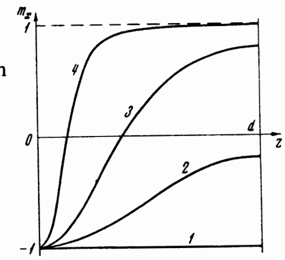

FIG. 2. Distribution of magnetization over the film thickness. Curve 1 ($k^2 = 0$) corresponds to $H_0 < H_u$, curves 2 ($k^2 = 0.4$), 3 ($k^2 = 0.9$), and 4 ($k^2 = 1-10^{-5}$) correspond to a sequence of increasing fields at $H_0 > H_u$.

become much simpler if $|h| \ll |4\pi m_x|$, after which the substitution

$$\mu_y=m_x v \tag{1.17}$$

reduces the equation to the form

$$\frac{\partial^2 v}{\partial\eta^2}+\left(\frac{\sigma^2}{4\pi h}-i\xi\frac{\sigma}{h}-m_x\right)v=-\frac{a}{h}m_x \tag{1.18}$$

with boundary conditions

$$v|_{z=0}=0,\qquad \left.\frac{\partial v}{\partial z}\right|_{z=d}=0. \tag{1.19}$$

CONDITIONS FOR MAGNETIC RESONANCE

To derive the resonance conditions let us consider the homogeneous equation corresponding to (1.18). In the region $h \leq h_u$ this is an equation with constant coefficients. The eigenfunctions satisfying the boundary conditions (1.19) are

$$v_m=\sin(2m-1)\frac{\pi z}{2d},\qquad m=1,2,3,\ldots \tag{2.1}$$

and correspond to the dispersion equation

$$\frac{\sigma^2}{4\pi}=-h+(2m-1)^2\left(\frac{\pi}{2}\right)^2\frac{\alpha}{d^2}=-h+(2m-1)^2 h_u. \tag{2.2}$$

When $h \geq h_u$ the equation goes over into a Lame equation. The eigenfunctions satisfying the boundary conditions (1.19) are the Lame functions[7]

$$v_m=\mathrm{Ec}_1^{2m-1}(\eta), \tag{2.3}$$

which goes over into (2.1) when $k \to 0$, i.e., $h \to h_u$. The Lame function corresponding to m = 1 is the elliptic sine

$$v_1=\mathrm{sn}(\eta). \tag{2.4}$$

This functions corresponds to the eigenvalue (at $\xi = 0$)

$$\sigma^2/4\pi=hk^2(h), \tag{2.5}$$

which determines the resonance condition for the first resonant mode. Here k is the modulus of the elliptic integral, and depends on the magnetic field in accord with (1.14). Using the usual expansions of K(k) at $k \approx 0$ and $k \approx 1$, we write down for the limiting cases of (2.5):

$$\frac{\sigma^2}{4\pi}\approx\begin{cases}2h(h/h_u-1), & h\gtrsim h_u\quad (k^2\approx 0)\\ h\{1-16\exp[-2(hd^2/\alpha)^{1/2}]\}, & h\gg h_u\quad (k^2\approx 1).\end{cases} \tag{2.6}$$

Let us examine in greater detail the behavior of the first resonant mode, the behavior of which at $h \leq h_u$ is described by condition (2.2) with m = 1, and at $h \geq h_u$ by condition (2.5) (Fig. 3). The dashed lines in Fig. 3 show the homogeneous ferromagnetic resonance that would be observed in the absence of surface fixing at the frequency

$$\sigma^2/4\pi=|h|. \tag{2.7}$$

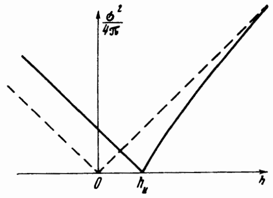

FIG. 3. Solid line—first resonant mode; dashed—"homogeneous FMR".

In our case, there is no homogeneous resonance, the frequency of the first spin-wave mode behaves in the usual manner at $h \geq h_u$, and approaches asymptotically the frequency of the homogeneous resonance with increasing h when $h > h_u$.

Such a result is physically understandable. The effective exchange-interaction radius (width of the transition layer) is in our case $\sim(\alpha/h)^{1/2}$ and decreases with increasing h. This is seen clearly in Fig. 2. Curve 4, which corresponds to large h, consists of a narrow transition layer near z = 0 and of the remainder of the film, which is practically homogeneously magnetized and in which the influence of the surface fixing of the magnetization is no longer felt. It is natural to expect this part of the film to behave like a free system also in the dynamic regime, i.e., to resonate at the frequency of the homogeneous magnetic resonance.

We were unable to represent the resonance conditions for the next resonance modes in a closed analytic form similar to (2.5). They can always be determined numerically from the characteristic equation obtained by Ince[8] for the natural frequencies, in the form of a continuous fraction

$$G = 1 + 4k^2 + \frac{3(n-2)(n+3)k^2/2\cdot 2^2}{9/16 + k^2 - G/16} + \frac{5(n-4)(n+5)k^2/4\cdot 6^2}{25/36 + k^2 - G/36} + \frac{7(n-6)(n+7)k^2/6\cdot 8^2}{49/64 + k^2 - G/64} + \ldots \quad (2.8)$$

In our case we must put n = 1,

$$G = \frac{\sigma^2}{4\pi h} - i\xi\frac{\sigma}{h} + 1. \quad (2.9)$$

To obtain an idea of the resonance frequencies of these modes, let us consider the limiting case of fields not much stronger than h_u, i.e., $k \approx 0$. In this case we ca can use in lieu of the solution of the characteristic equation (2.8) the fact that when $k \to 0$ the Lame equation goes over approximately into a Mathieu equation in the form

$$\frac{1}{4}\frac{\partial^2 v}{\partial \eta^2} + (\gamma - 4q\cos 2\eta)v = 0, \quad (2.10)$$

where in our case

$$\gamma = \left(\frac{K}{\pi}\right)^2\left[\frac{\sigma^2}{4\pi h} - i\xi\frac{\sigma}{h} + 1 - k^2\right], \quad q = -\left(\frac{kK}{2\pi}\right)^2. \quad (2.11)$$

On the other hand, the eigenvalues of the parameter γ of the Mathieu equation are known[9]; after substituting in them the approximate dependence of k on h at $k \approx 0$, we obtain the following resonance conditions (which are valid from $h = h_u$ to $h \lesssim 2h_u$)

$$\frac{\sigma^2}{4\pi} = -h + (2m-1)^2 h_u + 2h\left(\frac{h}{h_u} - 1\right). \quad (2.12)$$

Thus, all the resonant modes have at $h - h_u$ a clearly pronounced angle singularity; starting with m = 2 and for all the succeeding modes, this singularity is the same and corresponds to a rotation of the plots of $\sigma^2/4\pi$ against h through $\pi/2$. With further increase of h, the curves bend to the right, and the subsequent approximations (2.10)—(2.12) no longer hold, and it becomes necessary to analyze the solutions of the characteristic equation (2.8).

A resonance of the first mode (m = 1) in such a two-layer system was observed experimentally in[4]. The frequency dependence of the position of the resonant peak corresponds to Fig. 3, i.e., it is described by the condition (2.2) when $h \leq h_u$ and by the condition (2.5) when $h \geq h_u$. The hysteresis phenomena on the resonance curve, observed in[4], are manifestations of static hysteresis due to the usual magnetic anisotropy, which we have neglected in this case.

3. CONCLUSION

Analyzing the resonant conditions obtained in this paper for ferromagnetic films on antiferromagnetic substrates, we can propose the following two directions to follow in the experimental investigations:

1. Study of the exchange-interaction constant α of the ferromagnetic material. The two-layer systems considered here are superior to ordinary films in that the spins of the ferromagnet are actually rigidly fixed in them, and consequently there is no need to determine a quantity difficult to measure, namely the surface anisotropy β', which ensures partial fixing of the spins on the surface of an ordinary film. (The accuracy with which α is measured would be even larger if there are antiferromagnetic layers on both sides of the ferromagnetic film; it is easy to extend the results obtained here to such a case). It is clear that for such investigations the most convenient is the region $h \leq h_u$, where the film is magnetized homogeneously and the resonance conditions are simple (2.1) and have been known for a long time. It is possible that an important role in the increase of the accuracy of the measurements of α is played by the here-investigated angle singularity of the frequency curves at $h = h_u$.

2. An investigation of the homogeneity of the properties of the films. This can be done by comparing the dependence of the resonant frequency on the magnetic field at $h > h_u$ with the theoretical curve (2.5), calculated for ideally homogeneous films. It is clear that only films that satisfy this homogeneity criterion are suitable for fundamental research such as the measurement of α.

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