ON THE STRUCTURE OF THE TRANSITION LAYER IN A HIGH-FREQUENCY GAS DISCHARGE

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The structure of an equilibrium high-frequency gas discharge under conditions of strong skin effect is investigated. An analytical dependence of the plasma temperature on input power is derived. It is shown that the power required to heat the plasma to some temperature depends on the thermal and electrical conductivities of the gas at a given temperature and, under conditions of strong skin effect, does not depend on the dimensions and geometry of the discharge. In the transition region at the boundary of the discharge, the temperature, the high-frequency energy flux density and the power absorbed in the discharge are expressible in terms of a universal function of the coordinates. The results are applicable to induction discharges as well as to discharges in volume resonators.

1. INTRODUCTION

INTEREST in high-frequency electrodeless discharges in gases at high pressures--of the order of one atmosphere--has lately heightened considerably. However, in the available theoretical calculations (see, for example, the review[1]) no proper use is made of the smallness of the temperature of the plasma in the discharge in comparison with the ionization potential. The greatest advance in the construction of the theory of gas discharge has been made by Gruzdev, Rovinskiĭ, and Sobolev[2]. But even they have not determined the exact structure of the boundary of the discharge.

Assuming local thermodynamic equilibrium, we construct in the present paper a theory of heat exchange in a high-frequency gas discharge under conditions of strong normal skin effect. The smallness of of plasma temperature in comparison with the ionization potential allows us to find an analytical dependence of the parameters of the discharge on the input power.

Under the conditions of strong skin effect, when the depth to which the field penetrates into the plasma is small in comparison with both the dimensions of the discharge and the distance to the walls of the vessel, the conversion of the high-frequency power into Joule heat occurs, in the main, in a small (of the order of the penetration depth) transition layer on the surface of the discharge. Outside the discharge, the equilibrium electron concentration (and, hence, the conductivity σ) exponentially decreases as the temperature decreases. Inside the discharge, because of the strong skin effect, the electric vector $\mathbf{E}$ of the high-frequency field maintaining the discharge tends rapidly to zero. Therefore, the power absorbed by a unit volume of the plasma $\frac{1}{2}\sigma|\mathbf{E}|^2$ rapidly vanishes with increasing distance from the boundary into or away from the discharge. Because of the smallness of the temperature compared with the ionization potential, the temperature itself changes little in a region of considerable heat release. This allows us to consider separately the region outside the discharge in which the temperature varies significantly, but no heat is released, and the overlapping region of strong heat release in which the temperature differs little from its maximum value inside the discharge.

In the foregoing assumptions, the plasma temperature in the transition region, the high-frequency energy flux density, and the power absorbed in the discharge can be expressed in terms of a universal function of the coordinates. By matching this solution with the solution outside the discharge, we can construct the temperature profile and determine the dimensions of the discharge as well. The obtained formulas are applicable to induction discharges as well as to discharges in cavity resonators.

2. CONDITIONS FOR LOCAL THERMODYNAMIC EQUILIBRIUM

We shall assume that local thermodynamic equilibrium sets in in the plasma of a discharge, so that the thermal and electrical conductivities are known functions of the equilibrium temperature. A necessary condition for local thermodynamic equilibrium is the equality of the temperature of the electrons to the temperature of the ions and the neutral atoms-which, in any case, will occur at a sufficiently high pressure. According to Ginzburg and Gurevich[3], the difference between the electron temperature and the temperature of the ions and atoms can be neglected if the amplitude of the high-frequency field E is small compared with the characteristic "plasma" field E_p:

$$E \ll E_p = \sqrt{3mkTe^{-2}\delta(\omega^2+\nu_{\text{eff}}^2)}. \tag{2.1}$$

Here T is the equilibrium temperature of the plasma, ω the frequency of the high-frequency field, ν_{eff} the effective number of collisions, δ the fraction of the energy transferred by electrons during collisions with heavy particles (in elastic collisions $\delta = \delta_{el} = 2m/M$), m and e the mass and charge of an electron, and k the Boltzmann constant.

It is convenient for what follows to introduce, in place of the "plasma" field E_p, the critical energy flux $S_p = \sigma E_p^2 \delta$. Here,

$$\delta = c/\sqrt{8\pi\omega\sigma} \tag{2.2}$$

is the depth of penetration of the field into the plasma, so that the necessary condition for local thermodynamic equilibrium (2.1) imposes the following limitation on the high-frequency energy flux per unit area of the surface of the discharge S_0: $S_0 \ll S_p$.

3. FORMULATION OF THE PROBLEM. THE BASIC EQUATIONS

Let us consider a high-frequency discharge in a gas in a stationary regime, when the Joule heat released in the plasma by the electromagnetic field is transferred to the cooled walls of the vessel by heat conduction. We shall neglect radiation losses, which, as follows from numerical computations[4], are small at temperatures below 10,000°K. In the absence of convection the complex amplitude of the high-frequency field **E** and the equilibrium temperature T satisfy the wave and heat equations:

$$\Delta \mathbf{E} + \left(\frac{\omega^2}{c^2} - \frac{4\pi i\omega\sigma}{c^2}\right)\mathbf{E} = 0, \tag{3.1}$$

$$\operatorname{div}(\varkappa \operatorname{grad} T) + {}^1\!/_2\sigma|\mathbf{E}^2| = 0 \tag{3.2}$$

(a time dependence of the form $e^{i\omega t}$ is assumed). The thermal conductivity κ and the electrical conductivity σ of the gas entering into Eqs. (3.1) and (3.2), in the presence of local thermodynamic equilibrium, are known functions of the equilibrium temperature (depending, generally speaking, on the coordinates). The electron concentration, as a function of the temperature, is found from the ionization equilibrium formulas ([5], Sec. 106). If the degree of ionization of the gas is small, then the conductivity exponentially depends on the temperature[1]:

$$\sigma \sim \exp(-I/2T). \tag{3.3}$$

The thermal conductivity of gases at temperatures of the order of 5,000–10,000°K are less well known.

The boundary conditions for Eqs. (3.1) and (3.2) depend on the concrete apparatus used for obtaining the discharge, on the mode of excitation of the high-frequency oscillations, and also on the conditions of heat exchange at the surface of the vessel. If the temperature of the walls is maintained at a constant value, we may take, as a boundary condition on the temperature, $T = T_0 \approx 0$.

Let us denote by T_m the maximum temperature inside the discharge. We shall assume that $T_m \ll I$. Under this condition, the conductivity of the gas (3.3) decreases substantially when the temperature decreases by a small (in comparison with T_m) amount $\Delta T = T_m - T \sim T_m^2/I \ll T_m$. This permits us to split the solution of the problem into solutions in different regions.

In the region

$$T_m - T \gg T_m^2/I$$

the conductivity is exponentially small and the terms containing σ in Eqs. (3.1) and (3.2) may be neglected:

$$\Delta \mathbf{E} + \frac{\omega^2}{c^2}\mathbf{E} = 0. \tag{3.4}$$

Equation (3.4), with the corresponding boundary conditions, determines the electromagnetic energy flux density $S(\mathbf{r})$. Under the condition of smallness of the depth of penetration (2.2) of the field into the plasma (at the temperature T_m) in comparison with both the dimensions of the discharge r_0 and the distance to the walls of the container $R - r_0$:

$$\delta_m \ll r_0,\ \delta_m \ll R - r_0, \tag{3.5}$$

the high-frequency energy flux density near the discharge in the region

$$R - r_0 \gg r - r_0 \gg \delta_m \tag{3.6}$$

may be assumed to be independent of the coordinate **r** and equal to S_0. If the flux of the electromagnetic energy is totally absorbed inside the discharge, then, in accordance with (3.2), we find:

$$\varkappa \operatorname{grad} T = -S(\mathbf{r}).$$

Integrating this equation with the boundary condition $T = T_0$ at the surface of the vessel, we find the temperature profile outside the discharge. If the discharge is an infinitely long cylinder, then $S(r)r = S_0 r_0 = \text{const}$ and the dependence of the temperature on the radius in the region $r - r_0 \gg \delta_m$ takes the form

$$\int_{T_0}^{T} \varkappa\, dT = S_0 r_0 \ln\frac{R}{r}, \quad T = T_0 \text{ for } r = R. \tag{3.7}$$

Let us now investigate the asymptotic behavior of the solution at points far from the transition layer inside the discharge. In a region far from the boundary

$$T_m - T \ll T_m^2/I$$

the temperature differs so little from T_m, that the conductivity σ in Eqs. (3.1) and (3.2) can be assumed to be a constant and equal to $\sigma_m = \sigma(T_m)$. Equation (3.1) with the constant σ_m describes the skin effect in the plasma. Neglecting the displacement current in comparison with the conduction current, we find that in the interior of the discharge the magnitude of the electric field E decreases exponentially:

$$E \sim \exp[-(r_0 - r)/2\delta_m], \quad r_0 - r \gg \delta_m. \tag{3.8}$$

Substituting (3.8) into (3.2), we find that deep inside the discharge the temperature exponentially approaches T_m:

$$T_m - T \sim \exp[-(r_0 - r)/\delta_m]. \tag{3.9}$$

4. THEORY OF THE TRANSITION LAYER

Let us now consider the transition region at the boundary of the discharge. Owing to the condition (3.5), we can assume the boundary of the discharge to be plane. We shall assume that the electric vector **E** is parallel to the surface of the discharge and that it depends only on the distance to the surface. Neglecting in (3.1) the displacement current in comparison with the conduction current, we write Eqs. (3.1) and (3.2) and the complex conjugate of (3.1) in the form:

[1] Formula (3.3) is valid if the main contribution to the resistance is made by collisions of electrons with neutral atoms. This condition is more stringent than the simple requirement of smallness of the degree of ionization. For hydrogen at atmospheric pressure it is fulfilled at $T < 8{,}000$°K.

$$\frac{d^2E}{dr^2}-\frac{4\pi i\omega\sigma}{c^2}E=0, \tag{4.1}$$

$$\frac{d^2E^*}{dr^2}+\frac{4\pi i\omega\sigma}{c^2}E^*=0, \tag{4.2}$$

$$\frac{d}{dr}\left(\varkappa\frac{dT}{dr}\right)+\frac{1}{2}\sigma|E^2|=0. \tag{4.3}$$

It is convenient for the solution of the problem to eliminate from the equations the electric field E. Multiplying (4.1) by E^* and (4.2) by E, and then successively adding and subtracting the obtained equations, we arrive at the relations

$$d^2|E^2|/dr^2-2|E'^2|=0, \tag{4.4}$$

$$\frac{d}{dr}(E^*E'-EE^{*\prime})-\frac{8\pi i\omega\sigma}{c^2}|E^2|=0. \tag{4.5}$$

Now let us multiply (4.1) by $E^{*\prime}$, (4.2) by E' and add the obtained equations. We obtain as a result

$$\frac{d|E'^2|}{dr}+\frac{4\pi i\omega\sigma}{c^2}(E^*E'-EE^{*\prime})=0. \tag{4.6}$$

Eliminating the functions $|E'|^2$ and $(E^*E' - EE^{*\prime})$ from Eqs. (4.4)–(4.6), we find for $|E^2|$ the equation:

$$\frac{d}{dr}\frac{1}{\sigma}\frac{d^3|E^2|}{dr^3}-\frac{64\pi^2\omega^2\sigma}{c^4}|E^2|=0. \tag{4.7}$$

Let us express $|E^2|$ from Eq. (4.3) in terms of the temperature and substitute it into (4.7). After integration with respect to r, taking into account the fact that the derivatives with respect to the temperature tend to zero inside the discharge (on the side $r < r_0$), we obtain the following equation for the temperature in the transition layer:

$$\frac{d^3}{dr^3}\frac{1}{\sigma}\frac{d}{dr}\varkappa\frac{dT}{dr}-\frac{64\pi^2\omega^2\sigma}{c^4}\varkappa\frac{dT}{dr}=0. \tag{4.8}$$

Of interest to us is the solution to this equation for a given flux outside the discharge:

$$\varkappa dT/dr=-S_0,\quad r-r_0\gg\delta_m, \tag{4.9}$$

which tends to some (a priori unknown) limit T_m inside the discharge:

$$T\to T_m,\quad r_0-r\gg\delta_m. \tag{4.10}$$

(for the moment we conditionally take r_0 as the coordinate of the transition layer).

Taking the temperature dependence of the conductivity (3.3) into consideration, we find it convenient to introduce in place of T the dimensionless temperature Θ measured from T_m:

$$T=T_m-2T_m^2\Theta/I, \tag{4.11}$$

and measure the distance to the surface layer in units of the penetration depth:

$$r=-\delta_m\zeta.$$

Limiting ourselves to the region $T_m - T \ll T_m$ and neglecting the temperature dependence of the thermal conductivity, we write Eq. (4.8) in terms of the dimensionless variables in the form

$$\frac{d^3}{d\zeta^3}e^{\Theta}\frac{d^2\Theta}{d\zeta^2}-e^{-\Theta}\frac{d\Theta}{d\zeta}=0. \tag{4.12}$$

The relation (4.9) in terms of the dimensionless variables takes the form ($\kappa_m = \kappa(T_m)$):

$$\Theta'=-IS_0\delta_m/2\varkappa_mT_m^2,\quad \zeta_0-\zeta\gg1. \tag{4.13}$$

It is, however, not convenient to use it as a boundary condition since the temperature in the discharge T_m is not known a priori. Noting that, in accordance with (3.9), the temperature deep inside the discharge should exponentially tend to T_m, we find the following asymptotic expression for Θ:

$$\Theta\approx e^{-(\zeta-\zeta_0)},\quad \zeta-\zeta_0\gg1. \tag{4.14}$$

Since ζ enters into Eq. (4.12) only via the derivatives, while Eq. (4.12) itself does not contain any parameters whatsoever, the solution to it, having the asymptotic form (4.14), is some universal function of $(\zeta - \zeta_0)$:

$$\Theta=\Theta(\zeta-\zeta_0), \tag{4.15}$$

while $\Theta'(\zeta - \zeta_0)$ for $\zeta - \zeta \gg 1$ is some constant of the order of unity. A numerical integration yields

$$\Theta'(\zeta-\zeta_0)=-1.57,\quad \zeta_0-\zeta\gg1. \tag{4.16}$$

The results of the numerical integration of Eq. (4.12) are shown in Fig. 1, a, b and c. In Fig. 1a is shown the dependence of the dimensionless temperature (4.15) on the dimensionless coordinate. Figure 1b shows the dependence of the high-frequency energy flux S(r) on the coordinate in the transition layer ($|\Theta'| = I\delta_m S/2\kappa_m T_m^2$), while Fig. 1c shows the heat evolution in the transition layer ($\Theta'' = I\delta_m^2\sigma|E^2|/4\kappa_m T_m^2$).

The electron concentration in the discharge as a function of the coordinate can also be expressed in terms of the dimensionless function (4.15).

$$N(r)=N_m\exp\left\{-\Theta\left(\frac{r-r_0}{\delta_m}\right)\right\}.$$

Here, N_m is the equilibrium electron concentration at the temperature T_m. The dependence of the electron concentration on the coordinate is shown in Fig. 2.

We find from the relations (4.13) and (4.16) the dependence of the temperature T_m in the discharge on the energy flux S_0 and, inversely, the energy flux required to heat the plasma to the temperature T_m:

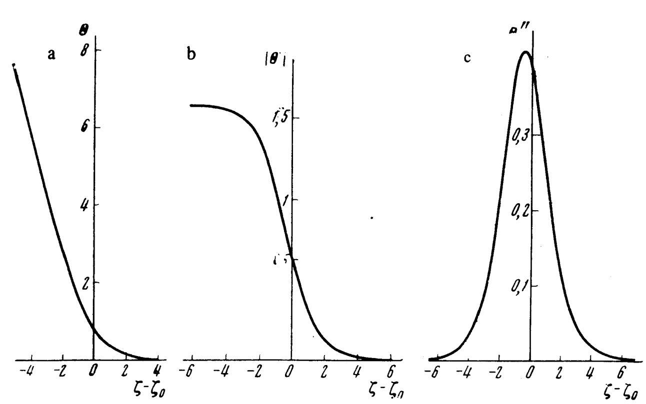

FIG. 1. Results of the numerical integration of Eq. (4.12): a–graph of the function (4.15)–the dependence of the dimensionless temperature on the dimensionless coordinate: b–dependence of the high-frequency energy flux S on the coordinate in the transition layer ($|\Theta'| = I\delta_m S/2\kappa T_m^2$): c–graph of heat evolution in the transition layer ($\Theta'' = I\delta_m^2\sigma|E^2|/4\kappa_m T_m^2$).

$$S_0(T_m) = 3.14\varkappa_m T_m^2 / I\delta_m. \qquad (4.17)$$

Thus, the energy flux required to heat the plasma to the temperature T_m is determined by the thermal and electrical conductivities of the gas at the given temperature and, under the conditions of strong skin effect (3.5), does not depend on the dimensions and geometry of the discharge.

For the description of the electrical characteristics of the discharge under the conditions (3.5), we may introduce the surface impedance Z just as in the case of a sharp boundary:

$$Z = \frac{4\pi i\omega}{c^2}\frac{E(r)}{E'(r)}, \quad R - r_0 \gg r - r_0 \gg \delta_m. \qquad (4.18)$$

By this definition, the active part R_Z of the surface impedance does not depend on r and has the meaning of a resistance of the discharge per unit surface area. Let us calculate the dependence of the surface impedance of the discharge on temperature and power. Let us multiply the numerator and denominator of (4.18) by $E^{*\prime}(r)$ and reduce with the hlep of the relation (4.6) the expression for R_Z to the form

$$R_z = \operatorname{Re} Z = \frac{1}{2\sigma}\frac{|E'^2|'}{|E'^2|}. \qquad (4.19)$$

With the aid of the relations (4.3)–(4.7), we find the identity

$$|E'^2|' = \frac{64\pi^2\omega^2\sigma}{c^4} S(r), \qquad (4.20)$$

where S(r) is the electromagnetic energy flux density. Substituting (4.20) into (4.19) and noting that outside the transition layer $\sigma S(r)$ tends rapidly to zero, we can verify that the active part of the surface impedance (4.19) does not depend on r and is equal to

$$R_z = \frac{2\pi\omega\delta_m}{c^2}\frac{|\Theta'(-\infty)|}{\int_0^\infty e^{-\Theta}\,d\Theta} = \frac{2\pi\omega\delta_m}{c^2}|\Theta'(-\infty)|. \qquad (4.21)$$

Substituting (4.16) into (4.21), we finally obtain:

$$R_z = 3.14\pi\omega c^{-2}\delta_m = 3.14\pi 10^{-9}\omega\delta_m \ [\text{ohm}]. \qquad (4.22)$$

Notice that the surface resistance R_Z for a half-space with the temperature T_m and a sharp boundary, after a specular reflection, is equal to $R_Z = 4\pi\omega c^{-2}\delta_m$, which is roughly 20% higher than the correct formula (4.22).

As another electrical characteristic of the discharge, we give a formula for the coefficient of reflection of a normally incident wave from the plane boundary of the discharge: $\mathscr{R} = 1 - 6.28\,\omega c^{-1}\delta_m$.

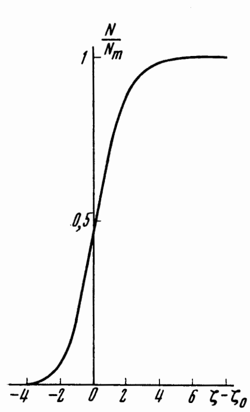

FIG. The electron concentration distribution.

5. INFINITE CYLINDRICAL DISCHARGE

The constant of integration ζ_0 in (4.15) is determined from the condition that in the temperature region

$$T_m^2/I \ll T_m - T \ll T_m \qquad (5.1)$$

the expression (4.11) coincides with the solution of the heat equation outside the discharge. In the case of an infinitely long cylindrical discharge, the dependence of the temperature on the coordinate (3.7) in the region (5.1) can be written in the form (here $\delta_m \ll r - r_0 \ll r_0$):

$$\int_{T_0}^{T_m} \varkappa\, dT - \varkappa_m(T_m - T) = -r_0 S_0 \ln\frac{r_0}{R} - S_0(r - r_0). \qquad (5.2)$$

Noting that when $\zeta_0 - \zeta \gg 1$, we have $\Theta(\zeta - \zeta_0) = -1.57(\zeta - \zeta_0)$ and find with the aid of (4.11) the following expression for the temperature in the region (5.1):

$$\varkappa_m(T_m - T) = S_0(r - r_0), \qquad (5.3)$$

where $r = -\delta_m\zeta$ and $r_0 = -\delta_m\zeta_0$. Comparing (5.2) with (5.3), we find

$$r_0 = R\exp\left\{-\frac{1}{S_0 r_0}\int_{T_0}^{T_m}\varkappa\, dT\right\}. \qquad (5.4)$$

The relation (5.4) together with (4.17) determines the dependence of the radius of the discharge r_0 on the input energy flux S_0 or on the maximum temperature T_m.

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