Current-Voltage Characteristics of Bounded Semiconductors Possessing a Negative Differential Conductivity

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The effect of inelastic surface mechanisms of energy absorption on the shape of the current-voltage characteristic is investigated. Stable solutions are found for the distribution of the electron temperature over the cross section of the sample. It is shown that the decreasing part of the current-voltage characteristic vanishes in the case of thin samples in the presence of sufficiently strong surface mechanisms. In the case of bulk samples the current-voltage characteristic is multivalued as usual; however, the presence of inelastic surface absorption mechanisms of sufficient intensity leads to the disappearance of one of the two hysteresis sections on the characteristic.

THE influence of sample size on the shape of the current-voltage characteristic (CVC) was investigated in[1] for media in which, under certain conditions (in sufficiently thin samples), there is an S-shaped dependence of the electron temperature on the magnitude of the electric field. The absence of inelastic mechanisms of energy absorption on the surfaces of the sample was assumed in the cited article, that is, it was assumed that the electron temperature Θ satisfies the following conditions on the boundaries: $d\Theta/dz = 0$ (the one-dimensional problem was considered in[1]). In real samples, as experiments show (see, for example,[2]), it is evident that inelastic surface mechanisms of energy absorption always exist. Therefore it is of interest to take their influence on the shape of the CVC into account. The investigation of this question is therefore the goal of the present article.

1. FORMULATION OF THE PROBLEM

Let us consider a semiconducting sample having the shape of a parallelepiped, to which is applied a constant electric field E along the x axis, and where as usual the absence of inelastic mechanisms of energy absorption is assumed on the boundary planes y = 0, b. If it is assumed that $b < l_{c\,\min}$ (the expression for $l_{c\,\min}$ is given in[1]), then the electron temperature does not depend on y, that is, the problem is one-dimensional. In the direction of the z axis the sample may have an arbitrary thickness with boundary conditions on the planes (z = 0, a), which take into consideration the presence on them of inelastic mechanisms of energy scattering. In what follows for simplicity we consider the case when inelastic mechanisms exist only on the plane z = a, that is, the boundary conditions have the form[3]

$$\left.\frac{d\Theta}{dz}\right|_{z=0}=0,\quad \varkappa(\Theta)\left.\frac{d\Theta}{dz}\right|_{z=a}=-\eta f(\Theta)(\Theta-T)\,|_{z=a}. \tag{1}$$

Here $\kappa(\Theta)$ is the electronic thermal conductivity, T is the fixed temperature of the lattice, $\eta f(\Theta)$ is a function which takes into consideration the strength of the inelastic surface mechanisms, η is the parameter characterizing this mechanism, where η and $f(\Theta)$ are intrinsically positive quantities.

In dimensionless units the equation for the electron temperature has the form[1]

$$\frac{d^2w}{dz^2}+\frac{dU(w)}{dw}=0,$$

$$U(w)=[T\varkappa(T)]^{-1}\int^{w}[\sigma(w)E^2-A(w)]\,dw. \tag{2}$$

Here $w = [T\kappa(T)]^{-1}\int^{\Theta}\kappa(\Theta)d\Theta$ is the dimensionless electron temperature, $\sigma[\Theta(w)]$ is the static conductivity, and $A[\Theta(w)]$ is the quantity describing the transfer of heat from the electron subsystem to the lattice.

The boundary conditions (1) for w are written as follows:

$$\left.\frac{dw}{dz}\right|_{z=0}=0,\quad \left.\frac{dw}{dz}\right|_{z=a}=-\eta F(w)\,|_{z=a}, \tag{3}$$

where $F(w) = [T\kappa(T)]^{-1}f[\Theta(w)]\cdot[\Theta(w)-T] > 0$; we shall also assume that $dF(w)/dw > 0$.

As already mentioned in[1], Eq. (2) coincides in outward appearance with the equation of motion of a particle in a potential field, where the function U(w) has the meaning of the potential energy of the "particle," and the roles of the time and of the coordinate are played by z and w, respectively. As follows from the expression for U(w) (see formula (2)), the potential energy depends on the field E as a parameter. If the quantity w is an S-shaped function of E (Fig. 1) in sufficiently thin samples for $\eta = 0$, then the function

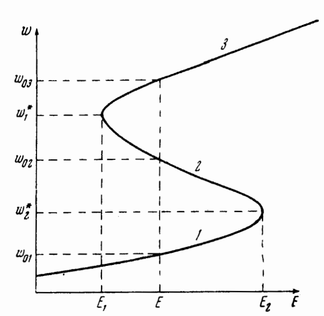

FIG. 1

U(w) has a maximum in fields $E < E_1$ and $E > E_2$. In fields $E_1 < E < E_2$ it has two maxima and a minimum; finally, in the field $E = E_1$ ($E = E_2$) it has one maximum and an inflection point with a zero first derivative on the right (left) of the maximum point (for more details, see[1]).

The solution of Eq. (2) with the boundary conditions (3) is written down in quadratures, and determines the dependence of w on z in implicit form. To determine this dependence explicitly, we confine the investigation to certain limiting cases.

2. CLASSIFICATION OF THE SOLUTIONS AND THEIR STABILITY

In fields $E < E_1$ ($E > E_2$) for $\eta = 0$ the single maximum of the function U(w) is located at the point w_{01} (w_{03}); therefore Eq. (2) has a single homogeneous solution $w_1 = w_{01}$ ($w_3 = w_{03}$).

If the field E lies in the interval $E_1 < E < E_2$, then the maxima of the function U(w) are located at the points w_{01} and w_{03}, and the minimum is located at the point w_{02}. As shown in[1], in a field E from this interval and for $\eta = 0$ Eq. (2) has no less than three and no more than four stable solutions. These are the two solutions $w_{1,2} = w_{01,3}$ which are homogeneous in z and which are stable for arbitrary values of $\alpha_{1,3}a$, where

$$\alpha_{1,3} = \sqrt{-\left.\frac{d^2U(w)}{dw^2}\right|_{w_{01,3}}}, \quad \left.\frac{d^2U}{dw^2}\right|_{w_{01,3}} < 0.$$

The solutions $w_{1,3}$ correspond to the two branches of w(E) which increase with increasing values of E (branches 1 and 3 on Fig. 1).

If $\alpha_2 a \leq \pi$,

$$\alpha_2 = \sqrt{\left.\frac{d^2U(w)}{dw^2}\right|_{w_{02}}}, \quad \left.\frac{d^2U}{dw^2}\right|_{w_{02}} > 0,$$

then there is only a single stable solution $w_2 = w_{02}$. However, if $\alpha_2 a > \pi$, then the solution $w_2 = w_{02}$ is unstable, but the two monotonic solutions are stable: $w_2^{(+1)}(z)$ (reaching a maximum at the point $z = 0$) and $w_2^{(-1)}(z)$, (reaching a maximum at the point $z = a$) which are connected by the relation $w_2^{(-1)}(z) = w_2^{(+1)}(a - z)$. The solutions w_2 correspond to the branch of w(E) which decreases with increasing values of E (branch 2 on Fig. 1).

There are two stable solutions in the field $E_1(E_2)$ for $\eta = 0$ (Fig. 1): the homogeneous solutions $w_1 = w_{01}$ and $w_1^* = w_{02} \equiv w_{03}$ ($w_3 = w_{03}$ and $w_2^* = w_{01} \equiv w_{02}$).

In this section it will be shown that for $\eta \neq 0$ only a single solution exists in fields $E \leq E_1$; not more than three stable solutions $w_{1,2,3}$ exist in the interval of fields $E_1 < E \leq E_2$. It will also be shown that for arbitrarily small values of η an interval of fields is found with the left boundary at the point E_2, in which three solutions exist (we recall that for $\eta = 0$ there is only one solution w_{03} in fields $E > E_2$).

Let us fix the field E in the interval $E_1 \leq E \leq E_2$. First let us consider how the solutions w_1 and w_3 are modified when the parameter η is sufficiently small. Then it is obvious that the solutions of Eq. (2) for $w_{1,3}$ differ from $w_{01,3}$ weakly (for reasonably small values of η); therefore one can seek these solutions in the form $w = w_{01,3} + w'$ where $|w'| \ll w_{01,3}$. Linearization of Eq. (2) and of the boundary conditions (3) with respect to w' gives

$$w_{1,3} = w_{01,3} - \frac{\eta F_{1,3}}{\alpha_{1,3}\,\mathrm{sh}\,\alpha_{1,3}a}\,\mathrm{ch}\,\alpha_{1,3}z. \tag{4}$$

Here $F_{1,3} \equiv F(w_{01,3})$. Formulas (4) are valid when η satisfies the following condition:

$$\eta \ll F_{1,3}^{-1}\, w_{01,3}\alpha_{1,3}\,\mathrm{th}\,\alpha_{1,3}a. \tag{5}$$

We note that the solution w_1 (w_3) is also valid in fields $E < E_1$ ($E > E_2$).

Now let us consider how the solution w_2 changes when η is different from zero. If $\alpha_2 a \leq \pi$, then for $\eta = 0$ the quantity $w_2 = w_{02}$. Therefore, by considering the parameter η to be sufficiently small and proceeding in analogy to what was done when finding the solutions $w_{1,3}(z)$, we define $w_2(z)$ as follows:

$$w_2 = w_{02} + \frac{\eta F_2}{\alpha_2 \sin \alpha_2 a}\cos \alpha_2 z, \tag{6}$$

where $F_2 \equiv F(w_{02})$.

The criterion for the validity of formula (6) is given by the inequality

$$\eta \ll F_2^{-1} w_{02}\alpha_2 \sin \alpha_2 a. \tag{7}$$

We note that $\alpha_{1,2} = 0$ in the field $E = E_2$; therefore the criteria (5) and (7) are not satisfied for finite values of a (in this connection α_3 does not vanish, and the criterion (5) is satisfied for w_3).

In order to construct the solutions w_1 and w_2 in the field $E = E_2$, it is necessary to keep the term of order w'^2 upon linearization of Eq. (2). Thus, the equation for w_1 and w_2 takes the following form in the field $E = E_2$:

$$\frac{d^2 w_{1,2}'}{dz^2} + B_2^* w_{1,2}'^2 = 0, \quad B_2^* = B|_{w_2^*}, \quad B = \frac{1}{2}\left.\frac{d^3U(w)}{dw^3}\right|_{w_{02}}. \tag{8}$$

Simple investigation of the function U(w) shows that $B_2^* > 0$. If for simplicity it is assumed that $\sqrt{B_2^* a} \ll 1$, then the temperature distribution over the cross section of the sample is almost homogeneous, that is,

$$w_{1,2} = w_2^* \mp \sqrt{\eta F(w_2^*)/B_2^* a.} \tag{9}$$

From formula (9) it follows that in the field $E = E_2$ the temperatures corresponding to the first and second branches do not coincide (we emphasize that at $\eta = 0$ in this field, as is evident from Fig. 1, there was a transition from an increasing branch of w(E) to the decreasing branch). It is natural to conjecture that for $\eta \neq 0$ the solutions $w_{1,2}$ exist in fields $E > E_2$. In fact, to the right of E_2 the function U(w) has a point of inflection with the first derivative $dU(w)/dw > 0$. Near E_2 this derivative is small (at the very point E_2 it is equal to zero). Therefore near E_2 in the expansion of Eq. (2) at the inflection point in powers of small deviations of the temperature from this point, one can neglect the zero-order term in the expansion of the function $dU(w)/dw$. Then Eq. (2) takes the same form as Eq. (8) in which it is necessary to replace w_2^* by the temperature w_i, defining the inflection point. Its solution is written as follows:

$$w_{1,2} = w_i \mp \sqrt{\frac{\eta F(w_i)}{\bar{B}a}}, \quad \bar{B} = \frac{1}{2}\left.\frac{d^3U(w)}{dw^3}\right|_{w_i} \tag{10}$$

Thus, for arbitrarily small values of η there is an interval of field strengths with the left boundary at the point E_2, and three solutions exist within this interval.

Conditions (5) and (7) are also violated in the field $E = E_1$, because $\alpha_{2,3}$ vanishes in this field. (The quantity α_1 does not vanish in the field E_1; therefore for w_1 the solution (4) is valid as usual.) It is obvious that in the field $E = E_1$ the solution $w_{2,3}$ is described by formula (9) with $B_1^* = B \mid w_1^*$. However, the quantity B_1^* is negative, as one can easily verify; therefore the solutions $w_{2,3}$ are absent in the field $E = E_1$.

We note that for any small (but finite) value of η one can always select a suitable neighborhood near the field E_1 ($E > E_1$) such that the condition $\alpha_2 w_2' \ll B w_2'^2$ is satisfied in this neighborhood. Equation (8) with $B < 0$ will be valid in this interval. Therefore the solution w_2 is absent (for $\eta \neq 0$) even in fields close to E_1. Similar arguments are valid for the solution w_3, because we have $d^3U(w)/dw^3 < 0$ at the point w_{03} in fields close to E_1. Therefore, there exists a finite interval of field strengths with the left boundary at the point E_1, and only the single solution w_1 exists in this interval.

As to condition (7), it ceases to be satisfied not only for $\alpha_2 a \to 0$ but also for $\alpha_2 a \to \pi$. Therefore, in order to construct the solution w_2 for $\alpha_2 a \leq \pi$ in the case when $\alpha_2 a$ is close to π, let us proceed in the following manner. As usual we shall seek the solution for w_2 in the form $w_2 = w_{02} + w_2'$, where we choose the parameter η in such a way that the condition $|w_2'| \ll w_{02}$ is satisfied. Let us linearize Eq. (2) with respect to w_2' correct to terms of order $w_2'^3$:

$$d^2w_2'/dz^2 + \alpha_2^2 w_2' + Bw_2'^2 - \gamma^2 w_2'^3 = 0, \tag{11}$$

where $\gamma^2 = |d^4U(w)/dw^4|_{w_{02}}$. A simple investigation of the form of the function $U(w)$ shows that the quantity $d^4U(w)/dw^4|_{w_{02}} < 0$ over the entire range of fields $E_1 \leq E \leq E_2$.

The solution of Eq. (11) has the form[4]

$$w_2' = A\cos[(\alpha_2 - \delta A^2)z + \psi], \tag{12}$$

where $\delta = 3\gamma^2/8\alpha_2 + 5B^2/12\alpha_2^3 > 0$, and A and ψ denote the amplitude and the initial phase; from the boundary conditions (3) we obtain the following system of equations for the determination of the latter quantities:

$$\begin{gathered}\sin\psi = 0,\\ A[(\pi\delta A^2 + \alpha_2^2 a_1)\cos\psi - \alpha_2\sin\psi] = \eta F_2,\end{gathered} \tag{13}$$

where $a_1 = \pi/\alpha_2 - a$, $\alpha_2 a_1 \ll \pi$.

From the possible solutions $\psi = k\pi$ (k = 0, 1, 2, ...) of the first equation in (13) we select only the two values $\psi = 0$ and $\psi = \pi$, which lead to different equations for A:

$$A^3 = -\alpha_2^2 a_1 A/\pi\delta + \eta F_2/\pi\delta, \tag{14}$$

$$A^3 = -\alpha_2^2 a_1 A/\pi\delta - \eta F_2/\pi\delta. \tag{14a}$$

Graphical investigation of Eqs. (14) and (14a) (see Fig. 2a) shows that Eq. (14) has one positive solution A_1 (the amplitude A is positive by definition), whereas Eq. (14a) does not have any positive solutions for any nonzero values of the parameter η. Therefore the expression for w_2 will be described by formula (12) with $\psi = 0$.

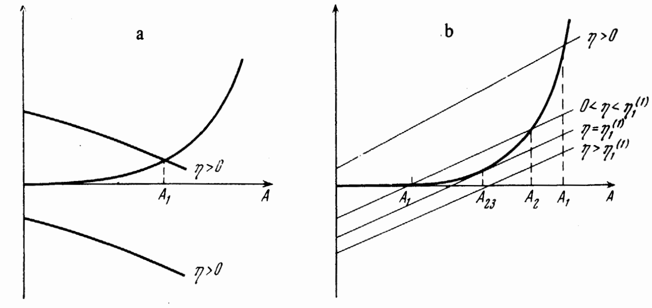

FIG. 2

Now let $\alpha_2 a > \pi$. First let us consider the case when $\alpha_2 a = \pi + \alpha_2 a_1$, where $\alpha_2 a_1 \ll \pi$. For a sufficiently small value of η, the solution w_2 is as usual close to w_{02}. Therefore the expression for w_2 is given by formula (12) with the amplitude A determined from Eq. (14) ($\psi = 0$) and Eq. (14a) ($\psi = \pi$), and where in Eqs. (14) and (14a) it is necessary to change the sign in front of the terms containing a_1:

$$A^3 = \alpha_2^2 a_1 A/\pi\delta + \eta F_2/\pi\delta \text{ for } \psi = 0, \tag{15}$$

$$A^3 = \alpha_2^2 a_1 A/\pi\delta - \eta F_2/\pi\delta \text{ for } \psi = \pi. \tag{15a}$$

A graphical investigation of Eqs. (15) and (15a) is presented in Fig. 2b. As follows from Fig. 2, Eq. (15) has a single positive solution A_1. Equation (15a) does not have any positive solutions for $\eta > \eta_1^{(1)}$, it has one solution (A_{23}) for $\eta = \eta_1^{(1)}$, and finally it has two solutions (A_2 and $A_3 < A_2$) for $\eta < \eta_1^{(1)}$ ($\eta_1^{(1)} = 2\pi\delta(\alpha_2^2 a_1/3\pi\delta)^{3/2} F_2^{-1}$; with regard to the meaning of the employment of upper and lower indices, see below).

Thus, for $\eta_1^{(1)}$ there is a unique monotonic solution $w_2 = \overline{w}_2^{(1)}$ (formula (12) with $\psi = 0$) which goes over into the stable solution $w_2^{(+1)}(z)$ as $\eta \to 0$.

For $\eta < \eta_1^{(1)}$ two more solutions w_2 ($\overline{w}_2^{(2)}$ and $\overline{w}_2^{(3)}$) exist, described by formula (2) with $\psi = \pi$ and with A equal to A_2 and A_3, respectively. Both solutions, $\overline{w}_2^{(2)}$ and $\overline{w}_2^{(3)}$ have maxima inside the sample. The solution $\overline{w}_2^{(3)}$ goes over into the unstable homogeneous solution w_{02} as $\eta \to 0$. The solution $\overline{w}_2^{(2)}$ goes over into the stable solution $w_2^{(-1)}(z)$ as $\eta \to 0$; as $\eta \to \eta_1^{(1)}$ it goes over into the solution $\overline{w}_2^{(2)}$.

It is convenient to carry out further investigation of the case $\alpha_2 a > \pi$ in terms of the "energy levels."

As is shown in[1], when $\eta = 0$ and $\alpha_2 a > \pi$ there are, besides the level $U(w_{02}) \equiv U^{02}$ which corresponds to the homogeneous solution w_{02}, also $p = [\alpha_2 a/\pi]$ (the square brackets denote the integer part of the argument if $\alpha_2 a/\pi$ is not equal to an integer[1)]) doubly degenerate energy levels $U^{(p)}(w_2^{(\pm p)}(0)) \equiv U^{(p)}$, where $w_2^{(\pm p)}(0) = w_2^{(\pm p)}(z)$ for z = 0. The ± signs in front of p indicate that there are two solutions corresponding to each level $U^{(p)}$, where one of these solutions ($w_2^{(+p)}$) has a maximum at the point z = 0 and the other ($w_2^{(-p)}$) has a minimum. The following simple relation exists between the solutions $w_2^{(+p)}$ and $w_2^{(-p)}$: $w_2^{(-p)}(z) = w_2^{(+p)}(a - z)$. As has been indicated above, the two monotonic solutions $w_2^{(\pm p)}(z)$, corresponding to the level $U^{(p)}$ with p = 1, are stable.

When $\eta \neq 0$ it is not difficult to formulate the re-

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1)If $\alpha_2 a/\pi$ is an integer, then the square brackets denote the number which is smaller by one.

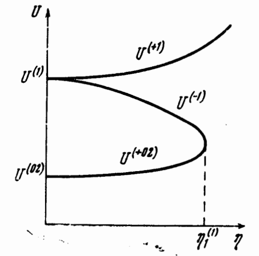

FIG. 3

sults obtained for the case $\alpha_2 a = \pi + \alpha_2 a_1$ ($\alpha_2 a_1 \ll \pi$) in terms of energy levels, since in the parabolicity band there is an explicit (quadratic) dependence of the value of the energy level on the amplitude. The dependence of the levels $U^{(+1)}$, $U^{(-1)}$, and $U^{(02)}$, corresponding to the solutions $\overline{w}_2^{(1)}$, $\overline{w}_2^{(2)}$, and $\overline{w}_2^{(3)}$, on η is shown on Fig. 3. We note that, as follows from Fig. 3, the presence of an inelastic mechanism for the absorption of energy on one of the walls of the sample removes the degeneracy of the level $U^{(1)}$.

It is not difficult now to represent the dependence of the magnitudes of the energy levels on η when $\alpha_2 a = n\pi + \alpha_2 a_1$ ($\alpha_2 a_1 < \pi$, n = 1, 2, ...), that is, when n levels $U^{(p)}$ exist ($p \leq n$) for $\eta = 0$. It is obvious that the level $U^{(-p)}$ merges with the level $U^{(+[p-1])}$ for $\eta = \eta_n^{(p)}$. Here the subscript indicates how many levels (excuding the level $U^{(02)}$) exist for a given value of $\alpha_2 a$ in the case when $\eta = 0$, and the superscript p indicates that the levels $U^{(-p)}$ and $U^{(+[p-1])}$ merge together. We note that $\eta_n^{(p-1)} > \eta_n^{(p)}$ (see Fig. 4).

Following[1], it is not difficult to show that the solution w_2, corresponding to the level $U^{(+1)}$, is stable with respect to small perturbations of the temperature in the regime of a given current, but the solutions w_2 corresponding to the levels $U^{(-1)}$, $U^{(+02)}$, and $U^{(\pm p)}$ with p = 2, 3, ..., n are unstable.

Thus, if there is a solution w_2 for $\eta \neq 0$ in the interval of field strengths $E_1 < E \leq E_2$, then the monotonic solution corresponding to the level $U^{(+1)}$ that increases with increasing values of η is stable.

From what has been said above it follows that whereas for $\eta = 0$ two stable solutions $w^{(\pm 1)}$ would exist when $\alpha_2 a > \pi$, the presence of an arbitrarily weak inelastic mechanism for the absorption of energy on one of the walls leaves one stable solution w_2.[2)]

3. THE CURRENT-VOLTAGE CHARACTERISTICS

Let us use the results obtained in Sec. 2 to construct the CVC. The temperature $\overline{w}_{1,3}$ averaged over the cross section ($\overline{w} = 1/a \int_0^a w(z)dz$), corresponding to the increasing branches of the function w(E) (see

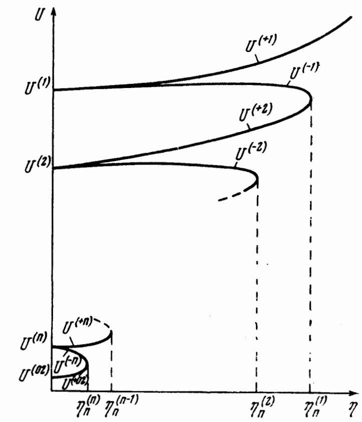

FIG. 4

Fig. 1), fall with increasing values of η (see formulas (4) and (9)), and $\overline{w}_2$, corresponding to the decreasing branch, increases with increasing values of η (see formulas (6), (9), (10), (12), (14) and (15)). The change of $\overline{w}_{1,2,3}$ with increase of η is stronger the thinner the sample.

First let us consider the limiting case of thin samples.

If the parameter η is sufficiently small then, as follows from Sec. 2, near the field E_1 ($E > E_1$) there exists an interval of field strengths in which the solutions w_2 and w_3 are not present, and moreover this interval is larger the bigger the value of η. In terms of energy levels, one can understand this situation in the following way. Let us fix the field $E = \widetilde{E}_1$ ($\widetilde{E}_1 > E_1$, $(\widetilde{E}_1 - E_1)/E_1 \ll 1$). On the one hand, if $\eta = 0$ the levels corresponding to the homogeneous solutions w_{02} and w_{03} in this field are rather close together. On the other hand, if $\eta \neq 0$ the levels corresponding to the solutions w_2 and w_3 approach each other as η increases. It is clear that for a certain sufficiently small value of $\eta = \eta_{cr}$ (for $\widetilde{E}_1$ sufficiently close to E_1) in the given field, the levels merge together (in the same way as this occurred for the levels $U^{(-p)}$ and $U^{(+[p-1])}$ when $\eta = \eta_n^{(p)}$, that is, for $\eta = \eta_{cr}$ the field intensity $\widetilde{E}_1 \equiv \widetilde{E}_1^{(\eta)}$ is the field in which a continuous transition from the upper increasing branch to the decreasing branch is realized). For $\eta > \eta_{cr}$ the solutions w_2 and w_3 in the field $\widetilde{E}_1$ are absent (the branches 2 and 3 join together in a field $E > \widetilde{E}_1$).

When $\eta = 0$ the continuous transition from branch 1 to branch 2 occurs in the field $E = E_2$ (see Fig. 1). For arbitrarily small values of η, the solutions w_1 and w_2 corresponding to the lower increasing and decreasing branches exist in a narrow (for sufficiently small η) interval of fields larger than E_2 (see formula (10)). It is obvious that a field $E_2^{(\eta)} > E_2$ exists (the larger the value of η, the larger this field will be) in which $\overline{w}_1$ and $\overline{w}_2$ coincide, that is, branches 1 and 2 join together.

Thus, the field dependence of the temperature averaged over the cross section (and hence the dependence of E of the current $\overline{j} = (E/a) \int_0^a \sigma(w(z))dz$ averaged over the cross section) remains S-shaped for thin samples and small values of η; however, the

[2)]In the two-dimensional problem with axial symmetry (see[5]) where two stable solutions w_2 exist for $\eta = 0$, the one being obtained from the other by replacing ρ by $R - \rho$ (R denotes the radius of the cylinder), the presence of an arbitrarily small inelastic mechanism for the absorption of energy on the surface of a cylindrical sample leads to the result that the temperature distribution over the cross section has a maximum on the axis of the cylinder (where η vanishes by definition). Therefore in the two-dimensional problem for those sample dimensions which were considered in[5], under actual conditions a "hot filament in a cold plasma" always exists, but not vice versa.

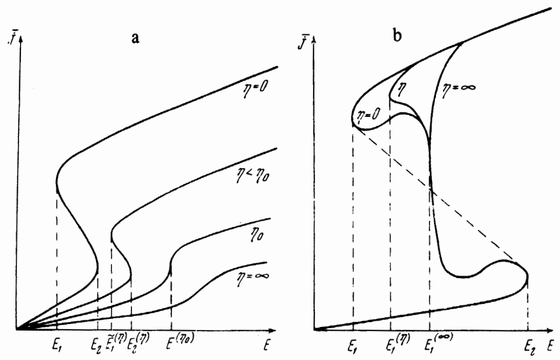

FIG. 5

region of multivaluedness is shifted in proportion to the increase of η toward the side of larger fields. Physically the result of the deformation of the CVC for small values of η is quite clear. The presence on one of the walls of the sample of an additional (inelastic) mechanism for energy absorption must necessarily lead to the result that stronger fields are required in order to reach the superheating region.

However, if $\eta \to \infty$ (the temperature of the electrons on the boundary tends to the temperature of the lattice), then simple estimates show[3] that for sufficiently thin samples inelastic surface mechanisms are the basic mechanism for the removal of energy. This means that in Eq. (2) one can neglect the term $A(w)$ describing the transfer of energy from the electron subsystem to the lattice. In this case Eq. (2) has a single solution over the entire interval of field strengths, that is, the current is a single-valued function of the field over the entire range of fields.

In the intermediate case (when the parameter η is not very small, but also not very large) the presence of inelastic mechanisms on the surface leads to the result that, on the one hand the multivalued region of the CVC is displaced toward the side of larger fields, but on the other hand it becomes narrower. At a certain value $\eta = \eta_0$ the multivaluedness of the CVC disappears. It is not difficult to understand that for $\eta = \eta_0$ in a certain field $E^{(\eta_0)} \equiv E_{1,2}^{(\eta)}$ the single-valued monotonic CVC has a vertical tangent.

Current-voltage characteristics for thin samples and for different values of the parameter η are shown in Fig. 5a.

Let us proceed to the investigation of bulk samples. It is most convenient to start with the case $\eta \to \infty$. As long as the values of E are such that $U^{(01)} < U^{(03)}$ ($U^{(01,3)} \equiv U(w_{01,3})$), for very massive but finite samples the level corresponding to the solution w_1 (w_2) occurs slightly below (above) the level $U^{(01)}$, and the level corresponding to the solution w_3 occurs slightly below the level $U^{(03)}$. In this case the temperatures $\overline{w}_{1,2}$ averaged over the cross-section are close to w_{01}, where $\overline{w}_1 < \overline{w}_2$, and $\overline{w}_3$ is close to w_{03} so that $\overline{w}_3 < w_{03}$. It is obvious that in the range of fields under consideration ($U^{(01)} < U^{(03)}$) the strength of the surface mechanisms essentially has no influence on the values of the average temperatures.

Only the single solution w_1 exists in fields where $U^{(01)} > U^{(03)}$, and the solutions $w_{2,3}$ are absent. Their absence is connected with the impossibility of satisfying the boundary condition at the point $z = a$; the potential barrier hinders the motion of the particle from the point $w(z)|_{z=0}$ to the point $w(\Theta)|_{\Theta=T}$.

In the narrow range of fields where $U^{(01)} < U^{(03)}$ and moreover $(U^{(03)} - U^{(01)})/U^{(01)} \ll 1$, $\overline{w}_1$ is as usual close to w_{01}. However, the average temperature $\overline{w}_2$ in these fields may differ substantially from w_{01}, since during its motion the particle is located for a long "time" both near w_{01} as well as near w_{03}. The same arguments are also valid for the quantity $\overline{w}_3$, which also may differ substantially from w_{03}. If the field is changed such that $U^{(01)} \to U^{(03)}$, then the levels corresponding to the solutions w_2 and w_3 approach each other. In a certain field $E = E_1^{(\infty)}$ the levels will merge together, and the average temperature $\overline{w}_2 \equiv \overline{w}_3$ will be of the order of w_{02}, as is not difficult to understand. It is obvious that for finite η the branches 2 and 3 join together in a field $E_1^{(\eta)} < E_1^{(\infty)}$ ($E_1^{(\eta)} \to E_1$ as $\eta \to 0$).

The current-voltage characteristics for bulk samples and for various values of the parameter η are shown in Fig. 5 b.

We note that in contrast to the case of thin samples, in bulk samples the multivalued dependence of the current on the field is preserved even for infinitely large values of the parameter η. In this connection, in bulk samples one of the hysteresis regions vanishes as the value of η increases.

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