CROSS SECTION FOR DEUTERON DISINTEGRATION BY π MESONS NEAR THRESHOLD

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The cross section for the reaction $\pi d \to \pi pn$ near threshold is obtained by summing an infinite series of diagrams. The reaction amplitude takes into account multiple scattering of the π meson by the nucleons in the deuteron. Corrections to the amplitude due to the kinetic energy of the nucleons in the intermediate state and the p-wave amplitudes for πN scattering are taken into account in the terms corresponding to single and double scattering. The cross section for the reaction is $10.9\,\epsilon^2$ (microbarns), where ϵ denotes the excess energy of the π meson above its threshold value, divided by the deuteron binding energy.

1. INTRODUCTION

THE attempt to utilize the smallness of the ratio of the pion mass to the nucleon mass ($\mu/m \sim 1/7$) in order to describe the interaction between the π meson and the nucleon appears to be extremely attractive. Such attempts were first made by Brueckner[1,2] in order to obtain the amplitude for the elastic scattering of a π meson by a deuteron. It was assumed there that owing to the smallness of the ratio μ/m the nucleons will experience a small recoil when scattering the pion, and consequently the amplitude for the scattering of a pion by a deuteron can be obtained by averaging the amplitudes for scattering by a system of two fixed centers over the deuteron wave function.

The formula proposed by Brueckner took multiple scattering of the pion by the nucleons into consideration. In this connection, however, the accuracy of the approximation was not estimated. Such an estimate can be obtained if the nonrelativistic diagram technique, developed by I. S. Shapiro[3] for applications to nuclear reactions, is used in order to describe the reactions between the π mesons and the nuclei. This method was used in[4,5] to calculate the amplitude for elastic πd scattering. Here one was able to derive an expression for the πd-scattering amplitude, similar in form to the Brueckner formula, but taking the deuteron binding energy into account in an explicit way:

$$f_{\pi d}=\int [\Psi_d(\mathbf{r})]^2\left\{\frac{f_1+f_2+2f_1f_2r^{-1}e^{ikr+ipr}}{1-f_1f_2r^{-2}e^{2ipr}}\right\}d\mathbf{r};$$

here $\psi_d(\mathbf{r})$ is the normalized deuteron wave function; f_1 and f_2 are the amplitudes for the scattering of a π meson by a neutron and by a proton, respectively; $\mathbf{k}$ is the momentum of the incoming π meson, $p^2 = k^2 - 2\mu\epsilon_d$, and ϵ_d is the deuteron binding energy.

In the present article the method of summing multiple scattering diagrams, which was used in[4], is applied to the calculation of the amplitude for deuteron disintegration by a pion near threshold. We consider only the reaction without charge exchange of the π meson (i.e., $\pi d \to \pi pn$), since near threshold the cross section for the reaction $\pi^+ d \to \pi^0 pp$ is much smaller, owing to the smallness of the π-nucleon spin-flip scattering amplitude. The diagrams giving the major contribution to this amplitude will be summed in Sec. 2. The corrections to allow for the kinetic energy of the nucleons in the intermediate states and the contribution of the p-wave πN scattering to the amplitude for the reaction $\pi d \to \pi pn$ are considered in Sec. 3.

2. THE AMPLITUDE FOR THE REACTION $\pi d \to \pi pn$

The series of diagrams shown in Figs. 1 and 2 will be summed in the present section. Everywhere on these figures the solid lines represent nucleons, the double lines represent a deuteron, and dotted lines represent a π meson.

Let us first consider the pole diagram (diagram I in Fig. 1). This diagram corresponds to two invariant amplitudes M_1^a and M_1^b (the π meson is scattered by the proton and by the neutron). The invariant amplitude M_1^a has the form

$$M_1^a=\frac{i(-2im)M_0F(p)A_1}{\mathbf{p}^2-2m\varepsilon-i0}=\frac{mM_0A_1F(p_n)}{p_n^2+\alpha^2},$$

Here $M_0 = 8\pi\alpha/m^2$ ($\alpha^2 = m\epsilon_d$), A_1 (A_2) is the invariant amplitude for the scattering of a π meson by a proton (neutron); $F(q)$ is the deuteron form factor, where

$$\frac{F(\mathbf{q})}{\mathbf{q}^2+\alpha^2}=\frac{1}{\sqrt{8\pi\alpha}}\int e^{i\mathbf{qr}}\Psi_d(\mathbf{r})\,d\mathbf{r}. \quad (1)$$

Taking expression (1) into account, and also considering that

$$f_i=\frac{\mu m}{2\pi(m+\mu)}A_i \quad (i=1,2),$$

we can represent the amplitude M_1^a in the following

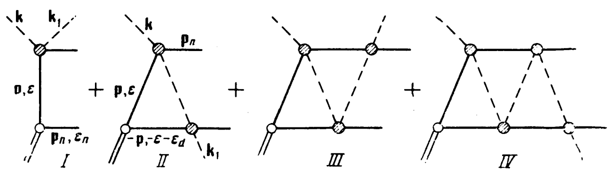
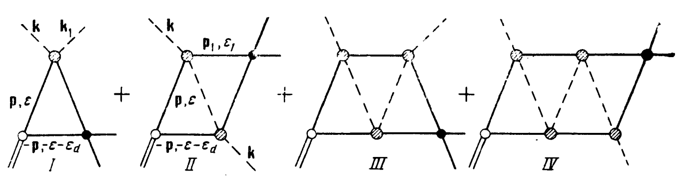


FIG. 1



form:

$$M_1^a = C\int e^{-i\Delta r}\langle\Psi_d(\mathbf{r})|\tilde{f}_1|e^{i\mathbf{q}\mathbf{r}}\rangle d\mathbf{r},$$

where it is convenient to introduce the variables

$$2\Delta = \mathbf{p}_n + \mathbf{p}_p, \quad 2\mathbf{q} = \mathbf{p}_p - \mathbf{p}_n;$$

and the quantities $\tilde{f}_i$ and C are defined by

$$\tilde{f}_i = \frac{m+\mu}{m} f_i, \quad C = \frac{mM_0}{\mu}\sqrt{\frac{\pi}{2\alpha}}.$$

We obtain a similar expression for the pole diagram corresponding to the process in which a π meson is scattered by a neutron:

$$M_1^b = C\int e^{-i\Delta r}\langle\Psi_d(\mathbf{r})|\tilde{f}_2|e^{-i\mathbf{q}\mathbf{r}}\rangle d\mathbf{r}.$$

Now let us go on to a consideration of double scattering (diagram II in Fig. 1). After integrating over the energy, the expression for this amplitude takes the form

$$M_2^a = \frac{i^2(-2im)^2(-2i\mu)M_0A_1A_2\pi i}{(2\pi)^4\cdot 2m} \times \int\frac{F(\mathbf{p})\,d\mathbf{p}}{(p^2+\alpha^2)[(\mathbf{k}+\mathbf{p}-\mathbf{p}_n)^2-2\mu(E_\pi-\varepsilon_d-E_n)+\mu p^2/m]}. \quad (2)$$

One can evaluate this integral exactly, but for convenience in summing the whole series of diagrams shown in Fig. 1 we shall evaluate it approximately, omitting the small terms $\mu p^2/m$ and $2\mu E_n$ in the denominator of the pion propagator. It turns out that the diagrams M_1 and M_2 make the major contribution to the sum of the series shown in Fig. 1; therefore in Sec. 3 we shall consider the contribution to the total amplitude coming from the corrections which must be made to account for the above-mentioned terms discarded in expression (2).

Using the transformation (1), and also using the fact that

$$\frac{1}{q^2-p^2-i0} = \frac{1}{4\pi}\int\frac{\exp(iqr+ipr)}{r}\,d\mathbf{r}, \quad (3)$$

we obtain the following expression for M_2:

$$M_2^a = C\int e^{-i\Delta r}\langle\Psi_d(\mathbf{r})|\tilde{f}_1\tilde{f}_2 r^{-1}e^{ikr+ipr}|e^{i\mathbf{q}\mathbf{r}}\rangle d\mathbf{r}. \quad (4)$$

For the same diagram, except that the positions of the proton and neutron have been interchanged, the expression has the form

$$M_2^b = C\int e^{-i\Delta r}\langle\Psi_d(\mathbf{r})|\tilde{f}_1\tilde{f}_2 r^{-1}e^{ikr+ipr}|e^{-i\mathbf{q}\mathbf{r}}\rangle d\mathbf{r}.$$

Proceeding in analogous fashion, one can show that the amplitudes represented by diagrams III and IV in Fig. 1 are given by the following formulas:

$$M_3^a = C\int e^{-i\Delta r}\langle\Psi_d(\mathbf{r})|\tilde{f}_1^2\tilde{f}_2 r^{-2}e^{2ipr}|e^{i\mathbf{q}\mathbf{r}}\rangle d\mathbf{r},$$

$$M_4^a = C\int e^{-i\Delta r}\langle\Psi_d(\mathbf{r})|\tilde{f}_1^2\tilde{f}_2^2 r^{-3}e^{ikr+3ipr}|e^{i\mathbf{q}\mathbf{r}}\rangle d\mathbf{r}.$$

Any arbitrary term of the series of diagrams shown in Fig. 1 is calculated in the same way.

It is now easily noted that this series of diagrams can be summed. In fact, we find

$$M^a = \sum_i M_i^a = C\int e^{-i\Delta r}\langle\Psi_d(\mathbf{r})|\tilde{f}_1 + \tilde{f}_1\tilde{f}_2 r^{-1}e^{ikr+ipr} + \tilde{f}_1^2\tilde{f}_2 r^{-2}e^{2ipr} + \ldots|e^{i\mathbf{q}\mathbf{r}}\rangle d\mathbf{r} = C\int e^{-i\Delta r}\langle\Psi_d(\mathbf{r})|\Phi_1(\mathbf{r},p)|e^{i\mathbf{q}\mathbf{r}}\rangle d\mathbf{r},$$

$$M^b = C\int e^{-i\Delta r}\langle\Psi_d(\mathbf{r})|\Phi_2(\mathbf{r},p)|e^{-i\mathbf{q}\mathbf{r}}\rangle d\mathbf{r},$$

where

$$\Phi_1(\mathbf{r},p) = [\tilde{f}_1 + \tilde{f}_1\tilde{f}_2 r^{-1}e^{ikr+ipr}][1-\tilde{f}_1\tilde{f}_2 r^{-2}e^{2ipr}]^{-1},$$
$$\Phi_2(\mathbf{r},p) = [\tilde{f}_2 + \tilde{f}_2\tilde{f}_1 r^{-1}e^{ikr+ipr}][1-\tilde{f}_1\tilde{f}_2 r^{-2}e^{2ipr}]^{-1}.$$

Now let us proceed to the calculation of the diagrams shown in Fig. 2. Diagram I of Fig. 2 corresponds to the following expression for the amplitude $\tilde{M}_1$ (the integration over the energy has already been carried out):

$$\tilde{M}_1^a = \frac{i^2(-2im)^3M_0A_1\pi i\cdot 4\pi}{(2\pi)^4\cdot 4m^2}\int\frac{F(\mathbf{p})f_{NN}(|\mathbf{p}+\Delta|,q)\,d\mathbf{p}}{(p^2+\alpha^2)[(\mathbf{p}+\Delta)^2-q^2-i0]}. \quad (5)$$

Here $f_{NN}(p, q)$ denotes the nucleon-nucleon scattering amplitude. Since $|\mathbf{p} + \Delta|$ is not equal to q in general, then the amplitude f_{NN} in expression (5) is not taken on the energy surface. It is well-known that one can represent it in the following form:

$$\frac{f_{NN}(k,q)}{k^2-q^2-i0} = \frac{1}{4\pi}\int e^{-i\mathbf{k}\mathbf{r}}\Psi_{\mathbf{q}}^+(\mathbf{r})\,d\mathbf{r} - 2\pi^2\delta(\mathbf{q}-\mathbf{k}), \quad (6)$$

where $\psi_q^+(\mathbf{r})$ is the exact solution of the Schrödinger equation for a proton and a neutron with the following boundary condition at infinity:

$$\Psi_{\mathbf{q}}^+(\mathbf{r}) \to e^{i\mathbf{q}\mathbf{r}} + \frac{f_{NN}(q)}{r}e^{iqr}, \quad r\to\infty.$$

(q denotes the relative momentum of the two nucleons, and $f_{NN}(q)$ is the nucleon-nucleon scattering amplitude on the energy surface). One can easily verify the correctness of the normalization factor in Eq. (6) by evaluating the wave function of the np-system in the approximation of zero-range forces. Using Eq. (6) we obtain the following expression for the amplitude $\tilde{M}_1^a$:

$$\tilde{M}_1^a = C\int e^{-i\Delta r}\langle\Psi_d(\mathbf{r})|\tilde{f}_1|\tilde{\Psi}_q^+(r)\rangle d\mathbf{r},$$

$$\tilde{\Psi}_q^+(r) = \Psi_q^+(\mathbf{r}) - e^{i\mathbf{q}\mathbf{r}}.$$

We obtain a similar expression for the amplitude $\tilde{M}_1^b$ corresponding to the process in which a pion is scattered by a neutron:

$$\tilde{M}_1^b = C\int e^{-i\Delta r}\langle\Psi_d(\mathbf{r})|\tilde{f}_2|\tilde{\Psi}_q^+(r)\rangle d\mathbf{r}. \quad (7)$$

Now let us consider the amplitude $\tilde{M}_2$ which corresponds to diagram II of Fig. 2:

$$\tilde{M}_2^a = \frac{i^3(-2im)^4(-2i\mu)M_0A_1A_2(\pi i)^2\cdot 4\pi}{(2\pi)^8\cdot 4m^3} \quad (8)$$
$$\times\int\frac{F(\mathbf{p})f_{NN}(|\Delta-\mathbf{p}_1|,q)\,d\mathbf{p}\,d\mathbf{p}_1}{(p^2+\alpha^2)[(\mathbf{k}+\mathbf{p}-\mathbf{p}_1)^2-2\mu(E-\varepsilon_d)+\mu p^2/m+\mu p_1^2/m][(\Delta-\mathbf{p}_1)^2-q^2]}$$

Omitting the terms $\mu p^2/m$ and $\mu p_1^2/m$ in the pion propagator and using the transformations (1), (3), and (6), we obtain the final expression for $\tilde{M}_2^a$:

$$\tilde{M}_2^a = C\int e^{-i\Delta r}\langle\Psi_d(\mathbf{r})|\tilde{f}_1\tilde{f}_2 r^{-1}e^{ikr+ipr}|\tilde{\Psi}_q^+(r)\rangle d\mathbf{r}. \quad (9)$$

The amplitude $\tilde{M}_2^b$ is equal to the amplitude $\tilde{M}_2^a$. One can calculate the contributions from any other term of the series shown in Fig. 2 in exactly the same way.

By summing all of these amplitudes we obtain

$$\tilde{M}^a = \sum_i \tilde{M}_i^a = C\int e^{-i\Delta r}\langle\Psi_d(\mathbf{r})|\Phi_1(\mathbf{r},p)|\tilde{\Psi}_q^+(r)\rangle d\mathbf{r}.$$

It is convenient to represent the expression for the amplitude of the reaction $\pi d \to \pi pn$ as the sum of two terms $\mathfrak{M}^a$ and $\mathfrak{M}^b$, where

$$\mathfrak{M}^a = M^a + \tilde{M}^a = C\int e^{-i\Delta r}\langle\Psi_d(\mathbf{r})|\Phi_1(\mathbf{r},p)|\Psi_q^+(\mathbf{r})\rangle d\mathbf{r}, \quad (10)$$

$$\mathfrak{M}^b = C\int e^{-i\Delta r}\langle\Psi_d(\mathbf{r})|\Phi_2(\mathbf{r},p)|\Psi_{-\mathbf{q}}^+(\mathbf{r})\rangle d\mathbf{r}. \quad (11)$$

Thus, the amplitude of the inelastic process $\pi d \to \pi pn$

is expressed in terms of the matrix element for the exact amplitude for the scattering by a two-center system (with the deuteron binding energy taken into account), the matrix element being taken between the deuteron wave function and the wave function of a neutron and a proton in the continuum.

In addition to the diagrams summed in Fig. 1 and Fig. 2, diagrams containing a rescattering of the nucleons in the intermediate state (for example, the diagram shown in Fig. 3) must also contribute to the amplitude for the reaction $\pi d \to \pi pn$. However, as shown in[4], this contribution is small. In the first place, any diagram involving a rescattering of the nucleons contains the factor $\sqrt{\mu/m}$. In the second place, such a diagram has a smallness of order $(b_0/b_1)^2$ in comparison with the diagram for double scattering of a pion by nucleons, where b_0 and b_1 are the isoscalar and isovector pion-nucleon scattering lengths:

$$b_{\pi N} = b_0 + b_1(\mathbf{t}\boldsymbol{\tau}). \qquad (12)$$

Here $b_0 = -0.017 \pm 0.006$ F, $b_1 = -0.137 \pm 0.010$ F; **t** and τ are the isospin operators of the pion and nucleon. The point is that, after rescattering of the nucleons, the pion can be scattered by both the neutron and by the proton. The sum of the diagrams enters into the answer, giving the value $2b_0$. One can also make the same assertion about the first scattering of a pion. Taken all together this leads to the result that the contribution from the diagrams involving rescattering of the nucleons does not amount to more than 2–3% of the contribution from the diagrams shown in Figs. 1 and 2.

3. CONSIDERATION OF THE TERMS CORRESPONDING TO THE KINETIC ENERGY OF THE NUCLEONS IN THE INTERMEDIATE STATE AND THE p-WAVE PART OF THE πN SCATTERING

By discarding the diagrams involving rescattering of the nucleons, we make an error of the order of a few percent. Therefore it makes sense to also calculate the series of diagrams shown in Figs. 1 and 2 to within an accuracy of a few percent. However, by discarding the terms of the form $\mu p^2/m$ in the pion propagator (these terms correspond to the kinetic energy of the nucleons in the intermediate states), we have made a much larger error. Now we shall take the contribution from these terms into account.

As shown by the calculations, the terms associated with single and double scattering of the pion on the nucleons give the major contribution to the integrals (10) and (11). Therefore, in the final answer for the amplitude of the reaction $\pi d \to \pi pn$ we must take account of the corrections to the amplitudes M_2 and $\widetilde{M}_2$ due to the terms indicated above. First let us calculate the amplitude M_2 (given by formula (2)) at threshold, i.e., for $E_\pi = \epsilon_d$. Here $E_n = 0$ and $p_n = 0$. Comparing the exact value of the integral with the approximate formula (4), we find that the latter formula gives a result amounting

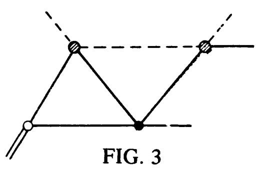


FIG. 3

to 74 per cent of the exact amplitude. This correction must be taken into account in the final answer. Evaluation of the integral (8) shows that in order to obtain the correct answer, it is sufficient to replace the terms $\mu p^2/m$ and $\mu p_i^2/m$ in this integral by $\mu p_{eff}^2/m$, where $p_{eff} = 110$ MeV/c.[1)] Having made this substitution in all of the amplitudes $\widetilde{M}_i$, we obtain the following expression for the amplitudes $\widetilde{M} = \widetilde{M}^a + \widetilde{M}^b$ at threshold:

$$\widetilde{M} = C \int e^{-i\mathbf{kr}/2} \langle \Psi_d(\mathbf{r}) | \Phi(\mathbf{r}, \varkappa) | \widetilde{\Psi}_0^+(r) \rangle \, d\mathbf{r},$$

where

$$\Phi(\mathbf{r}, \varkappa) = [\bar{f}_1 + \bar{f}_2 + 2\bar{f}_1\bar{f}_2 r^{-1} e^{i\mathbf{kr} - \varkappa r}][1 - \bar{f}_1\bar{f}_2 r^{-2} e^{-2\varkappa r}]^{-1}$$

and $\kappa^2 = 2\mu p_{eff}^2/m$.

Now let us turn our attention to the calculation of the corrections which are associated with taking the p-wave part of the pion-nucleon amplitudes into account. At low energies the p-wave amplitudes for πN scattering can be represented in the form $A_{\pi N}^p = (2\pi c_{\pi N}/\mu)\mathbf{q}\cdot\mathbf{q}'$, where the $c_{\pi N}$ are the p-wave scattering lengths, and **q** and **q**′ denote the relative momenta of the pion and nucleon before and after scattering. Here

$$c_{\pi N} = c_0 + c_1(\mathbf{t}\boldsymbol{\tau}),$$

where[2)] $c_0 = (0.208 \pm 0.008)\mu^{-3}$, $c_1 = (0.180 \pm 0.005)\mu^{-3}$. It is clear that at threshold the contribution from the pole diagram I of Fig. 1 vanishes—because the relative momentum of the pion and nucleon is equal to zero after scattering. Diagram II of Fig. 1 vanishes for the same reason, in the case when the second pion-nucleon scattering process occurs in the p-wave.

Neglecting terms of order μ/m, the expression for the diagrams of Fig. 1 containing a single p-wave πN-scattering amplitude has the form

$$M_2^a = \frac{i^2(-2im)^2(-2i\mu) M_0 \widetilde{A}_{\pi N}{}^p A_{\pi N} \pi i}{(2\pi)^4 \cdot 2m} \int \frac{F(\mathbf{p})[k^2/2 + \mathbf{kp} - \mu p^2/m]\, d\mathbf{p}}{(p^2+\alpha^2)(\mathbf{k}/2+\mathbf{p})^2}.$$

Here $\widetilde{A}_{\pi N}^p = 2\pi c_{\pi N}/\mu$. The calculation gives $M_2'^a = 0.165\, M_2^a$ and M_2^a is determined by expression (4). We shall not consider the corrections to the total amplitude due to taking account of the p-waves in the remaining diagrams of Fig. 1, since taking account of each p-wave vertex introduces an additional small factor μ/m.

Let us consider the corrections to the major terms of the series of diagrams shown in Fig. 2. With the p-wave part of the πN-scattering amplitude included, diagram I of Fig. 2 has the form

$$\widetilde{M}_1'^a = \frac{i^2(-2im)^3 M_0 \widetilde{A}_{\pi N}{}^p \pi i \cdot 4\pi}{(2\pi)^4 \cdot 4m^2} \times \int \frac{F(\mathbf{p})[\mu^2 p^2/m^2 - \mu\mathbf{pk}/m - \mu k^2/m] f_{NN}(|\mathbf{p}+\mathbf{k}/2|, 0)}{(p^2+\alpha^2)(\mathbf{p}+\mathbf{k}/2)^2}\, d\mathbf{p},$$

which amounts to -0.158 times the value of the integral (7). With a single p-wave vertex included, diagram II of Fig. 2 is determined by the expression

$$\widetilde{M}_2'^a = \frac{i^3(-2im)^4(-2i\mu) M_0 \widetilde{A}_{\pi N}{}^p A_{\pi N} (\pi i)^2 \cdot 4\pi}{(2\pi)^8 \cdot 4m^3}.$$

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1)Here the wave functions $\Psi_d(\mathbf{r})$ and $\widetilde{\Psi}_q^+(\mathbf{r})$ are taken in the form proposed by Hulthén, [6] with the parameter $\beta = 240$ MeV/c.

2)The values of the πN scattering lengths are taken from [7].

$$\times \int \frac{F(\mathbf{q})(\mathbf{k}-\mu\mathbf{q}/m)(\mathbf{k}/2+\mathbf{q}-\mathbf{q}_1) f_{NN}(q_1)\, d\mathbf{q}\, d\mathbf{q}_1}{(q^2+\alpha^2)[\mathbf{k}/2+\mathbf{q}-\mathbf{q}_1]^2 q_1^2},$$

which amounts to 26 percent of the value of the integral (9).

In order to obtain the final answer, we must take into consideration the fact that the πN-scattering amplitude is a matrix in isospin space. One can represent the amplitude of the reaction $\pi d \rightarrow \pi pn$ at threshold in the form

$$M = C \int e^{-i\mathbf{kr}/2} \langle \Psi_d(\mathbf{r}) | \hat{T}^{(1)}(\mathbf{r}) + \hat{T}^{(2)}(\mathbf{r}) | \Psi_0^+(\mathbf{r}) \rangle \, d\mathbf{r}, \qquad (13)$$

where $\hat{T}^{(1)}$ and $\hat{T}^{(2)}$ are operators obeying the equations

$$\begin{aligned} &[1-(b_0+b_1\mathbf{t}\boldsymbol{\tau}_1)(b_0+b_1\mathbf{t}\boldsymbol{\tau}_2)r^{-2}]\hat{T}^{(1)} \\ &= (b_0+b_1\mathbf{t}\boldsymbol{\tau}_1) + e^{ikr}r^{-1}(b_0+b_1\mathbf{t}\boldsymbol{\tau}_1)(b_0+b_1\mathbf{t}\boldsymbol{\tau}_2), \\ &[1-(b_0+b_1\mathbf{t}\boldsymbol{\tau}_2)(b_0+b_1\mathbf{t}\boldsymbol{\tau}_1)r^{-2}]\hat{T}^{(2)} \\ &= (b_0+b_1\mathbf{t}\boldsymbol{\tau}_2) + e^{ikr}r^{-1}(b_0+b_1\mathbf{t}\boldsymbol{\tau}_2)(b_0+b_1\mathbf{t}\boldsymbol{\tau}_1). \end{aligned} \qquad (14)$$

Let us denote the isospin wave functions of the pion by ω^1, ω^0, ω^{-1} (π^+, π^0, π^-), and the isospin wave functions of the two nucleons by χ_0^0 (isoscalar) and $\chi_1^{1,0,-1}$ (isovector). In order to be definite, let us consider the reaction with a π^+ meson. Introducing the functions $\psi_1 = \omega_1 \chi_0^0$ and $\psi_2 = (\omega^1 \chi_1^0 - \omega^0 \chi_1^1)/\sqrt{2}$, one can easily verify that the operators $\tilde{b}_0 + \tilde{b}_1 \mathbf{t}\cdot\boldsymbol{\tau}$ act on the column $\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$ like 2×2 matrices:

$$\tilde{b}_0 + \tilde{b}_1 \mathbf{t}\boldsymbol{\tau}_1 = \begin{pmatrix} \tilde{b}_0 & \sqrt{2}\tilde{b}_1 \\ \sqrt{2}\tilde{b}_1 & \tilde{b}_0 - \tilde{b}_1 \end{pmatrix} \qquad \tilde{b}_0 + \tilde{b}_1 \mathbf{t}\boldsymbol{\tau}_2 = \begin{pmatrix} \tilde{b}_0 & -\sqrt{2}\tilde{b}_1 \\ -\sqrt{2}\tilde{b}_1 & \tilde{b}_0 - \tilde{b}_1 \end{pmatrix}$$

Therefore, in order to determine $\hat{T}$ it is necessary to seek the matrix which is the inverse of the matrix $\hat{A}$:

$$\hat{A} = \begin{pmatrix} 1-(\tilde{b}_0^2 - 2\tilde{b}_1^2)r^{-2} & \sqrt{2}\tilde{b}_1^2 r^{-2} \\ -\sqrt{2}\tilde{b}_1^2 r^{-2} & 1-(\tilde{b}_0^2 - 2\tilde{b}_0\tilde{b}_1 - \tilde{b}_1^2)r^{-2} \end{pmatrix}.$$

By determining the matrix $\hat{A}^{-1}$ and multiplying (14) on the left by this matrix, we obtain expressions for the operators $\hat{T}$. In calculating the amplitude (13) we only need the "1 to 1" element of the T matrices.

The final expression for the amplitude of the reaction $\pi d \rightarrow \pi pn$ has the form

$$M = C \int e^{-i\mathbf{kr}/2} \langle \Psi_d(\mathbf{r}) | \hat{f} | \Psi_0^+(\mathbf{r}) \rangle \, d\mathbf{r}, \qquad (15)$$

where

$$\begin{aligned} \hat{f} = \{ &[2\tilde{b}_0 - 2r^{-2}(\tilde{b}_0^3 - 2\tilde{b}_0^2\tilde{b}_1 - \tilde{b}_0\tilde{b}_1^2 + 2\tilde{b}_1^2)] \\ &+ [2(\tilde{b}_0^2 - 2\tilde{b}_1^2) - 2r^{-2}[(\tilde{b}_0^2 - 2\tilde{b}_1^2)(\tilde{b}_0^2 - 2\tilde{b}_0\tilde{b}_1 - \tilde{b}_1^2) + 2\tilde{b}_1^4]]\} \cdot \\ &- r^{-2}(\tilde{b}_0^2 - 2\tilde{b}_1^2)][1 - r^{-2}(\tilde{b}_0^2 - 2\tilde{b}_0\tilde{b}_1 - \tilde{b}_1^2)] + 2r^{-4}\tilde{b}_1^4\}^{-1}. \end{aligned}$$

Near threshold the cross section for the reaction $\pi d \rightarrow \pi pn$ can be expressed in terms of the amplitude in the following way:

$$\sigma = \frac{1}{v_{\pi d}} \frac{|\tilde{M}|^2}{(2\pi)^5} V$$

where $v_{\pi d}$ is the relative velocity of the pion and deuteron at threshold. $\tilde{M}$ is the amplitude of the reaction $\pi d \rightarrow \pi pn$ at threshold, and V_3 is the phase space volume. The quantity $\tilde{M}$ is defined as the amplitude M (given by expression (15)) plus the corrections indicated in this section. This calculation of the cross section is accurate to within a few percent. However, the actual exactness of the resulting numerical value is smaller, due to the large errors in the values of the S-wave πN scattering lengths. The errors associated with the uncertainty in the values of the πN scattering lengths far exceed the error connected with the uncertainty in our choice of the wave functions ψ_d and ψ_0^+.

The final expression for the cross section of the reaction $\pi d \rightarrow \pi pn$ has the form

$$\sigma = 10.9 \left(\frac{E_\pi - \varepsilon_d}{\varepsilon_d} \right)^2 \ \mu\text{b}$$

The replacement of $\tilde{M}$ by a constant at threshold is valid for $E_\pi - \epsilon_d \lesssim \epsilon_d$. When the incident pion has larger energies, the value of $\tilde{M}$ begins to change due to the rapid growth of the p-wave, πN-scattering amplitudes.

In conclusion I express my gratitude to V. M. Kolybasov and I. S. Shapiro for helpful discussions.

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