EFFECT OF AN INTENSE ELECTROMAGNETIC WAVE ON THE PROPAGATION OF ULTRASOUND IN A QUANTIZED MAGNETIC FIELD

Yu. I. BALKAREĬ and É. M. ÉPSHTEĬN

Institute of Radio Engineering and Electronics, Academy of Sciences USSR

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The effect of a strong, high-frequency electric field of frequency Ω, directed along a quantized magnetic field, on ultrasonic propagation in a degenerate Fermi system is considered. Pronounced resonance singularities are found in the sound absorption coefficient at low temperatures under the condition that $qv_{0\nu} \sim s\Omega$ (q is the wave number of sound directed along the magnetic field, $v_{0\nu}$ is the Fermi velocity in the ν-th Landau sub-band, $s = 1, 2, 3, \ldots$). Near each positive peak corresponding to absorption, there is a negative peak corresponding to sound amplification.

1. The singularities of ultrasonic propagation in degenerate Fermi systems (metals, semimetals, semiconductors) in the presence of a magnetic field have been widely studied in recent years. A number of interesting physical effects have been discovered—acoustic cyclotron resonance,[1] giant geometric resonance,[2-3] and giant quantum oscillations of the sound absorption.[4] On the other hand, important singularities have recently been predicted, for sound propagation in electron-phonon systems, in the effect on the electrons of the field of a strong electromagnetic wave.[5,6]

Here we consider the effect of the combined action of the field of a strong electromagnetic wave and the quantized magnetic field on ultrasonic propagation. It will be shown that, for parallel orientation of the electric field of the wave, the static magnetic field and the wave vector **q** of the longitudinal sound wave in the neighborhood of values of q satisfying the condition $qv_{0\nu} \sim s\Omega$ ($v_{0\nu}$ is the Fermi velocity of the electrons in the ν-th Landau sub-band, Ω the frequency of the electromagnetic wave, $s = 1, 2, 3, \ldots$), upon satisfaction of certain conditions, strong resonance singularities appear in the damping and dispersion of the sound. The sound absorption coefficient can change sign, which corresponds to sound amplification by the high-frequency field. Such amplification can be very effective. We shall make clear the physical reason for the origin of these singularities.

Electrons take part in the interaction with sound traveling along a quantized magnetic field; these electrons execute one-dimensional free motion in this same direction. In such a situation, real absorption (emission) of phonons is virtually absent at low temperatures, since it is impossible to satisfy one dimensionally the conservation laws of energy and momentum in such processes near the Fermi surface. In the presence of the field of an electromagnetic wave of frequency Ω, which arises in the scattering of an electron by a phonon with wave number q, the transfer energy can be compensated by the energy of the photon Ω. If $\Omega \leq qv_0$, then absorption of a phonon $\omega_0(q)$ and a photon Ω is possible such that $\omega_0 + \Omega = qv_0$. In the case $\Omega \gtrsim qv_0$, the possibility opens up for a process of emission of a phonon in the absorption of a photon, such that $\Omega = \omega_0 = qv_0$.

Without a magnetic field, electrons in their interaction with phonons can be scattered at different angles, and the transfer energy takes on arbitrary values $qv_0 \cos\theta$ (θ is the angle between the initial momentum of the electron and the momentum of the phonon). Therefore, strong phonon-photon resonance is absent. All that has been said refers to the many-photon case, when $s\Omega \sim qv_0$.

2. Thus, let there be a degenerate electron-phonon system in a magnetic field. For definiteness, we shall assume that the magnetic field **H** is applied parallel to the surface of the sample along the z axis, and a plane electromagnetic wave with electric field intensity **E** and frequency Ω is incident perpendicular to the surface, such that $\mathbf{E} \parallel \mathbf{H}$. We shall assume the sound wave vector q to be arbitrarily oriented initially. Satisfaction of the following conditions is assumed:

$$1 \ll \frac{\Omega}{\Gamma} \ll \frac{c}{v_0}, \quad \frac{\Omega}{\omega_p} \ll \frac{c}{v_0}, \quad qv_0 \gg \Gamma. \tag{1}$$

Here Γ is the relaxation frequency of electrons in their momenta, ω_p the electron plasma frequency, c the speed of light, and v_0 the speed of electrons on the Fermi surface. In conjunction with the condition

$$\Omega / \omega_p > 1 \tag{2}$$

the conditions (1) guarantee penetration of the electromagnetic wave into the depth of the sample and mean that the wavelength is much greater than the electron mean free path, the screening radius, the phonon wavelength. Moreover, this wavelength is assumed to be large in comparison with the amplitude of oscillations of the electron in the field of the wave (for this, it is sufficient that the motion of the electron be nonrelativistic). It then follows that we can limit ourselves to consideration of the electron-phonon system in a spatially homogeneous, high-frequency wave, and also neglect the effect of the magnetic field of the wave.

If the inverse condition $\Omega < \omega_p$ is satisfied instead of (2), we can then assume the field of the wave to be homogeneous when the thickness of the sample is less than the penetration depth.

We assume the magnetic field to be quantized; i.e., we assume

$$T \ll \omega_c, \quad \Gamma \ll \omega_c, \tag{3}$$

where ω_c is the cyclotron frequency, T the temperature of the electrons. We shall neglect spin splitting of the levels of the electrons, assuming that it is smaller than Γ or T.

3. The Hamiltonian of a system of magnetically quantized electrons interacting with a high-frequency electric field and with sound can be written in the form

$$\hat{H}(t) = \sum_{\nu, p} \varepsilon_\nu \left(p - \frac{e}{c} A(t) \right) a_\nu^+(p) a_\nu(p)$$

$$+ \sum_{\nu,\nu',p,\mathbf{q}} C_{\nu\nu'}(\mathbf{q}) \{ a_\nu^+(p) a_{\nu'}(p-q) b(\mathbf{q}) + a_{\nu'}^+(p-q) a_\nu(p) b^+(-\mathbf{q}) \}$$

$$+ \sum_{\mathbf{q}} \omega_0(\mathbf{q}) b^+(\mathbf{q}) b(\mathbf{q}), \tag{4}$$

where $\epsilon_\nu(p) = \omega_c(\nu + 1/2) + p^2/2m$; $\nu = 0, 1, 2, \ldots$; p and q are the z components of the quasimomentum of the electron and of the phonon; $C_{\nu\nu'}(\mathbf{q})$ is the matrix element of electron-phonon interaction in the magnetic field; a^+ and a, b^+ and b are respectively the electron and phonon creation and annihilation operators; $\omega_0(\mathbf{q}) = u\mathbf{q}$, u is the sound speed. We choose the electric field E(t) in the form

$$E_0(t) = E_0 \sin \Omega t = -dA / cdt.$$

We have used a quadratic dispersion law for electrons in (4), keeping in mind the application of the results in what follows primarily to semiconductors or semimetals.

Using the canonical transformation, with the help of the unitary operator,

$$S(t) = \exp\left\{ if(t,p) \sum_{\nu,p} a_\nu^+(p) a_\nu(p) \right\}, \tag{5}$$

$$f(t,p) = \int_{-\infty}^{t} \left\{ \frac{e}{mc} A(t') p - \frac{e^2}{2mc^2} A^2(t') \right\} dt' = f_1(t,p) + f_2(t), \tag{6}$$

we shift the dependence on the electromagnetic field from the first term of the Hamiltonian (4) to the second:

$$\tilde{\hat{H}}(t) = S^+ \left\{ -i \frac{\partial}{\partial t} + \hat{H}(t) \right\} S = \sum_{\nu,p} \varepsilon_\nu(p) a_\nu^+(p) a_\nu(p)$$

$$+ \sum_{\nu,\nu',p,\mathbf{q}} C_{\nu\nu'}(\mathbf{q}) \{ \exp[if_1(t,q)] a_\nu^+(p) a_{\nu'}(p-q) b(\mathbf{q}) \tag{7}$$

$$+ \exp\{-if_1(t,q)\} a_{\nu'}^+(p-q) a_\nu(p) b^+(-\mathbf{q}) \} + \sum_{\mathbf{q}} \omega_0(\mathbf{q}) b^+(\mathbf{q}) b(\mathbf{q}).$$

Expanding $\exp\{if_1(t,q)\}$ in a series in the Bessel functions J_s, and assuming an adiabatically slow connection of the field for $t = -\infty$, we reduce the Hamiltonian to the form

$$\tilde{H}(t) = \sum_{\nu,p} \varepsilon_\nu(p) a_\nu^+(p) a_\nu(p)$$

$$+ \sum_{s=-\infty}^{\infty} \sum_{\nu,\nu',p,\mathbf{q}} C_{\nu\nu'}(\mathbf{q}) \{ J_s(aq) e^{is\Omega t} a_\nu^+(p) a_{\nu'}(p-q) b(\mathbf{q})$$

$$+ J_s(aq) e^{-is\Omega t} a_{\nu'}^+(p-q) a_\nu(p) b^+(-\mathbf{q}) \} + \sum_{\mathbf{q}} \omega_0(\mathbf{q}) b^+(\mathbf{q}) b(\mathbf{q}), \tag{8}$$

where $a = eE_0/m\Omega^2$ is the amplitude of the oscillations of the electron in the field of the wave.

By means of the Hamiltonian (8), and using the assumption on the smallness of the electron-phonon interaction, we write down the equation of motion for the averaged phonon operator $\langle b(\mathbf{q}) \rangle$ and the functions connected with it:

$$i \frac{d\langle b(\mathbf{q}) \rangle}{dt} = \omega_0(\mathbf{q}) \langle b(\mathbf{q}) \rangle$$

$$+ \sum_{s=-\infty} \sum_{\nu,\nu',p} C_{\nu\nu'}(\mathbf{q}) J_s(aq) e^{-is\Omega t} \langle a_{\nu'}(p) a_\nu^+(p-q) \rangle,$$

$$i \frac{d}{dt} \langle a_{\nu'}(p) a_\nu^+(p-q) \rangle = \{ \varepsilon_{\nu'}(p) - \varepsilon_\nu(p-q) \} \langle a_{\nu'}(p) a_\nu^+(p-q) \rangle$$

$$+ \sum_{s=-\infty} C_{\nu\nu'}(\mathbf{q}) J_s(aq) e^{is\Omega t} [n_\nu(p-q) - n_{\nu'}(p)] \langle b(\mathbf{q}) \rangle. \tag{9}$$

Here we use the notation $\langle X \rangle = \mathrm{Sp}\, X\rho(t)$, where $\rho(t)$ is the matrix density of the system, defined by the relation

$$i\partial\rho / \partial t = [\tilde{H}, \rho],$$

$n_\nu(p)$ is the population number of states with a given quasimomentum in the different Landau sub-bands.

The smallness of the electron-phonon interaction allows us to produce a splitting of the form

$$\langle a_\nu^+(p-q) a_{\nu'}(p) b(\mathbf{q}) \rangle \approx \langle a_\nu^+(p-q) a_{\nu'}(p) \rangle \langle b(\mathbf{q}) \rangle$$

in obtaining (9).[7,8] Of all the quantities $\langle a_\nu^+(p-q) a_{\nu'}(p) \rangle$, only the diagonal averages $\langle a_\nu^+(p) a_\nu(p) \rangle$ remain, which, for $\Omega\Gamma^{-1} \gg 1$, can be assumed not to depend explicitly on the time. In the approximation considered, the dependence of the population numbers $n_\nu(p)$ on the electromagnetic field is manifest, for example, through the electron temperature, which can differ materially from the lattice temperature in strong fields.

Transforming from the functions $\langle b(\mathbf{q}) \rangle$ to their Fourier transforms $B(\mathbf{q}, \omega)$, we get the following from the set of equations (9), using the explicit form of the matrix elements $C_{\nu\nu'}(\mathbf{q})$,

$$[\omega - \omega_0(\mathbf{q})] B(\mathbf{q}, \omega) = -\sum_{s,s'=-\infty}^{\infty} \lambda^2 \omega_0(\mathbf{q}) \frac{m\omega_c}{2\pi^2} J_s(aq)$$

$$\times J_{s'}(aq) \Pi(\nu, \nu', q_\perp, q, \omega + s\Omega) B(\mathbf{q}, \omega + (s-s')\Omega), \tag{10}$$

$$\Pi(\nu, \nu', q_\perp, q, \omega) = \sum_{\nu,\nu'} \int_{-\infty}^{\infty} dp \Lambda_{\nu\nu'} \left(\frac{q_\perp}{(2m\omega_c)^{1/2}} \right) \frac{n_\nu(p-q) - n_{\nu'}(p)}{\varepsilon_\nu(p-q) - \varepsilon_{\nu'}(p) + \omega - i\delta} \tag{11}$$

($q_\perp$ is the component of the phonon momentum perpendicular to the magnetic field, λ the constant of the electron-phonon interaction.

The polarization operator $\Gamma(\nu, \nu', \mathbf{q}, \omega)$ was investigated in detail by Akhiezer.[9] In particular, the function $\Lambda_{\nu\nu'}(x)$ is expressed in the form of an integral

$$\Lambda_{\nu\nu'}(x) = \int_0^\infty ds J_0(2x\sqrt{s}) L_\nu(s) L_{\nu'}(s) e^{-s}$$

($L_\nu(s)$ is the Laguerre polynomial); for small values of the argument $x \ll 1$ it has the form

$$\Lambda_{\nu\nu'}(x) \approx \delta_{\nu\nu'} + x^2 \{ (\nu+1)\delta_{\nu+1,\nu'} - (2\nu+1)\delta_{\nu\nu'} + \nu\delta_{\nu-1,\nu'} \}. \tag{12}$$

It is not difficult to establish the fact that the terms on the right side of Eq. (10) with $s \neq s'$ give a contribution that is small in the parameter $\zeta\omega_c\omega_0(\mathbf{q})/\Omega^2 \ll 1$ ($\zeta = \lambda^2 m p_0 / 2\pi^2 \ll 1$ is the dimensionless constant of electron-phonon interaction, p_0 is the Fermi momentum in the absence of a magnetic field). Thus we obtain the following dispersion equation for the phonons:

$$\omega = \omega_0(\mathbf{q})\left\{1-\lambda^2\frac{m\omega_c}{2\pi^2}\sum_{s=-\infty}^{\infty}J_s^2(aq)\Pi(\nu,\nu',q_\perp,q,\omega+s\Omega)\right\}. \quad (13)$$

The electric field appears in this equation only as a parameter.

4. We consider in more detail the case of sound propagation in a direction close to the direction of the magnetic field. Then, leaving only terms of zero order in the parameter $q_\perp/(2m\omega_c)^{1/2}$, we have

$$\omega = \omega_0(\mathbf{q})\left\{1-\lambda^2\frac{m\omega_c}{2\pi^2}\sum_{s=-\infty}^{\infty}J_s^2(aq)\cdot\right.$$

$$\left.\sum_\nu\int_{-\infty}^{\infty}dp\frac{n_\nu(p-q)-n_\nu(p)}{\varepsilon_\nu(p-q)-\varepsilon_\nu(p)+\omega+s\Omega-i\delta}\right\} \quad (14)$$

Separating out the imaginary part Im ω, we find the sound absorption coefficient:

$$\alpha(q)=\frac{2}{u}\frac{m\omega_c}{2\pi^2}\omega_0(\mathbf{q})\lambda^2\pi\sum_{s=-\infty}^{\infty}J_s^2(aq)\sum_\nu\int dp[n_\nu(p-q)-n_\nu(p)]\cdot$$

$$\cdot\delta\left(-\frac{pq}{m}+\frac{q^2}{2m}+uq+s\Omega\right). \quad (15)$$

After integration over p, using the parity of the functions $n_\nu(p)$, we find

$$\alpha(q)=\frac{m^2\omega_c}{\pi u}\frac{\omega_0(q)}{q}\lambda^2\sum_\nu\left\{J_0^2(aq)\left[n_\nu\left(-\frac{q}{2}+mu\right)-n_\nu\left(\frac{q}{2}+mu\right)\right]\right.$$

$$+\sum_{s=1}^{\infty}J_s^2(aq)\left[n_\nu\left(\frac{q}{2}-mu-\frac{sm\Omega}{q}\right)-n_\nu\left(\frac{q}{2}+mu-\frac{sm\Omega}{q}\right)\right.$$

$$\left.\left.+n_\nu\left(\frac{q}{2}-mu+\frac{sm\Omega}{q}\right)-n_\nu\left(\frac{q}{2}+mu+\frac{sm\Omega}{q}\right)\right]\right\}. \quad (16)$$

Using the inequalities q, $mu \ll p_{0\nu}$, $m\Omega/q$, $\sqrt{mT}$, we reduce $\alpha(q)$ to the form

$$\alpha(q)=-\frac{1}{\pi}m^2\omega_0(\mathbf{q})\omega_c\lambda^2\left[J_0^2(aq)\sum_\nu n_\nu''(0)\right.$$

$$\left.+2\sum_{s=1}^{\infty}J_s^2(aq)\sum_\nu n_\nu''\left(\frac{s\Omega m}{q}\right)\right]. \quad (17)$$

The first term in $\alpha(q)$ gives an exponentially small contribution, since $n_\nu''(0)\approx(mT)^{-1}\exp(-p_{0\nu}^2/2mT)$ and we assume that for all ν we have, $T\ll p_{0\nu}^2/2m$. This term is always present, even if the high-frequency electric field vanishes. Near the values $q\sim sm\Omega/v_{0\nu}$, terms which contain $n_\nu''(sm\Omega/q)$, under the condition $T\ll q\omega_c/p_{0\nu}$, give strong singularities of a resonance type with the resonance width of the order of T, since at low temperatures the Fermi functions $n_\nu(p)$ are close to step functions and have very well known inflection points. It is characteristic that each resonance with indices ν and s, proportional to the second derivative of the function $n_\nu(p)$, contains a pair of peaks, one of which is always negative, which corresponds to sound amplification. The sharpness of the resonance increases with decrease in temperature. At the extremal points, which are given by $q_e = sm\Omega(p_{0\nu}^2 + 2mTz)^{-1/2}$, where $z = \ln(2\pm\sqrt{3}\approx\pm1.3$, the absorption (amplification) coefficient $\alpha(q_0)$ is equal to (if it is limited to a single term in the sum over ν in the vicinity of each resonance)

$$\alpha_{\nu,s}(q_s)\approx\frac{\pi}{5}\zeta\frac{\omega_c}{\varepsilon_F}\frac{\omega_0(q_s)}{T}\frac{p_{0\nu}^2}{mT}p_0J_s^2(aq_s) \quad (18)$$

(ϵ_F is the Fermi energy). For not too large field strengths, when $aq_e \ll 1$, the Bessel function can be expanded in a series, and then the value of each peak is proportional to $(aq_e)^{2s}$.

Resonance singularities in the sound dispersion ought also to be observed in the region of the absorption anomalies.

5. We proceed to numerical estimates. From the viewpoint of sound amplification, the use of a semimetal or of a degenerate semiconductor with small effective carrier mass and electron concentration $n\sim10^{17}$–10^{18} cm^{-3} is most common, since here one can, on the one hand satisfy the condition $\Omega>\omega_p$, which guarantees the penetration of the electromagnetic wave into the bulk of the sample, and on the other obtain a comparatively large amplification coefficient of the sound wave for attainable intensities of the electromagnetic radiation.

Let $n\sim10^{18}$ cm^{-3}, $m\sim10^{-2}m_0$ (m_0 is the mass of the free electron), $\omega_p\sim10^{13}$ sec^{-1}, $\Omega\sim10^{14}$ sec^{-1} (CO_2 laser), $v_0\sim10^8$ cm/sec, $\omega_c\sim10^{12}$ sec^{-1}, $T\sim1$°K, $\zeta\sim10^{-2}-10^{-1}$, s = 1, $E_0\sim10^4$ V/cm. Then the resonant values are $q\sim\Omega/v_0\sim10^6$ cm^{-1} and the sound should be amplified with $\omega\sim10^{11}$ sec^{-1}. For the given values of the parameters, we obtain $\alpha_{\nu s}(q_e)\sim10^3$–$10^4$ cm^{-1}. Then high-frequency sound amplification by means of an electromagnetic wave can be very effective under the described conditions. For the given value of Ω, absorption and amplification have an oscillatory character with sharp spikes in the change of the magnetic field intensity.

If the electromagnetic field penetrates to a finite depth in the sample, amplification of the sound will take place in the region of penetration. In such a case, it is convenient to strengthen the surface sound waves, for which the same picture ought to be observed as for bulk waves.

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