APPEARANCE OF TURBULENCE DURING THE INTERACTION OF A "MONOENERGETIC" BEAM WITH A PLASMA

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A nonlinear theory of the hydrodynamic instability of a beam in a plasma is constructed. The transition from a monochromatic-wave excitation to a "multimode" regime is studied with the aid of an electronic computer, and "phase" mixing in the beam in such a regime is investigated. It is shown that the spectra of the oscillations excited during the beam instability can be controlled by a preliminary modulation of the beam.

1. It is known that the process of relaxation of an initially "monoenergetic" beam in a plasma can be divided into two stages[1]. During the first-hydrodynamic-stage, a narrow wave packet of width $\Delta(\omega_k/k) \sim v_0(n_1/n_0)^{1/3}$ with characteristic values of the growth constant $\gamma_1 \sim \omega_p(n_1/n_0)^{1/3}$ (ω_k and k are the frequency and wave vector of the oscillations, v_0 is the beam velocity, ω_p is the plasma frequency, and n_1 and n_0 are respectively the beam and plasma densities; $(n_1/n_0)^{1/3} \ll 1$) is excited in the beam. Subsequently, as a result of the "spread" of the velocity distribution function of the beam, the growth constant decreases to the value $\gamma_2 \sim \omega_p n_1/n_0$, while the spectral width $\Delta(\omega_k/k)$ becomes of the order of v_0 (the kinetic stage ends in the one-dimensional model with the formation of a plateau in the distribution function).

In a paper by one of the authors[1] the hydrodynamic stage of the relaxation was investigated on the basis of the equation of the quasilinear approximation, under the assumption that the beam excites during this stage many waves with random phases. In this case the primary effect of the reaction of the waves on the beam is the diffusion of the particles in velocity space, leading in the hydrodynamic stage to the appearance of a thermal spread:

$$\Delta v \sim \gamma_1/k \sim v_0(n_1/n_0)^{1/3}. \qquad (1)$$

The investigation of the hydrodynamic beam instability in a plasma carried out in[1] is, strictly speaking, qualitative since the possibility of the beam particles being trapped by the excited waves is neglected in it. During the hydrodynamic instability, when the spectral width is of the same order of magnitude as the oscillation velocity of the trapped particles, the effect of the trapping can be important. The influence of the effect on the dynamics of the instability is most sharply manifested in the excitation of a monochromatic wave[2,3]. The trapped particles, which execute phase oscillations with respect to the waves that have trapped them, do not exchange energy with the waves on the average during the period $\tau = 2\pi/\Omega$ of the oscillations. As a result, upon fulfilment of the condition

$$(e\varphi_0/m)^{1/2} \sim v_0(n_1/n_0)^{1/3} \qquad (2)$$

(φ_0 is the amplitude of the potential in the wave), when the beam splits into bunches of particles trapped by the wave, the exponential growth of the field amplitude during the instability is replaced by amplitude oscillations of frequency $\sim\Omega = k(e\varphi_0/m)^{1/2}$. The resulting state of the plasma + beam (of particles trapped by the wave) system is unstable with respect to excitation in the vibrational spectrum of satellites displaced in phase velocity by the amount $\lesssim \Omega/k$ [4,5].

In the present paper we investigate the transition to the "multimode" regime induced by this instability during the hydrodynamic beam instability in the plasma. With that end in view we solve in Sec. 2 the nonlinear problem of the interaction of the finite perturbations excited by the beam in the plasma. We consider the one-dimensional case: the perturbations propagate in the direction of motion of the beam. It is shown that for small phase-velocity "detuning" for neighboring harmonics of the spectrum $\delta(\omega_k/k) \ll v_0(n_1/n_0)^{1/3}$, only the "multimode" regime of the instability is possible. When the "detuning" is increased, a monochromatic wave is excited which corresponds to the maximum of the linear increment; the excitation of this wave eventually goes over into the "multimode" regime. In such a regime as a result of the concurrent action of several waves on the beam, there occurs a randomization of the motion of the beam particles and a "mixing" in the phase plane that corresponds to a thermal velocity spread in the beam of the order of magnitude given by the relation (1).

In Sec. 3 of the present paper the interaction of the perturbations is considered for the case when they are spatially intensified by an electron beam injected into the plasma. It is shown that by setting off at the entrance of the beam a signal of fixed frequency and with an amplitude substantially exceeding the fluctuation amplitude makes it possible to distinguish this frequency in the vibrational spectrum and that the wave amplified by the beam remains monochromatic at sufficiently large distances. This confirms the possibility suggested earlier by one of the authors[6] that the vibrational spectrum can be controlled by a preliminary modulation of the beam.

2. A "monoenergetic" beam excites in a plasma a narrow wave packet with $k \sim \omega_p/v_0$. The growth in time of the amplitude of the individual harmonics of this packet is determined in the linear theory from the relation

$$E_k(t) \sim E_k(0)\exp\left[\gamma_{max}t\left(1 - \frac{2^{5/6}}{9}\frac{(k-k_0)^2}{k_0^2}\left(\frac{n_0}{n_1}\right)^{2/3}\right)\right],$$

where $k_0 = \omega_p/v_0$ is the wave number corresponding to the maximum of the increment, $\gamma_{max} = 3^{1/2}\omega_p(n_1/16n_0)^{1/3}$. It follows from (3) that the amplitude E_k is comparable with E_{k_0} in the wave-number range

$$\Delta k \approx k_0\left(\frac{n_1}{n_0}\right)^{1/3}\frac{3}{(2^{5/6}\gamma_{max}t)^{1/2}} \sim k_0\left(\frac{n_1}{n_0}\right)^{1/3} \Big/ \left[\ln\frac{E_{max}}{E(0)}\right]^{1/2}; \quad (4)$$

where $E_{max} \sim (4\pi n_1 m v_0^2 (n_1/n_0)^{1/3})^{1/2}$ is the amplitude of the electric field, determined from the condition (2), at which the nonlinear effects become important, $E(0)$ is the initial value of the amplitude, i.e.,

$$\frac{E^2(0)}{4\pi} \sim \frac{T}{\lambda_D^2}\frac{\omega_p}{v_0}\left(\frac{n_1}{n_0}\right)^{1/3}$$

is the energy of the thermal fluctuations of the field for the wave-number range Δk, T is the plasma temperature, and λ_D is the Debye radius. Thus,

$$\frac{E_{max}}{E(0)} \sim \left(n_1\frac{v_0^3}{\omega_p^3}\right)^{1/2}. \quad (5)$$

We investigate below the nonlinear dynamics of the wave packet excited during the hydrodynamic beam instability in the plasma. With this end in view let us consider the interaction of perturbations with wave numbers k and k_0 belonging to the packet of the unstable oscillations. As a result of the interaction in the plasma there develop perturbations with wave numbers

$$k_j \approx k_0 + j(k-k_0), \quad j = 0, \pm 1, \pm 2\ldots, \quad (6)$$

the linear increment of which is also close to γ_{max}. We seek the electric field of the individual perturbations in the form

$$E_j(t, z) = E_j(t)\sin[k_j(z - v_0 t) + \alpha_j(t)]. \quad (7)$$

When $(n_1/n_0)^{1/3} \ll 1$ the amplitudes of the waves excited by the beam and given by (2) satisfy the condition $e\varphi_0 \ll mv_0^2$. At such amplitudes the oscillations of the plasma particles remain linear. Under these same conditions we can also neglect the excitation of higher harmonics of the electric field with frequencies $s\omega_p$ ($s \ge 2$). According to[2,3], their amplitude $E_s \sim \gamma E_1/\omega_p \ll E_1$.

Integrating the equations of the plasma particle oscillations in the linear—with respect to amplitude—approximation, substituting the result into the Poisson equation, and applying to this equation the method of harmonic analysis, we obtain for the amplitude and phase of the perturbations the following system of equations:

$$\frac{dE_j}{dt} = \frac{4\pi e\omega_p}{k_j L}\int_{-L/2}^{L/2}\sin[k_j z' + \alpha_j] n_b(t, z')dz', \quad (8)$$

$$E_j\left(\frac{d\alpha_j}{dt} + 1 - \frac{k_j v_0}{\omega_p}\right) = \frac{4\pi e\omega_p}{k_j L}\int_{-L/2}^{L/2}\cos[k_j z' + \alpha_j] n_b(t, z')dz'. \quad (9)$$

In these equations $z' = z - v_0 t$, L is the general spatial period of the perturbations, i.e., $k_0 = 2\pi m/L$, $k = 2\pi n/L$, m and n are whole numbers, and $n_b = \int dv f_b(t, z', v)$ is the electron density in the beam. The use of Liouville's theorem of phase-volume conservation

$$dz'dv = dz(0)dv(0),$$

and the condition of preservation of the distribution function along the particle trajectories[1)]

$$f_b(t, z', v) = f_b{}^0(v(0)) = n_1\delta[v(0) - v_0]$$

allows us to reduce Eqs. (8) and (9) to the following form:

$$\frac{dE_j}{dt} = \frac{4\pi e}{k_j L}\omega_p n_1\int_{-L/2}^{L/2}\sin[k_j z'(t, z(0)) + \alpha_j]dz(0), \quad (10)$$

$$E_j\left(\frac{d\alpha_j}{dt} + 1 - \frac{k_j v_0}{\omega_p}\right) = \frac{4\pi e}{k_j L}\omega_p n_1\int_{-L/2}^{L/2}\cos[k_j z'(t, z(0)) + \alpha_j]dz(0). \quad (11)$$

The trajectories of the beam particles $z'(t, z(0))$ are determined by integrating the equations of motion

$$\frac{d^2 z'}{dt^2} = -\frac{e}{m}\sum_{j=-N}^{N} E_j \sin[k_j z' + \alpha_j]. \quad (12)$$

Equations (10)—(12) form a closed system that describes the nonlinear dynamics of a packet of 2N + 1 unstable perturbations in the "monoenergetic" beam plasma system. In terms of the dimensionless variables

$$\tau = \omega_p t\left(\frac{n_1}{n_0}\right)^{1/3}, \quad \zeta = \frac{z'}{L}, \quad \nu = \frac{v - v_0}{v_0(n_1/n_0)^{1/3}}, \quad (13)$$

$$\delta = \frac{kv_0 - \omega_p}{\omega_p(n_1/n_0)^{1/3}}, \quad \varepsilon_j = E_j\left[4\pi n_1 m v_0^2\left(\frac{n_1}{n_0}\right)^{1/3}\right]^{-1/2}$$

(with $e\varphi_{0j} = \epsilon_j m v_0^2 (n_1/n_0)^{2/3}$) the system of equations reduces to a universal system containing as a parameter only the phase "detuning" of the unstable waves $\delta_j = j\delta$ (it is assumed that the wave with $k_j = k_0$ has a zero "detuning" $k_0 v_0 = \omega_p$):

$$\frac{d\nu}{d\tau} = -\sum_{j=-N}^{N}\varepsilon_j\sin[2\pi(m + j(n-m))\zeta + \alpha_j], \quad \frac{d\zeta}{d\tau} = \frac{\nu}{2\pi m}, \quad (14)$$

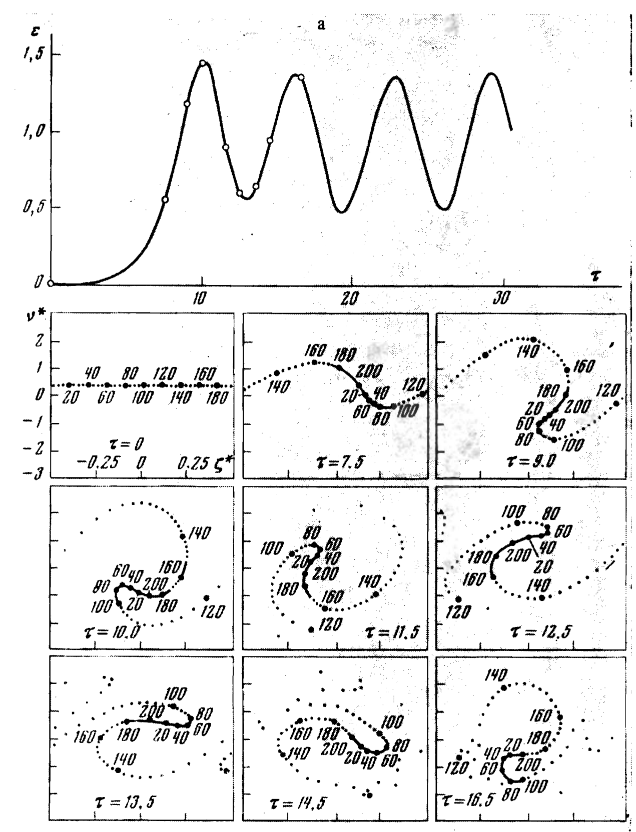
$$\frac{d\varepsilon_j}{d\tau} = \int_{-1/2}^{1/2}\sin[2\pi(m + j(n-m))\zeta(\tau, \zeta_0) + \alpha_j]d\zeta_0, \quad (15)$$

$$\varepsilon_j\left(\frac{d\alpha_j}{d\tau} - j\delta\right) = \int_{-1/2}^{1/2}\cos[2\pi(m + j(n-m))\zeta(\tau, \zeta_0) + \alpha_j]d\zeta_0. \quad (16)$$

The system (14)—(16) was integrated with an electronic computer. Figure 1 shows the results of the integration for the case of a monochromatic wave (N = 0) in the form of: a) the dependence of the dimensionless amplitude of the field ϵ on τ, and b) the dynamics of the phase plane of the beam. The wave excited by the beam leads to the bunching of its particles in the retarding-phase region of the field. To such a bunching correspond the steepening and break-up of the profile of the function $\nu(\zeta)$. The major portion of the particles of the beam then forms a bunch (the heavy-line spiral in Fig. 1) that rotates in the phase plane with the frequency of oscillation of the trapped particles. The periodic displacements of the bunch from the retarding to the accelerating phase of the field, and from the accelerating to the retarding phase lead to oscillations in time of the field amplitude ϵ.

The results presented for a monochromatic wave were obtained by integrating M = 200 equations of motion of the beam particles with initial coordinates in the interval $-\pi/k_0 < z(0) < \pi/k_0$; the results practically cease to depend on M for $M > 50$. In the case of interaction among several unstable perturbations

1) Here, z(0) and v(0) are the initial coordinates of the particle on the trajectory passing at time t through the point (z, v) of phase space. The initial perturbation of the equilibrium distribution function of the beam can be neglected when computing the right hand side of Eqs. (8) and (9).



b

FIG. 1. Results of the integration for the case of a monochromatic wave: a) the function $\epsilon(\tau)$; b) dynamics of the phase plane of the beam. The plane is shown in the reference frame of the wave for the moments of time corresponding to the points on the curve $\epsilon(\tau)$; $\nu^* = \nu + d\alpha/d\tau$, $\zeta^* = \zeta + \alpha/2\pi$, $\zeta = k_0 z/2\pi$.

($N \neq 0$) the initial coordinates of the particles are given in the interval $-\pi m/k_0 < z(0) < \pi m/k_0$, $m = (n - m)/\delta(n_1/n_0)^{1/3} \gg 1$, so that the number of particles in the beam should be substantially increased. We integrated the system (14)–(16) for three ($N = 1$), five ($N = 2$), and seven ($N = 3$) interacting waves with $m = 9$, $n = 10$, and $M = 450$. When the number of particles in the beam was varied up to $M = 900$ the divergence of the results for $\epsilon_j(\tau)$ in the range $\tau \lesssim 30$ did not exceed 5×10^{-2}; the variation of m or n at fixed δ also did not qualitatively affect the result. The accuracy of the computations was monitored with the aid of the energy integral of Eqs. (14)–(16):

$$\frac{1}{2}\sum_{j=-N}^{N} \varepsilon_j^2(\tau) + \int_{-1/2}^{1/2} d\zeta_0 \nu(\zeta_0, \tau) = \text{const.} \qquad (17)$$

The change in this quantity in the course of the computations was not more than 1%.

The results of the computations for $\epsilon(0) = 10^{-2}$ and different values of the "detuning" δ carried out for three waves are shown in Fig. 2, and for five waves and the same values of $\epsilon(0)$ and δ—in Fig. 3. It can be seen from Fig. 2 that when $\delta < 1$ the fundamental wave ($k_j = k_0$) and the slow satellite ($k_j = k$) grow simultaneously, the amplitude of the slow perturbation being somewhat greater than that of the principal wave in the nonlinear regime. This result remains valid for all possible values of the initial amplitude $\epsilon(0)$. According to (1), the difference between the rise times of the fundamental wave and the satellite sufficiently slowly (logarithmically) depends on the initial amplitude:

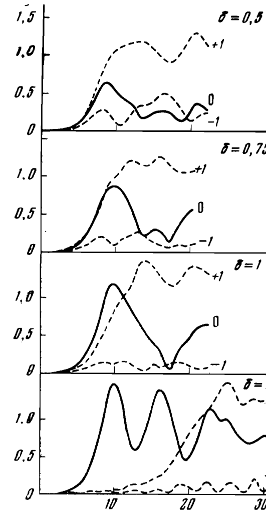


FIG. 2

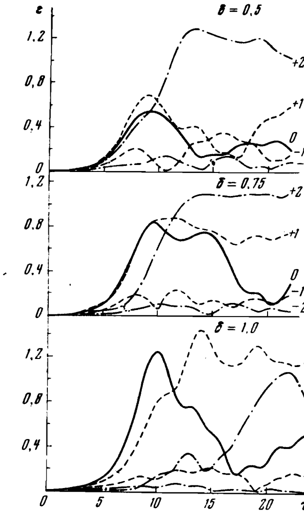


FIG. 3

FIG. 2. The function $\epsilon(\tau)$ for $\epsilon(0) = 10^{-2}$ and different values of the detuning δ for the case of three waves; the j = 0, 1, and −1 are for the fundamental, slow and fast waves respectively.

FIG. 3. The function $\epsilon(\tau)$ for $\epsilon(0) = 10^{-2}$ and different values of the detuning δ for the case of five waves; the j = 0 curve is for the fundamental wave, j = 1 and 2 are for slow waves, and j = −1 and −2 are for fast waves.

$$t_k - t_{k_0} \sim \frac{1}{\gamma_{\max}} \frac{(\Delta k)^2}{k_0^2} \left(\frac{n_0}{n_1}\right)^{2/3} \ln \frac{E_{\max}}{E(0)}.$$

This is confirmed by integration of the system (14)–(16) with different values of $\epsilon(0)$ (see Fig. 4).

The motion of the bunches into which the beam breaks up during the instability becomes randomized in the field of the excited waves, and the amplitudes of the individual harmonics become random functions of the time. Comparison of Figs. 2 and 3 shows that the vibrational spectrum broadens in time towards smaller phase velocities. As the number of harmonics in the spectrum increases, the vibrational energy becomes more evenly distributed among the harmonics. For a sufficiently large number of harmonics ($N \geq 2$) the total vibrational energy

$$\sim \sum_{j=-N}^{N} \varepsilon_j^2$$

is approximately constant in the nonlinear regime if N is fixed, and increases with N.

The excitation of a monochromatic wave having a maximum linear growth constant is possible only in the case of a sufficiently rarefied vibrational spectrum, when the minimum value of the "detuning" $\delta_{min} > 1$. This case is shown in Fig. 2d, which corresponds to

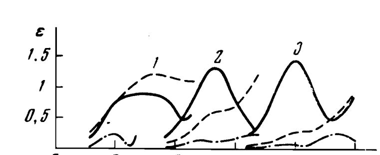


FIG. 4. The function $\epsilon(\tau)$ for $\delta = 1$ and different values of $\epsilon(0)$ for the case of three waves: 1) $\epsilon(0) = 10^{-1}$, 2) $\epsilon(0) = 10^{-3}$ 3) $\epsilon(0) = 10^{-5}$.

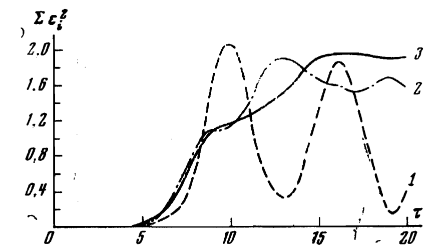


FIG. 5. The total vibrational energy $\sim\Sigma\epsilon^2{}_{kj}$ as a function of the time τ for five harmonics (curve 2) and seven harmonics (curve 3) when $\delta = 0.5$. The dashed curve is the plot of the function $\epsilon^2(\tau)$ for the case of a monochromatic wave.

the value $\delta = 2$. For times $\tau < 20$, the excited wave remains monochromatic, the beam breaks up into bunches that rotate synchronously in the phase plane (the dynamics of the phase plane of the beam for this case is shown in Fig. 6). For large values of the time the amplitudes of the fundamental wave and the slow perturbation are comparable. The distinctive features in the phase plane that correspond to the bunches of trapped particles then even out and a practically complete "phase mixing" eventually (for $\tau \gtrsim 30$) occurs in the beam. The same "mixing" takes place at smaller values of δ provided the total width of the spectrum is sufficiently large: $N\delta \gtrsim 1.5-2$, The dynamics of the phase plane of the beam for $\delta = 0.75$ are depicted in Fig. 7 for the case of three waves—a), b), c), d)—and for five waves—e). The destruction of the bunches in the phase plane and the "mixing" in the beam occur only in the last case.

"Mixing" corresponds to the appearance of a particle-velocity spread $\Delta v \sim v_0(n_1/n_0)^{1/3}$ in the beam. The same spread can be obtained in the framework of the quasilinear approximation. In that case the state of the beam is characterized by a sufficiently "smooth" function $f_0(t, v)$ whose variation in time is determined from the diffusion equation

$$\frac{\partial f_0}{\partial t} = \frac{e^2}{m^2}\frac{\partial}{\partial v}\left[\sum_{kj}\frac{\gamma_k |E_{kj}|^2}{(k_j v - \omega_k)^2 + \gamma_k^2}\frac{\partial f_0}{\partial v}\right]. \tag{18}$$

It follows from this equation that the width of the distribution function f_0

$$\Delta v = \left[\frac{1}{n_1}\int f_0 (v - v_0)^2 \, dv\right]^{1/2}$$

is related to the vibrational energy

$$\frac{1}{4\pi}\sum_{kj} |E_{kj}|^2$$

by the relation[1]

$$(\Delta v)^2 = \frac{1}{4\pi n_1 m}\left(\frac{4n_1}{n_0}\right)^{1/3}\sum_{kj} |E_{kj}|^2. \tag{19}$$

When $\Sigma\,|\,E_{kj}\,|^2 \sim 4\pi n_1 m v_0^2 (n_1/n_0)^{1/3}$ ($\Sigma\epsilon_j^2 \sim 1$) the velocity spread $\Delta v \sim v_0(n_1/n_0)^{1/3}$. The broadening of the spectrum and the increase of $\Sigma\,|\,E_{kj}\,|^2$ lead to the growth of Δv, and the instability develops into the kinetic phase.

Thus, the investigation of the nonlinear dynamics of the hydrodynamic instability carried out on the basis of Eqs. (14)–(16) confirms the qualitative picture of the process developed in[1] with the aid of the equations of the quasilinear approximation.

3. Let us turn now to the study of the interaction of

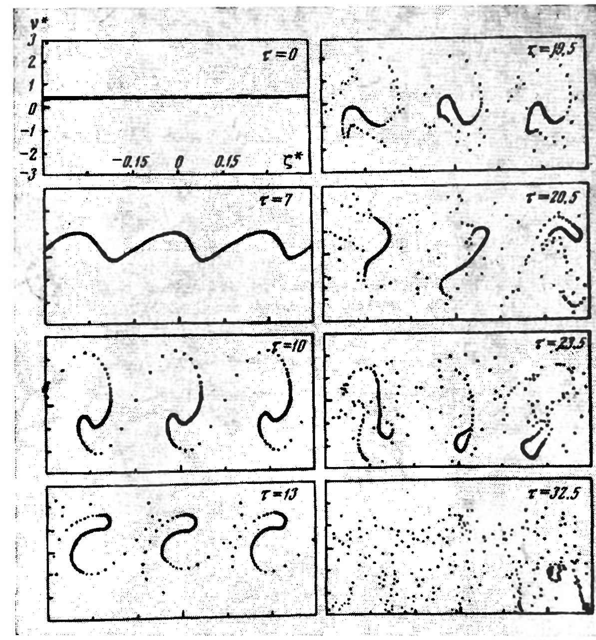


FIG. 6. Dynamics of the phase plane of the beam for the case of three interacting waves for δ - 2 and different τ. The phase plane is drawn in the reference frame of the fundamental wave: $\nu^* = \nu + d\alpha_0/d\tau$, $\zeta^* = \zeta + (2\pi)^{-1}\alpha_0$, $\zeta = z/L$.

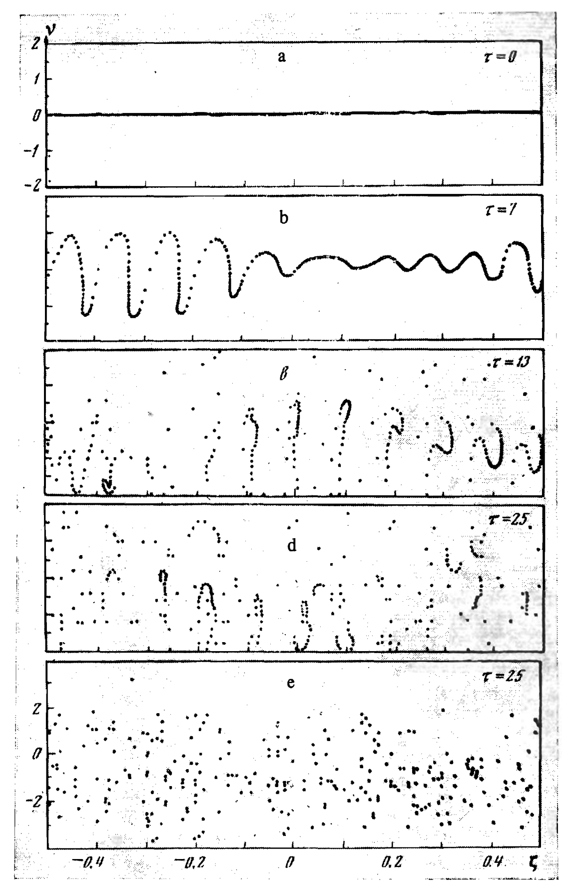


FIG. 7. Dynamics of the phase plane of the beam for $\delta = 0.75$ and different τ; a), b), c) and d) are for the case of three waves, e) for five waves. The phase plane is shown in the reference frame moving with the initial velocity v_0 of the beam.

the perturbations in the modulated beam-plasma system. In this case oscillations of frequency ω_0 and amplitude substantially exceeding the fluctuation amplitude are set up at the entrance ($z = 0$) where the beam enters the plasma. We shall assume that the modulation frequency ω_0 coincides with the resonant plasma frequency, i.e., that it satisfies the condition

$$\varepsilon_p\left(\omega_0, k=\frac{\omega_0}{v_0}\right)=1-\frac{\omega_p^2}{\omega_0^2}\left(1+\frac{v_g}{v_0}\right)=0, \tag{20}$$

where $\epsilon_p(\omega, k)$ is the dielectric constant of the plasma and $v_g = 3v_T^2 k/\omega \approx 3v_T^2/v_0$ is the group velocity of the plasma oscillations.

Let us consider the interaction of the wave generated when the beam is modulated with a test wave of frequency ω close to ω_0. As a result of the interaction there develop in the plasma perturbations of frequencies

$$\omega_j=\omega_0+j(\omega-\omega_0),\quad j=0, \pm 1, \ldots \tag{21}$$

also belonging to the packet of the most unstable oscillations with a linear amplification factor close to the maximum:

$$\varkappa_{\max}=3^{1/2}\frac{\omega_p}{v_0}\left(\frac{n_1 v_0}{16 n_0 v_g}\right)^{1/3}. \tag{22}$$

We now seek the electric fields of the individual perturbations in the form

$$E_j(t, z)=E_j(z)\sin\left[\omega_j(z/v_0-t)+\alpha_j(z)\right]. \tag{23}$$

For the amplitude $E_j(z)$ and phase $\alpha_j(z)$ of the perturbations we obtain, under the same conditions as in the preceding section, i.e., assuming the plasma oscillations to be linear and neglecting, for $(n_1 v_0/n_0 v_g)^{1/3} \ll 1$, the excitation of higher harmonics of the electric field with frequencies $s\omega_j$ ($s \geq 2$), the following system of equations:

$$v_g\frac{dE_j}{dz}=\frac{4\pi e n_1 v_0}{T}\int_{-T/2}^{T/2}\sin[\alpha_j(z)-\omega t'(z, t(0))]\,dt(0), \tag{24}$$

$$v_g\left[\frac{d\alpha_j}{dz}-j\frac{\omega-\omega_0}{v_g}\right]E_j=\frac{4\pi e n_1 v_0}{T}\int_{-T/2}^{T/2}\cos[\alpha_j(z)-\omega t'(z, t(0))]\,dt(0). \tag{25}$$

In these equations $t' = t - z/v_0$, $t(0)$ is the time of flight into the plasma of beam particles which are at the point z by the time t, T is the general period of the perturbations (it is assumed that the frequencies of the perturbations can be represented in the form $\omega_0 = 2\pi m/T$, $\omega = 2\pi n/T$, where m and n are integers). We neglected in deriving Eqs. (24) and (25) small quantities $\sim v'/v \sim \kappa v_0/\omega_0 \ll 1$ (v' is the vibrational velocity of the beam particles). The equation of motion of the beam particles

$$m\frac{dv}{dt}=-e\sum_j E_j\sin(\alpha_j-\omega t')$$

can be written with the same accuracy in the form

$$\frac{d^2t'}{dz^2}=-\frac{e}{m v_0^3}\sum_{j=-N}^{N}E_j\sin[\alpha_j-\omega t']. \tag{26}$$

Equations (24)—(26) describe the spatial amplification of a packet of $2N + 1$ unstable oscillations under a steady injection of an electron beam into the plasma. In terms of the dimensionless variables

$$\zeta=\frac{\omega_p z}{v_0}\left(\frac{n_1 v_0}{n_0 v_g}\right)^{1/3},\quad \tau=-\frac{t'}{T},\quad \tau_0=-\frac{t(0)}{T},$$

$$\delta=\frac{\omega-\omega_0}{\omega_0(n_1 v_g^2/n_0 v_0^2)^{1/3}},\quad \nu=2\pi m\frac{d\tau}{d\zeta}=\frac{v-v_0}{v_0(n_1 v_0/n_0 v_g)^{1/3}}, \tag{27}$$

$$\varepsilon_j=E_j\left[4\pi n_1 m v_0^2\left(\frac{n_1 v_0^4}{n_0 v_g^4}\right)^{1/3}\right]^{-1/2}$$

the system (24)—(26) reduces to the universal system (14)—(16) when the dimensionless time and coordinate are interchanged, $\tau \longleftrightarrow \zeta$. The dimensionless units for the electric field are determined from the condition that in the nonlinear regime (i.e., for $\epsilon_j \sim 1$) the beam particles should be trapped in the potential wells created by the excited waves. In that case

$$(e\varphi_0/m)^{1/2} \sim |v_0-\omega/k| \sim v_0\varkappa/k.$$

The energy of the vibrations excited by the beam in the stationary problem is equal to

$$\sum_k\frac{|E_k|^2}{4\pi}=n_1 m v_0^2\left(\frac{n_1 v_0}{n_0 v_g}\right)^{1/3}\frac{v_0}{v_g}\sum_j\varepsilon_j^2. \tag{28}$$

The factor v_0/v_g in this formula reflects the possibility of a buildup of the vibrations during the injection of the beam into the plasma, on account of which the vibrational energy density may, for $v_g \ll v_0$, substantially exceed the energy density in the beam[7].

We present the results of the solution of Eqs. (14)—(16) for the case when, owing to the modulation, the initial amplitude $\epsilon_{\omega_0}(0)$ of the fundamental wave substantially exceeds the initial amplitudes of the test waves. The test waves arise as a result of thermal fluctuations in the plasma and, for them,

$$\varepsilon(0) \sim \left(\frac{\omega_p^3}{n_1 v_0^3}\frac{v_g}{v_0}\right)^{1/2} \sim 10^{-3}-10^{-4}.$$

Figures 8a and 8b show the nonlinear dynamics of three unstable perturbations in the case when the initial amplitude of the fundamental wave exceeds the amplitudes of the test waves by only an order of magnitude. Under these conditions a monochromatic wave of frequency equal to the modulation frequency is excited at any value of the "detuning" δ. The wave leads to the breaking up of the beam into bunches of trapped particles synchronously rotating in the phase plane, the

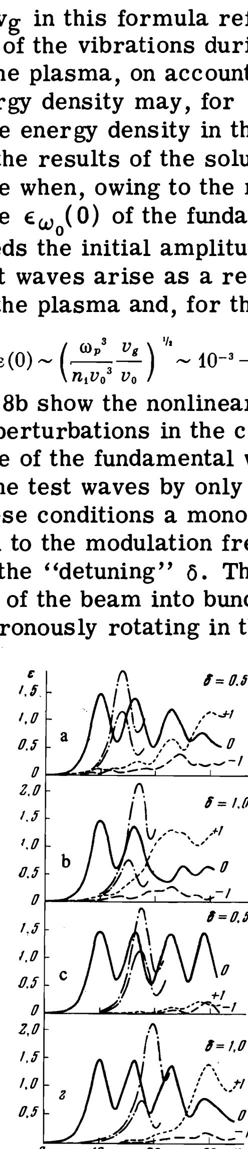


FIG. 8. The function $\epsilon(\zeta)$ for the case of three waves when the beam is initially modulated. In a) and b) $\epsilon_{j=0}(0) = 10^{-2}$, $\epsilon_{j=\pm 1}(0) = 10^{-3}$; in c) and d) $\epsilon_{j=0}(0) = 10^{-2}$, $\epsilon_{j=\pm 1}(0) = 10^{-4}$. The dot-dash curves show the dependence on ζ of the amplitudes $\epsilon_{j=\pm 1}(\zeta)$ of the test waves for the case when the fundamental wave is absent.

test perturbations being in this case suppressed (for comparison the growth of the test waves in the absence of the fundamental wave is shown by the dot-dash curves in Fig. 8). Only at sufficiently large distances $l_0 \sim (20-30)\kappa^{-1}$ (depending on the value of δ) from the point where the beam enters the plasma does the instability of the monochromatic wave become significant and the excitation of the slow satellite take place. For a larger difference between the initial amplitudes of the fundamental and test waves the distance over which the wave excited by the beam remains monochromatic can be considerably increased (in Figs. 8c and 8d), when $\epsilon_{\omega_0}(0) = 10^{-2}$ and $\epsilon_{\omega}(0) = \epsilon_{2\omega_0-\omega}(0) = 10^{-4}$, the distance is $l_0 \sim (30-40)\kappa^{-1}$).

The result obtained confirm the possibility of controlling the spectrum of the vibrations excited in the plasma by the beam upon a preliminary modulation of the beam by vibrations of sufficiently small amplitude.

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