

## EXCITED MOLECULES IN A MEDIUM WITH NEGATIVE PERMITTIVITY

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The behavior of excited molecules in a medium with negative permittivity is considered. It is shown that the fields of the individual molecules can add up coherently if the concentration of the active molecules is maintained at a sufficiently high level and the electromagnetic energy is removed by dissipation in the medium or emission into free space. The expounded method of producing coherent electromagnetic fields has the advantage that in a medium with negative permittivity spontaneous emission is strongly suppressed and, hence, intense transitions can be used as working transitions. Various media are discussed whose permittivity becomes negative in isolated regions of the spectrum, and the possibility of using a periodic structure with an opacity band as such a medium is also demonstrated.

THE spectrum of the electromagnetic oscillations of free space is, as is well known, continuous. Although an optical (or open) resonator leaves the spectrum continuous, it makes the amplitudes of the waves of the continuous spectrum critically dependent on frequency. Such a spectrum is sometimes called quasi-discrete. One usually tries to make it such that the peaks of the field amplitude are as widely spaced and as narrow as possible. In this case, however, in the spectrum remain many resonances with  $Q$  of the order of unity. In these resonances—into these low- $Q$  oscillations—occurs the spontaneous emission of excited molecules.

We can, generally speaking, effect much more drastic changes in the electromagnetic spectrum. Thus, for example, we can create in the spectrum the so-called opacity band i.e., a frequency band in which there are no waves of the continuous spectrum at all. An example of such a band may be the opacity band of a periodic structure or the opacity band of a medium with negative permittivity. The present work is devoted to the investigation of such type of systems.

The individual molecule whose transition frequency falls into the opacity zone goes over, after excitation, into the so-called dynamical state<sup>[1,2]</sup>. An ensemble of molecules in such a medium will behave somewhat differently. The behavior of the molecules depends on whether or not the fields connected with the dynamical state of the individual molecules can add up coherently. The condition for the coherent addition of the fields is the presence of an electrodynamic resonance, i.e., of a natural oscillation. If an electrodynamic resonance is absent, then the fields of the dynamical states of the individual molecules will extinguish each other, and out of these fields will remain only small fluctuations. If, on the other hand, the fields add up coherently, then interacting with these fields, the excited molecules produce a variety of the coherent Dicke state<sup>[3]</sup>. In the Dicke case the fields can radiate into free space and, therefore, the coherent state of the emitting molecules decays quite rapidly. In our case the radiation is trapped in the medium and, consequently, a steady state, i.e., a prolonged existence of the coherent state, is possible.

The presence of a natural oscillation in the opacity zone is possible only in the case when, thanks to their own contribution to the permittivity  $\epsilon$ , the excited mole-

cules produce a small opacity band in which with the appropriate dimensions and shape of the region a resonance can be produced. This natural oscillation will be completely isolated with respect to the spectrum and has other interesting distinctive features, which we shall consider below.

The most well-known example of a medium whose  $\epsilon$  can become negative is a plasma. In fact, its permittivity is negative at frequencies less than the plasma frequency. There are other examples of such media, but we shall return to this question later.

We shall first elucidate the condition for the appearance of a resonance, and then discuss the results and make some estimates to demonstrate the possibility of an experimental realization of the regime in question.

## THE APPEARANCE OF A RESONANCE

Let us consider a system consisting of a large number of two-level molecules placed in a medium with  $\epsilon < 0$  and interacting with one another through an electromagnetic field. For a large number of molecules in the system, the electromagnetic field can be regarded as unquantized and interacting with the macroscopic dipole moment of the system of molecules. The motion of the macroscopic dipole moment  $\mathbf{P}$  of the active molecules contained in a unit volume is described by the equations<sup>[4]</sup>

$$\frac{\partial^2 \mathbf{P}}{\partial t^2} + \frac{2}{T_2} \frac{\partial \mathbf{P}}{\partial t} + \omega_0^2 \mathbf{P} = -2NE' \frac{\omega_0 |\mu|^2}{\hbar}, \quad (1)$$

$$\frac{\partial N}{\partial t} + \frac{1}{T_1} (N - N_0) = \frac{2}{\hbar \omega_0} \mathbf{E}' \frac{\partial \mathbf{P}}{\partial t}, \quad (2)$$

where  $\mathbf{E}'$  is the effective field acting on the dipole,  $N$  and  $N_0$  are the inverse population densities in the presence and absence of the field, and  $\omega_0$  and  $\mu$  are the transition frequency and dipole moment of the molecules. Equations (1) and (2) take into account the presence in the system of molecules of longitudinal and transverse relaxations (the respective relaxation times are  $T_1$  and  $T_2$ ). During the times  $T_1$  and  $T_2$  we neglect spontaneous emission, which, in a medium with  $\epsilon < 0$ , is strongly suppressed. The small amount of spontaneous radiation is an important feature of the regime under consideration, for in a number of cases this radiation is a serious source of energy losses. In the

investigations of the spontaneous radiation and the dynamical state in a periodic structure<sup>[1,2]</sup>, spontaneous emission was the sole relaxation process. The presence of other sources of relaxation constitutes an additional difference between the problem being considered now and that investigated in<sup>[1,2]</sup>.

The presence of relaxation implies that the maintenance of some steady state calls for replenishment of the relaxation losses, i.e., pumping. Pumping is allowed for in Eqs. (1) and (2), or, more precisely, it is assumed that  $N_0 > 0$  in the system of molecules under consideration. The medium is then an amplifying medium. For  $N_0 < 0$  it is an absorbing medium.

In Eqs. (1) and (2) the primed quantity stands for the effective electric field acting on a molecule. Its connection with the electric field  $\mathbf{E}$  entering into the Maxwell equations is, as is well-known<sup>[4]</sup>, determined by the relation

$$\mathbf{E}' = \mathbf{E} \left( 1 + \frac{\epsilon - 1}{3} \right) + \frac{4\pi}{3} \mathbf{P}, \quad (3)$$

where  $\epsilon < 0$  is the permittivity of the medium,  $\mathbf{P}$  is the polarization due to the active molecules. We shall find  $\mathbf{P}$  later from Eqs. (1) and (2).

We shall assume that the active molecules are concentrated near the origin of coordinates. Although the physical results do not depend on how the concentration of the active molecules falls off with distance from the origin, it is convenient for the mathematical investigation of the problem to assume that the concentration is constant in a sphere of radius  $R$  and equal to zero outside this sphere (Fig. 1).

Since we are interested in those regimes when  $\mathbf{E}'$  and  $\mathbf{P}$  depend on the time harmonically or almost harmonically, the right-hand side of Eq. (2) will contain two terms: one which is constant and a second which oscillates with twice the transition frequency. Correspondingly,  $N$  will also have two components—a constant and an oscillating component; the second will, however, be smaller than the first by a factor  $\omega T_1 \sin \varphi \gg 1$  ( $\varphi$  is the phase difference between  $\mathbf{E}'$  and  $\mathbf{P}$ ; as we shall subsequently see,  $\varphi \sim \pi/2$  in the interesting cases). Since the oscillating part is small, we shall neglect it and, consequently, in this approximation the inverted population  $N$  is constant. Then Eqs. (1) and (2) are linear, and their harmonic solution is easily found. Clearly, the same approximation is suitable in those cases when  $N$  is a slowly varying function of the time; this will be the case when the frequency has a small imaginary correction.

When the time dependence of the fields is of the form  $e^{-i\omega t}$ , the Maxwell equations containing the curl assume the following form:

$$\text{rot } \mathbf{E} = ik\mathbf{H}, \quad \text{rot } \mathbf{H} = -ik\epsilon_{1,2}\mathbf{E}; \quad (4)$$

$$\epsilon_1 = \epsilon_0 + i\epsilon' + 4\pi\chi \quad \text{for } r < R. \quad (5)$$

$$\epsilon_2 = \epsilon_0 < 0 \quad \text{for } r > R, \quad (6)$$

where  $\chi$  is the susceptibility of the active molecules. The equations with the divergence are, for a harmonic dependence on the time, automatically satisfied.

Since the system we are considering possesses spherical symmetry, the Eqs. (4) are satisfied by the following fields (of the electric type):

$$\begin{aligned} E_r &= \frac{n(n+1)}{iker} UP_n^{(m)}(\cos \vartheta) e^{im\varphi}, & E_\vartheta &= \frac{1}{iker} \frac{d(rU)}{dr} \frac{\partial}{\partial \vartheta} [P_n^{(m)}(\cos \vartheta)] e^{im\varphi}, \\ E_\varphi &= \frac{im}{iker \sin \vartheta} \frac{d(rU)}{dr} P_n^{(m)}(\cos \vartheta) e^{im\varphi}, & H_r &= 0, \\ H_\vartheta &= -\frac{im}{\sin \vartheta} UP_n^{(m)}(\cos \vartheta) e^{im\varphi}, & H_\varphi &= U \frac{\partial}{\partial \vartheta} [P_n^{(m)}(\cos \vartheta)] e^{im\varphi}, \end{aligned} \quad (7)$$

where  $P_n^{(m)}$  are Legendre polynomials and  $U = U(r)$  is a function satisfying the equation

$$\frac{d^2 U}{dr^2} + \frac{2}{r} \frac{dU}{dr} - \frac{n(n+1)}{r^2} U + k^2 \epsilon_{1,2} U = 0. \quad (8)$$

The solution to this equation has the form

$$U = r^{-1/2} \Phi_{n+1/2}, \quad (9)$$

where  $\Phi_{n+1/2}$  is the Bessel function of half-integer order. For  $n = 0$  we obtain for the fields a solution that vanishes identically. Therefore, we set  $n = 1$  (and  $m = 0$ ) and restrict ourselves to this case. The solution to Eq. (8) for  $n = 1$  and  $r < R$  without any singularity at the origin has the following form<sup>[5]</sup>:

$$U_1 = \frac{U'}{r} \left( \frac{\sin k \sqrt{\epsilon_1} r}{k \sqrt{\epsilon_1} r} - \cos k \sqrt{\epsilon_1} r \right). \quad (10)$$

In this expression the quantity  $k\sqrt{\epsilon_1}$  can be complex. For  $r > R$  ( $\epsilon_0 < 0$ ), we have the solution<sup>[5]</sup>

$$U_2 = \frac{U''}{r} \exp(-k \sqrt{-\epsilon_2} r) \left( 1 + \frac{1}{k \sqrt{-\epsilon_2} r} \right). \quad (11)$$

According to the Maxwell equations, all the components of the fields, with the exception of the radial component of the electric field, are continuous on the surface of the sphere. On the surface of the sphere the radial component of the electric field is inversely proportional to the ratio of the permittivities of the media. As can be seen from the relation (7), these boundary conditions may be satisfied by requiring

$$U_1(k\sqrt{\epsilon_1}R) = U_2(k\sqrt{-\epsilon_2}R), \quad (12)$$

$$\frac{1}{\epsilon_1} \frac{d(rU_1)}{dr} \Big|_{r=R} = \frac{1}{\epsilon_2} \frac{d(rU_2)}{dr} \Big|_{r=R}. \quad (13)$$

Taking (10) and (11) into account, we obtain from these relations

$$U' \left( \frac{\sin k \sqrt{\epsilon_1} r}{k \sqrt{\epsilon_1} r} - \cos k \sqrt{\epsilon_1} r \right) = U'' \left[ \exp(-k \sqrt{-\epsilon_2} r) \left( 1 + \frac{1}{k \sqrt{-\epsilon_2} r} \right) \right], \quad (14)$$

$$\begin{aligned} & \frac{U'}{\epsilon_1} \frac{d}{dr} \left( \frac{\sin k \sqrt{\epsilon_1} r}{k \sqrt{\epsilon_1} r} - \cos k \sqrt{\epsilon_1} r \right) \Big|_{r=R} \\ &= \frac{U''}{\epsilon_2} \frac{d}{dr} \left[ \exp(-k \sqrt{-\epsilon_2} r) \left( 1 + \frac{1}{k \sqrt{-\epsilon_2} r} \right) \right] \Big|_{r=R}. \end{aligned} \quad (15)$$

The right-hand sides of these relations have been determined with a high degree of accuracy. Indeed, since the frequency can change only within the limits of the line width, and there are no rapidly varying functions

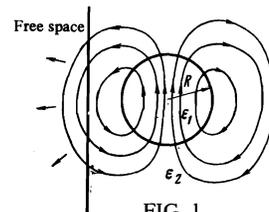


FIG. 1

on the right-hand sides, the relative correction to the right hand sides due to the indeterminacy in the frequency will be of the order of  $\Gamma/\omega \ll 1$ , where  $\Gamma$  is the line width. A similar argument is not applicable to the left-hand side, since this side contains the function  $\epsilon_1$  that varies rapidly with frequency. Small frequency changes within the limits of the line width can change considerably the quantity  $k\sqrt{\epsilon_1}r$ . Since any imaginary correction to the right-hand side must be extremely small, the left-hand side should be practically real. But the left-hand side takes on real values only when the arguments are real. Whence we can conclude that the root of the characteristic equation should be real, and the entire investigation of the characteristic equation can be carried out on the real axis.

Notice now that if the functions to be matched have the same sign, then their derivatives should have different signs ( $\epsilon_2 < 0$ ). Since the function  $rU_2$  has a negative derivative, it can be matched with the function  $rU_1$ , according to (14) and (15), only in the region to the left of the peak of the function  $rU_1$  (Fig. 2), where this function and its derivative are positive. Thus, the upper bound of the root is evident. We now give the lower bound. Let us introduce the dimensionless variable  $\rho = k\sqrt{\epsilon_1}r$  and divide Eq. (15) by (14). After obvious transformations we obtain

$$\frac{\rho - \text{tg } \rho + \rho^2 \text{tg } \rho}{\rho(\text{tg } \rho - \rho)} = \frac{1 + \alpha\rho + \alpha^2\rho^2}{\alpha(\alpha\rho + \alpha^2\rho^2)}, \quad (16)$$

where  $\alpha = \sqrt{-\epsilon_2}/\sqrt{\epsilon_1}$ . It is easy to see that the right-hand side of this equation decreases monotonically with increasing  $\alpha$ . We shall show that even for moderate  $\alpha$  ( $\alpha^2 = 10$ ) the left hand side is larger than the right when  $\rho < \pi/2$ , i.e., the root of the characteristic equation is larger than  $\pi/2$ . At larger values of  $\alpha$  the root will be even closer to the maximum of the function  $rU_1$  ( $\rho = \rho_m$ ).

For small  $\rho$  the left-hand side of Eq. (16) is equal to  $2/\rho$  and the left, to  $1/\alpha\rho$ , i.e., the left-hand side is greater than the right. We can convince ourselves by a direct verification that the left-hand side is larger than the right by a factor of at least up to  $\pi/2$ . Thus, the root of the characteristic equation lies between  $\pi/2$  and  $\rho_m$ . Consequently, the frequency corresponding to the natural oscillation is determined by the relation

$$k^2\epsilon_1 R^2 = \rho_0^2, \quad (17)$$

where  $\rho_0$  is the root of Eq. (16) ( $\pi/2 < \rho_0 < \rho_m$ ). The frequencies corresponding to the other roots of Eq. (16) do not fall in the amplification band of the active molecules and are therefore not of interest. In principle, we can, by varying  $R^2$ , tune in to another root, but then all the other roots will fall outside the amplification band, provided we are not dealing with a root of very high number.

According to (1) and (3), the polarization and the field

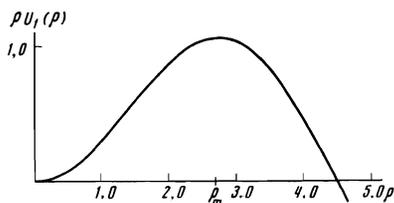


FIG. 2

are connected by the relation

$$\mathbf{P} = \frac{-QN[1 + 1/3(\epsilon_0 - 1)]}{(\omega_0^2 - \omega^2) + 1/3\pi QN - 2i\omega/T_2} \mathbf{E}, \quad (18)$$

where

$$Q = 2\omega_0 |\mu|^2 / \hbar > 0. \quad (19)$$

Consequently, we obtain for the permittivity the expression

$$\epsilon = \epsilon_0 + i\epsilon' - \frac{4\pi QN(1 + 1/3(\epsilon_0 - 1))}{(\omega_0^2 - \omega^2) + 1/3\pi QN - 2i\omega/T_2}. \quad (20)$$

The term  $1/3\pi QN$  in the denominator of (20) shifts the resonant frequency only insignificantly, and we shall neglect it. Further, by taking into account the fact that  $\omega \sim \omega_0$ , we can write (17) in the form

$$\bar{\omega}^2 \left( \epsilon_0 + i\epsilon' + \frac{\gamma}{\omega - \omega_1} \right) = \bar{\omega}^2, \quad (21)$$

where  $\gamma = 16/3\pi QN\omega_0^{-1}$ ,  $\bar{\omega} = \rho_0 cR^{-1}$ , and  $\omega_1 = \omega_0 - i/T_2 = \omega_0 - i\Gamma$ . To solve this equation we set  $\omega = \omega_1 + \Omega e^{i\varphi}$ . Then the equation assumes the form

$$\epsilon_0 + i\epsilon' + \frac{\gamma}{\Omega} (\cos \varphi - i \sin \varphi) = \frac{\bar{\omega}^2}{\omega_0^2}, \quad (22)$$

where instead of  $\omega$  we have taken  $\omega_0$  on the right-hand side, since the right hand side depends weakly on  $\omega$ . Equating separately the real and imaginary parts, we obtain the relations

$$\epsilon' = \frac{\gamma \sin \varphi}{\Omega}, \quad \epsilon_0 = \frac{\bar{\omega}^2}{\omega_0^2} - \Omega^{-1} \gamma \cos \varphi, \quad (23)$$

from which it is easy to find  $\Omega$  and  $\tan \varphi$ :

$$\Omega = \frac{\gamma}{\epsilon'} \left[ 1 + \frac{\xi^2}{\epsilon'^2} \right]^{-1/2}, \quad \text{tg } \varphi = \frac{\epsilon'}{\xi}, \quad \xi = \frac{\bar{\omega}^2}{\omega_0^2} - \epsilon_0. \quad (24)$$

The obtained equalities (24) essentially solve the set problem. They determine the frequency at which the radiations of the individual molecules add up coherently. We shall draw all subsequent conclusions precisely from these formulas.

For a steady or a rising regime to be possible,  $\gamma$  should be sufficiently high. Indeed, in such a regime we should have  $\Omega \sin \varphi > \Gamma$ . Even if  $\sin \varphi = 1$ ,  $\gamma$  should be sufficiently large for  $\Omega$  to be larger than  $\Gamma$ . This means that the concentration of the active molecules should exceed a certain minimum level. We obtain from the relations (24)

$$\Omega \sin \varphi = \frac{\gamma}{\epsilon'} \left[ 1 + \frac{\xi^2}{\epsilon'^2} \right]^{-1}. \quad (25)$$

An interesting peculiarity of this regime follows from this. For a steady or a rising regime to be possible, it is necessary to introduce losses. As the losses ( $\epsilon'$ ) decrease the quantity  $\Omega \sin \varphi$  decreases and becomes smaller than  $\Gamma$ . The quantity  $\Omega \sin \varphi - \Gamma$  is a maximum for the following losses:

$$\epsilon' = \xi \equiv \bar{\omega}^2 / \omega_0^2 - \epsilon_0. \quad (26)$$

We then have

$$\Omega \sin \varphi - \Gamma = \frac{\gamma}{2\xi} - \Gamma \quad (27)$$

and, consequently, the minimum value of  $\gamma$  at which the steady regime is still possible is equal to

$$\gamma = 2\Gamma\xi. \quad (28)$$

We must note that the losses (the quantity  $\epsilon'$ ) are a controlled quantity. Indeed, any connection with the outside space can be considered as losses, i.e., as growth of  $\epsilon'$ . Therefore, in those cases when the losses in the medium are too small, they must be increased by connection with the outside space. Of course, the term "losses" is in this case not quite appropriate; they are useful losses.

It is easy to show that the steady regime is a stable one. If for some reason the field  $\mathbf{E}$  increases, then, according to Eq. (2), this leads to a reduction in  $N$  and, consequently, to a decrease in the amplification. As a result, the field decreases to the steady-state value.

## DISCUSSION OF THE RESULTS

We show in the present paper that a medium with negative permittivity is of great importance for the investigation of the properties of radiating objects. In particular, we investigate in the paper the nature of the electromagnetic field accompanying a system of excited molecules. It is shown that the existence of a steady regime is uniquely connected with the presence of damping of the electromagnetic field, i.e., either with emission of electromagnetic waves into free space, or with their absorption in the medium.

Let us now consider specific examples of media whose permittivity can become negative. As has already been noted, a plasma is one of such media. However, at the densities attainable today, it can, as a medium with  $\epsilon < 0$ , be of interest only in the range of wavelengths longer than 1–0.5 mm. The other media with  $\epsilon < 0$  which are of interest may be divided into three classes: a) regions close to resonance lines in gases, b) metals and semiconductors, and c) artificial media with  $\epsilon < 0$ .

a) Opaque regions (regions with  $\epsilon < 0$ ) can be observed near resonance lines in gases. One of the first observations of such a region in mercury vapor was made by Wood<sup>[6]</sup>. As is known, in the vicinity of a resonance line the real and imaginary parts of  $\epsilon$  have the following form:

$$\operatorname{Re} \epsilon = \epsilon_0 - \frac{2\pi e^2 f N}{m\omega_0(\omega - \omega_0)}, \quad \operatorname{Im} \epsilon = \epsilon' + \frac{2\pi e^2 f N}{m\omega_0} \frac{\Gamma}{(\omega - \omega_0)^2},$$

where  $f$  is the oscillator strength of the transition at the frequency  $\omega_0$  and  $N$  is the molecule concentration. Let us estimate the distance (in  $\text{cm}^{-1}$ ) from the resonance at which  $\operatorname{Re} \epsilon$  becomes equal to zero. The transition  $6^1S_0 - 6^1P_1$  occurs in BaI at the wavelength  $\lambda = 5535 \text{ \AA}$  and with the oscillator strength  $f = 1.4$ <sup>[7]</sup>. For this transition (for  $N = 1.5 \times 10^{17} \text{ cm}^{-3}$ )  $\Delta(1/\lambda) \cong 0.52 \text{ cm}^{-1}$ . We recall that isotopic shifts are exactly equal to such values: 0.1–0.5  $\text{cm}^{-1}$ , and barium has five stable isotopes. Absorption at such a distance from the resonance is negligible:  $\operatorname{Im} \epsilon \sim \Gamma/\Delta\omega$ . The line width determined by collisions is roughly equal to  $6 \times 10^5 \text{ sec}^{-1}$  and, consequently,  $\operatorname{Im} \epsilon \sim 6 \times 10^{-6}$ . If  $\epsilon' \sim 0.1$ , then the concentration of the atoms of the active isotope should, according to (27), be equal to  $10^{14} - 10^{15} \text{ cm}^{-3}$ . The pumping power is then  $\sim N\hbar\omega_0\Gamma \sim 200 \text{ W}$ . Such pumping is easily feasible. Figure 3 shows the dependence of the permittivity on frequency and the position of the sole resonance in the case when an inversion is realized on the atoms of the impurity isotope.

Opacity bands can be observed not only in gases, but

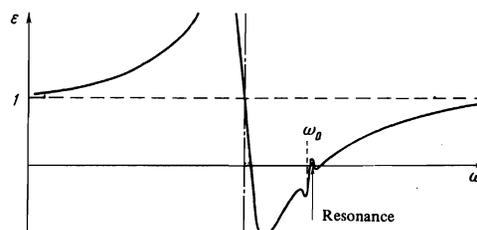


FIG. 3

also in crystals, where they should lie close to the absorption lines of the impurity atoms. We can expect the relaxation times  $T_1$  in crystals to be longer than in gases, especially at low temperatures.

b) Opacity bands connected with the presence of free carriers can also be observed in metals and semiconductors. Electrons in metals and semiconductors form a plasma of high concentration. The opacity band of metals extend from the lowest frequencies to the soft x-ray region. The realization in metals and semiconductors of the regime we have considered will apparently be difficult, due to their comparatively high resistance and the complications connected with pumping.

c) Of great interest is the creation of artificial media with effective  $\epsilon < 0$ . Among them are periodic dielectric structures. If the transition frequency of the active molecules falls within the opacity zone of the periodic structure, then the radiation of these molecules will, as in a medium with  $\epsilon < 0$  be trapped. The fields of the individual molecules may add up coherently, if the contribution made by the active molecules to the permittivity is so large that the periodic dependence of the permittivity on the coordinates vanishes or is sharply reduced in a small frequency range within the limits of the opacity band. Then in this small frequency range may arise a natural oscillation.

Three-dimensional periodic dielectric structures have not at present been investigated to such a degree that we can assert that opacity bands exist in them for all directions of propagation of electromagnetic waves. However, there hardly can be fundamental laws that will contradict such a possibility, for if there were such a contradiction, it would apply in the case of the opacity bands described in Secs. a) and b).

## CONCLUSIONS

1. It has been shown in the paper that media with negative permittivity are of great interest from the point of view of the investigation of the properties of radiating objects.
2. At a sufficiently high concentration, excited molecules occupying a region of appropriate dimensions and shape form a macroscopic dipole oscillator that oscillates together with the electromagnetic field with roughly the transition frequency of the molecules.
3. Steady or growing oscillations of such an oscillator are possible only when the electromagnetic field is damped, i.e., when it is emitted into free space, or when it is absorbed in the material.
4. It has been shown that from the experimental point of view, of great interest are frequency bands with  $\epsilon < 0$  in the vicinity of intense spectral lines in gases—in particular, the  $\lambda = 5535 \text{ \AA}$  transition in BaI and, possibly,

the  $\lambda = 1850 \text{ \AA}$  transition in HgI.

5. Attention is drawn to the three-dimensional periodic structures, which are artificial media with  $\epsilon < 0$ . If the presence in them of opacity bands is confirmed by subsequent theoretical investigations, then they will be the most flexible media of this sort.

6. The fact that the configuration considered in the paper of the active and passive molecules differs strongly from the traditional configuration (a mirror is an active substance) is of interest from the theoretical point of view. It is possible that there are other interesting configurations.

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130