THE NONLINEARITY MECHANISM OF THE VOLT-AMPERE CHARACTERISTICS OF POINT JUNCTIONS

N. I. BOGATINA and I. K. YANSON

Physico-technical Institute of Low Temperatures, Ukrainian Academy of Sciences

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Local breakdown of an oxide layer results in the short circuit of film tunnel junctions. Consequently, a narrow conducting bridge, which is a variant of a point junction, is formed between the two metal films. Nonlinearity is observed in the volt-ampere characteristics of such junctions in the region of voltages between 0 and 100 mV, and at temperatures above the T_c of the superconducting transitions The cause of the nonlinearity is that the free path length of the electrons accelerated up to the voltage V depends significantly on the voltage if $eV > kT$ and $eV > k\Theta_D$. In this case, the electrons are slowed rapidly when the drift velocity reaches the speed of sound in the metal.

WHEN a film tunnel junction is short-circuited as a result of the local breakdown of the oxide layer, a narrow conducting bridge, which is a variety of a point contact, develops between the two metal films. It was known previously that the volt-ampere characteristic of such contacts is nonlinear under the condition that one or both of the metals is in the superconducting state; however, upon transition to the normal state, the nonlinearity disappears.[1] Nonlinearities of similar types, associated with the residual superconductivity of some part of the point junction formed when a thin wire touches bulk metal, were apparently studied by Khaĭkin and Krasnopolin.[2] The vanishing of the nonlinearity at a temperature $T > 5-6°K$ and upon superposition of a magnetic field was indicative of this.

In the present paper we report observation of nonlinear volt-ampere characteristics of point contacts in the voltage range 0–100 mV, at temperatures above the T_c of the superconducting transition, which are maintained down to nitrogen temperatures. In contrast with the nonlinear volt-ampere characteristic curve described in[3], the observed characteristics cannot be attributed to thermal effects. The cause of the nonlinearity in our opinion is that the free path length of the electron accelerated to a voltage V depends significantly on the value of this voltage if $eV > kT$ and $eV > k\Theta_D$. The necessary condition for the observation of this effect is sufficient smallness of the radius of the contact r (the current density in the contact reaches values of 10^9-10^{11} A/cm^2).

Comparatively high-resistance bridges (~1 ohm) in film tunnel contacts of cross-shaped geometry served as the object of the investigation. These arise directly in liquid helium upon local breakdown of the sample. Films of width 1 mm and length 10 mm were condensed on a substrate cooled to 200°K in a vacuum ($10^{-7}-10^{-6}$ Torr). High purity lead and tin (99.999%) served as the initial material. The free path length at helium temperatures was approximately equal to the film thickness, and amounted to 500–1000 Å. Before the measurements, the films were heated to room temperature and maintained at it for the time necessary to mount the samples in a cryostat (~0.5 hr). The measurements were made at temperatures of 77 and 4.2°K for tin and 8°K for lead, which correspond to the normal states of the metals. The volt-ampere characteristics of the tunnel contacts were recorded with a two-pen potentiometer up to breakdown of the point contacts caused by the short-circuiting of the sample as a result of the breakdown. Furthermore, the temperature dependences of the film resistances in the temperature range $T_c < T < 300°K$ were obtained; these were necessary for the analysis of the experimental data given below.

Typical volt-ampere characteristics of the tin and lead contacts investigated by us are shown in Fig. 1 (curves 1 and 2). These characteristics differ essentially from the volt-ampere characteristics before breakdown, which reveal a decrease in the resistance with increase in the voltage (curve 3). Upon lowering of the temperature from 77 to 4.2°K, the resistance of the point contact falls off, while a small increase is typical of tunnel junctions.

It follows from the work of Sharvin[1] that, in the case when the radius of the contact is comparable with

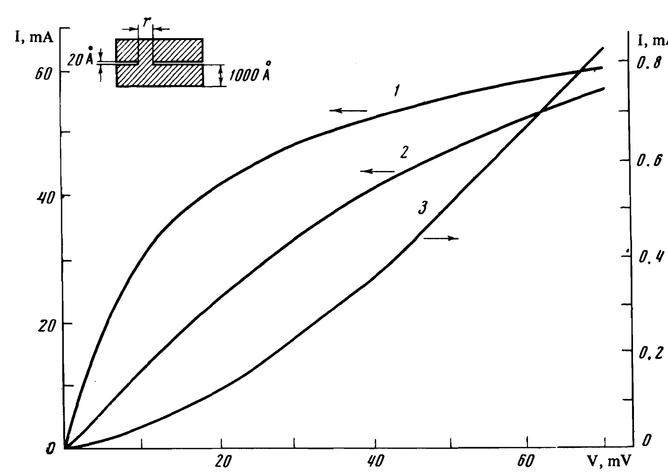

FIG. 1. Volt-ampere characteristics of point and tunnel contacts: 1–Pb point contact, T = 8°K; 2–Sn point contact, T = 77°K; 3–Sn tunnel contact, T = 4.2°K.

the free path length, the following relation is valid:

$$R \approx \frac{p_F}{ne^2 r}\left(\frac{1}{r}+\frac{1}{l}\right)=\frac{l}{\sigma}\left(\frac{1}{r}+\frac{1}{l}\right), \quad (1)$$

where R is the resistance of the point contact, p_F the Fermi momentum, r the dimension of the contact, n the density of electrons, l the mean free path length, and σ the conductivity. We set the ratio σ/l equal to 10^{11} ohm-cm^{-2} in the calculations for tin and lead.[5] The investigated contact had dimensions from 200 to 700 Å, which were determined from Eq. (1) and from the resistance R_0, measured at the temperature 4.2°K and $V < 10$ mV for tin and 8°K and $V < 5$ mV for lead. This simplifies Eq. (1), since the inequality $r \ll l$ is satisfied.

Figure 2 shows the dependences of the resistances of the point contacts on the voltage. The most important feature is the linear dependence of R(V) at high voltages. In spite of the great scatter in the values of V at which the linear dependence is reached, one can note that this quantity for tin contacts is on the average about twice the corresponding value for the lead contacts. Furthermore, both values lie very close to the Debye energies of the corresponding metals, which are denoted by the dashed lines in Fig. 2.

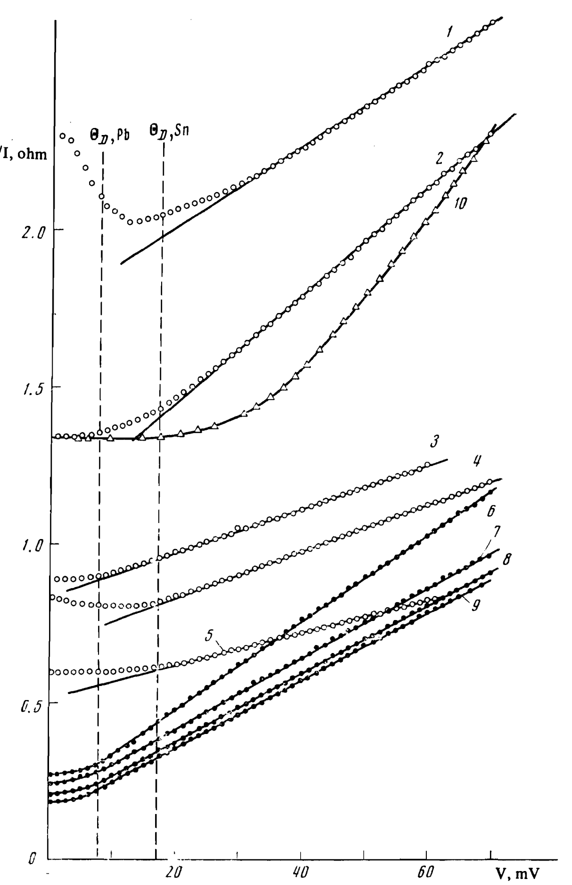

FIG. 2. Dependences of the resistances of point contacts on the voltage. 1–5–for tin; 1–r = 270Å, T = 77°K; 2–r = 270Å, T = 4.2°K; 3–r = 460Å, T = 77°K; 4–r = 490Å, T = 77°K; 5–r = 600Å, T = 77°K. Curves 6-9–for lead: 6–r = 610Å, 7–r = 650Å, 8–r = 700Å, 9–r = 740Å; T = 8°K. Curve 10 is the calculated dependence of the resistance of the Sn contact, r = 270Å, under the assumption that the nonlinear effects appear because of heating.

We shall now show that the observed characteristics cannot be attributed to heating of the metal in the contact region. This is especially important in connection with the fact that in previous experiments[3,6] the observed strong nonlinearities in the electrical conductivity of normal metals were attributed to heating effects. At first glance, and in our case, it would seem that we are dealing with heating of the sample. Actually, assuming that all the heat is released in a sphere of radius $\rho \sim l$ near the contact and is removed by means of the ordinary mechanism of thermal conduction of the metal, we obtain an estimate $\Delta T = q/4\pi k\rho$ for the heating, where $q = IV$ is the power dissipated in the junction, k the coefficient of thermal conductivity. If we substitute typical values $q = 0.002–10^2$W, $k = 1$ W-cm/deg and $\rho = 10^{-5}$ cm, we obtain $T = 20–100°$. Further, for $R(V) \gg R(0)$, $T > \Theta_D/3$, we have $R \sim \alpha\Delta T$ and, consequently, $R \sim \alpha V^2/4\pi k\rho R$, $R^2 \propto V^2$ and $R \propto V$.

However, it is necessary to emphasize that these estimates are valid only if $R(V) \gg R(0)$, $T \gg \Theta_D/3$. In the opposite case, it is necessary at least to take it into account that $R \approx R_0 + \alpha\Delta T$, and this in itself leads to a nonlinear dependence for R(V). Additional care must be taken with the substitution $R \to \alpha\Delta T$, since the R(T) dependence has a more complicated character in the intermediate region of temperatures. Moreover, the dimension of the region in which the heat is liberated is comparable with the thickness of the film, and the heat is carried away mainly by the liquid or gas surrounding the contact. Since the liquid (gas) near the contact may not be at rest, it is impossible to assume that such a situation can only worsen the heat transfer because the thermal conductivity of liquid (gaseous) helium is less than the thermal conductivity of the metal. The presence of convection can considerably improve the heat exchange and in the limiting case of constant replenishment of the liquid (gas) near the contact, calculation shows that the heating is negligibly small. In this connection, we note that in our experiments the film structures were mounted vertically in the cryostat, making convection easier. Thus we arrive at the necessity of a more detailed analysis of the heating effects in our case.

We now calculate the expected dependence of the contact resistance on the voltage, using as an example the tin contact Sn No. 1 (Fig. 2) under the assumption that the entire observed effect is connected with heating, and compare it with the curve observed in the experiment. Since the geometry of the point contacts is such that their length (~20 Å) is much less than the radius r, the dependence of the free path l on the temperature can be obtained by studying the temperature dependences of the resistance of the film $R_f(T)$. Similar information could also be obtained from the direct measurement of the temperature dependence of the resistance of point contacts at V = 0. However, upon increase in temperature, the film resistance in the region near the contact becomes comparable with the contact resistance and measurement with a four-point circuit becomes impossible. In the future, we plan to use thicker films to overcome this difficulty.

Several temperature dependences of the resistances of tin films of different thicknesses are shown in Fig.

3. With proper choice of l, these can all be reduced to a single dependence $l_{ep}(T)$, which enters into the formula

$$\frac{R_f\ (T)}{R_f\ (0)}=\frac{l_{ep}(T)+l_0}{l_{ep}(T)}. \quad (2)$$

Since the free path length in films is equal to

$$l^{-1}=l_0^{-1}+l_{ep}^{-1}(T), \quad (3)$$

the dependence of the resistance of a point junction on the temperature has the form

$$\frac{R(T)}{R(0)}=\frac{l_{ep}(T)+l_0'}{l_{ep}(T)}, \quad (4)$$

where

$$\frac{1}{l_0'}=\frac{1}{r}+\frac{1}{l_0}. \quad (5)$$

For the determination of the constant l_0' and of the proportionality coefficient between the power q released in the contact and the temperature of the heating ΔT, we make use of the fact that the resistance of 2.3 ohm for the contact Sn No. 1 under discussion should obviously be reached at $\Delta T = 77.5^\circ - 4.2^\circ = 73.3^\circ K$ and V = 0. The coefficient of proportionality between q and ΔT we shall assume to be independent of the temperature, since account of its temperature dependence can only increase the divergence and make the determined dependence R(V) even more nonlinear. We thus obtain the dependence of the temperature of heating ΔT on the voltage at the contact (Fig. 4). Knowing the dependence of the supposed temperature of the contact on the voltage, we construct the desired R(V) dependence (curve 10 in Fig. 2) from the $R_f(T)/R_f(0)$ dependences available to us, so choosing l_0' that it passes through the point R = 2.3 ohms at 77.5°K. Thus, in the example of tin contact No. 1, it is seen that the superheating explanation contradicts the R(V) dependence observed in experiment. Similar constructions were also carried out by us for lead and also led to an R(V) curve, that differed essentially from that observed for point contacts.

We now consider in more detail the results of Gayley, Langan, and Kim,[3] since similar nonlinearities in the volt-ampere characteristics were attributed by them to heating. Heating effects were observed in[3] for metallic bridges which did not satisfy the condition that the bridge length be much less than its radius. For example, for film bridges of indium, both the length and width of the bridge were very large and amounted to ~100 μ = 0.1 mm $\gg l$. Consequently, all the heating effects took place in the bridge itself, where the heat was released, and not in the region around the contact, in the abutting bulk pieces of the metal, as is the case in point contacts. The authors of [3] observed the melting and overheating of these bridges under a microscope. Experiments were mainly carried out at room and higher temperatures T_0. It is clear that under these conditions the heating effects are much more significant than at helium and nitrogen temperatures. Finally, from the formula obtained in[3], which connects the temperature of the heating T_m with the voltage at the junction V,

$$V^2 = 10.32 \cdot 10^{-8}\ (T_m^2 - T_0^2)\ \text{J-ohm-sec}^{-1}\text{deg}^{-2} \quad (6)$$

it follows that metals have a high melting temperature T_{melt} should have a more linear volt-ampere characteristic, since the point at which dI/dV = 0 is reached at $T_m = T_{melt}$. In our case, the opposite rule is observed for lead and tin, so that matters never reach the melting stage. Thus, although the shapes of the volt-ampere characteristics observed in the cited work[3] and in our work are similar in several respects (at $V < V(T_{melt})$, it is so for different reasons. Finally, we note that in[3] nonlinearities were also observed at low voltages V(< 0.02 V), which cannot be

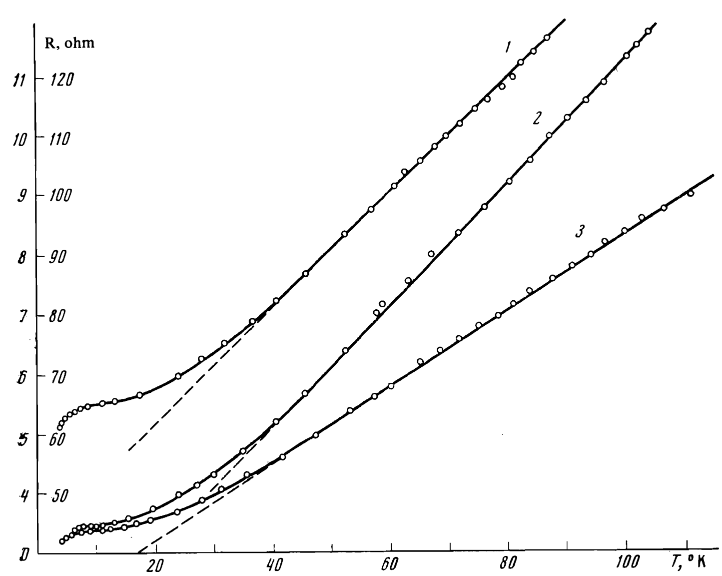

FIG. 3. Temperature dependences of the resistances of tin films of different thicknesses: 1–420Å, 2–1150Å, 3–580Å. The scale to the right of the ordinate corresponds to curves 1 and 3, that to the left, to curve 2.

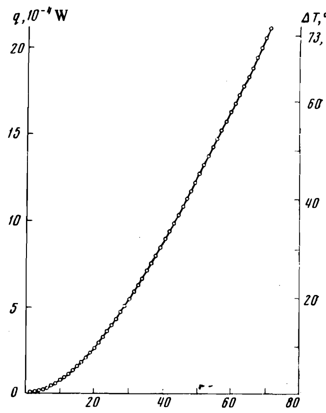

FIG. 4. Dependence of the assumed temperature of the heating ΔT and the power q on the voltage at the Sn contact, r = 270Å, T = 4.2°K.

attributed to the heating, and therefore other possible explanations of the nonlinearity mechanism were not discarded there.

As has been pointed out in[7], electrons emitted by a point contact acquire a large energy in the contact region, which facilitates the process of creation of other quasiparticles (phonons or electron-hole pairs) and leads to a decrease in the mean free path length width increase in the voltage and, consequently, to an increase in the resistance of the point contact. It is seen from Fig. 2 that for $eV > k\Theta_D$, the R(V) dependence acquires a linear character and the values of the energy, beginning with which nonlinearity appears, are about the same as the values of the Debye energies for tin and lead. This allows us to draw the conclusion that the principal process of electron scattering is the creation of phonons.

Using Eq. (1), we construct the dependence of the effective mean free path length of the "hot" electrons in the metal on their maximum energy (Fig. 5). For all samples of the same metal, and at the same temperature, we obtain straight lines that are close to one another and that are independent of the contact resistance; consequently, they are also independent of the power released in them. This also indicates the absence of strong heating in our case. Setting

$$1/l_{ep} = C_3 eV, \quad eV \gg k\Theta_D, \quad \Theta_D > T, \tag{7}$$

we obtain $C_3(\mathrm{Sn}) = 4.0 \times 10^6\ \mathrm{eV^{-1}\text{-}cm^{-1}}$ and $C_3(\mathrm{Pb}) = 8.5 \times 10^6\ \mathrm{eV^{-1}\text{-}cm^{-1}}$. The constant C_3 determined in this manner is proportional to the intensity of the electron-phonon interaction. By experiment, we obtain the correct ratio: $C_3(\mathrm{Pb})/C_3(\mathrm{Sn}) \approx 2$, which agrees with the ratio of the corresponding constants obtained, for example, from measurements of the temperature dependence of the electric conductivity.[8]

We note that the linear dependence of the contact resistance on the voltage applied to it means that for $eV \gg k\Theta_D$, the drift velocity of the electrons

$$v_{\text{др}} \approx eVl/p_F(l+r) = [p_F C_3(r+l)]^{-1}$$

approaches a limit $(p_F C_3 r)^{-1}$ approximately equal to 4×10^5 cm/sec for tin and 2×10^5 cm/sec for lead. In this estimate, we have used the following values: $p_F(\mathrm{Sn}) = 2 \times 10^{-19}$ g-cm/sec, $p_F(\mathrm{Pb}) = 1.6 \times 10^{-19}$ g-cm/sec, r ≈ 450 Å for tin and 670 Å for lead.

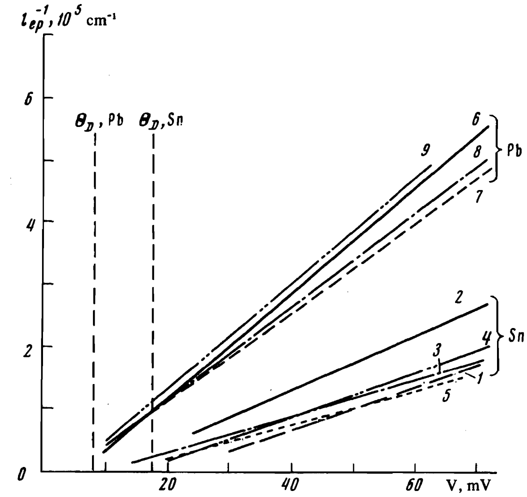

FIG. 5. Dependence of the inverse of the free path length l_{ep}^{-1} on the energy of the "hot" electrons in lead and in tin. The numbers correspond to the curves R(V) in Fig. 2.

Thus, in the case $eV > k\Theta_D$, there exists an intense retardation of the electrons when the drift velocity reaches the value of the sound velocity in the metal. This mechanism was discussed in[2]; however, in that research the energy of the electrons satisfied the opposite inequality.

The nonlinear dependences of $l_{ep}^{-1}(eV)$ obtained in the present research can be explained on the basis of the work of Kaganov, Lifshitz, and Tantarov,[9] in which the following expression was obtained for the amount of energy transferred by the electrons to the lattice per unit time per unit volume:

$$\bar{u} = \frac{\pi^2}{6}\frac{ms^2 n_0}{\tau(\Theta)}, \quad \Theta \gg T_0, \quad T \ll \Theta, \tag{8}$$

where Θ is the temperature of the electron gas, T the lattice and temperature, s the speed of sound, n_0 the density of electrons, and $\Theta_D = T_0$ the Debye temperature. In the given case, it is assumed that the electron gas is strongly heated relative to the lattice. $\tau(\Theta)$ is the time of free flight of the electrons under the condition that the lattice temperature is identical with the temperature of the electrons and is equal to Θ. In our experiments, the temperature of the electron gas in the contact region $\Theta \approx eV$, since, according to Ginzburg and Shabanskiĭ,[10] the equilibrium distribution of the electrons in energy in such a heated gas is established very rapidly. The transfer of the energy to the lattice takes place much more slowly; here the time $\tau(\Theta)$ is decisive. Since, for $eV > k\Theta_D$, $\tau(\Theta) \propto eV$, the contact resistance will depend linearly on the voltage.

Summing up, we emphasize that for point contacts, a real possibility exists for the experimental observation of strong nonlinear effects in the electrical conductivity of normal metals. Considering the heating of contacts of different radius under the condition that the voltage at the contact remains constant, we reach the conclusion that the heating falls off with decreasing contact radius:

$$q = V^2/R \propto V^2 r^2 \quad (r \ll l), \ \Delta T \propto r^2. \tag{9}$$

Thus, the problem of heat dissipation when strong nonlinear effects are observed in the electrical conductivity of normal metals can be solved by using point contacts of small radius and of such geometry that the region in which heat is liberated contains a flow of liquid or gas. The present research represents the first step on developing this solution.

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