Excitonic insulator in magnetic and electric fields

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The properties of an excitonic insulator are considered near the point of the transition into the dielectric phase. It is shown that a formal analogy exists with the properties of a Bose condensate of real excitons. The following characteristic effects are investigated for the case of crossed fields: condensation of the excitons into a state with nonzero momentum, the origin of the dipole moment, and the corresponding abrupt change in the value of the dielectric constant. The interaction of the excitons with acoustic phonons is discussed.

1. As is well known, the excitonic insulator is a phase that can exist at sufficiently low temperatures between the metallic and dielectric phases in a system with small forbidden gaps or overlaps between the valence band and the conduction band. In the present article we investigate the transition between the dielectric and excitonic insulator phases in the presence of crossed electric **E** and strong magnetic **H** fields. (We recall that the exciton phase was observed experimentally by Brandt and Chudinov[1] specifically in a strong magnetic field.) It is shown that the specific dispersion law of an exciton in crossed fields should lead to a noticeable thermodynamic effect: the appearance of an additional dipole moment below the transition point and a corresponding abrupt change in the dielectric constant. The magnitude of the discontinuity in the magnetic susceptibility is also determined.

The Hartree-Fock approximation has been used over the entire range of existence of this phase in all of the articles devoted to the theory of the excitonic insulator (see the review article by Halperin and Rice[2]). The applicability of this approach is obvious from a comparison with the theory of superconductivity only for the case of an excitonic insulator—semimetal transition with a weak interaction.[3] On the other hand, in the system we are considering the interaction is not small (the exciton's binding energy ϵ_0 is of the order of the width d of the forbidden gap), and as a result the Hartree-Fock approximation becomes inapplicable. In this work it is shown that nevertheless there is a region where the Landau theory of phase transitions is valid, and also the expansion in powers of the ordering parameter is analogous to the expansion in powers of the density in the Keldysh-Kozlov theory[4] of the Bose condensation of excitons. In view of the gaseous nature of the expansion, one is only able to obtain quantitative results near the transition point, where the density of the exciton background is small. In addition, the condition that the fluctuations of the ordering parameter—namely, the density of excitons—be small leads to a strong restriction on the transition temperature T_c, which must be small in comparison with the exciton's binding energy ϵ_0. All this restricts the applicability of the theory to the region where the width of the forbidden gap is close to ϵ_0.

2. We consider a semiconductor with a narrow indirect forbidden gap between the valence band v and the conduction band c. We shall assume the spectra of the electrons to be quadratic along the magnetic field. The magnetic field. The magnetic field must be strong enough so that the condition $\lambda \ll R$ is satisfied, where $\lambda = (c/eH)^{1/2}$ denotes the magnetic length, $R = \kappa^2/me^2$ is the Bohr radius of the exciton (κ is the static dielectric constant, m is the reduced longitudinal mass). As was shown by Elliott and Loudon,[5] this condition permits us to neglect transitions to higher Landau bands. Near the band extrema the electrons' spectrum will take the form

$$\varepsilon_{c,v}(p_z, p_x) = \pm 1/2 d \pm p_z^2/2m_{c,v} + Vp_x - m_{\perp c,v}V^2/2,$$

where d is the distance between extrema of the nearest Landau bands (with the spin taken into account), $m_{c,v}$ and $m_{\perp c,v}$ denote the longitudinal and transverse masses of electrons and holes, and V denotes the x-component of the Hall velocity $\mathbf{V} = c(\mathbf{E}\times\mathbf{H})/\mathbf{H}^2 = (V, 0, 0)$ (the z axis is directed along **H**, and the y axis is along **E**).

The exciton phase is characterized by the presence of an exciton background in the ground state, which manifests itself in an intermixing of the electronic states in the c and v bands and leads to the appearance of nondiagonal elements, G_{cv} and G_{vc}, of the one-particle Green's function. We have the following system of Dyson's equations for these nondiagonal elements and also for the renormalized diagonal elements G_c and G_v of the Green's function:

$$\begin{aligned} G_{cv} &= G_c^{(0)}\Sigma_c G_{cv} + G_c^{(0)}\Sigma_{cv}G_v \\ G_v &= G_v^{(0)} + G_v^{(0)}\Sigma_v G_v + G_v^{(0)}\Sigma_{vc}G_{cv}, \end{aligned} \tag{1}$$

and there are analogous equations for G_{vc} and G_c. Here $G_{c,v}^{(0)} = (\epsilon - \epsilon_{c,v}(p_x, p_z) \pm i\delta)^{-1}$ are the unrenormalized Green's functions, and $\Sigma_{\alpha\beta}$ are the matrix elements of the self-energy part. The Green's function is investigated in the representation of wave functions $\psi_{p_x p_z}^{(c,v)}(\mathbf{r})$ by using the effective mass approximation for the zero Landau bands of the electrons near the extrema of the c and v bands. For the time being we shall assume the electric field to be equal to zero.

The nondiagonal elements $\Sigma_{cv} = \Sigma_{vc}^*$ only appear in the exciton phase and, just as in the work by Kozlov and Maksimov,[6] they play the role of the order parameter. In general the diagonal elements Σ_c and Σ_v are different from zero even in the dielectric phase. In this connection they are either proportional to the number of thermally excited or impurity carriers or else are proportional to the square of the interaction U_{cc}^{vv}, which simultaneously transfers two electrons from one band to the other. We shall primarily consider the case $T = 0$ and neglect the influence of impurities. As for the interaction U_{cc}^{vv}, it is always small in the theory of large-radius excitons and in addition, being a short-range interaction, it acquires an additional degree of smallness associated with the scattering of electrons with parallel spins, which occurs in our case. As a result it turns out

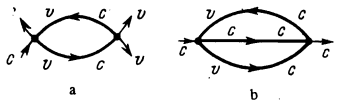
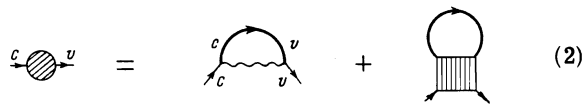


FIG. 1

that $U_{cc}^{vv}/U_{cv}^{cv} \approx (a_0/\lambda)^4$ and therefore this interaction can be neglected.[1)] Thus, one can assume that Σ_c and Σ_v are also different from zero only in the exciton phase.

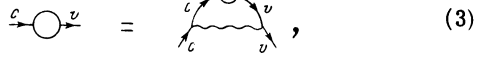
The exact equation for Σ_{cv} has the usual form:



(2)

In these diagrams the cross-hatched circle corresponds to the quantity Σ_{cv}, the heavy line corresponds to G_{cv}, a wavy line represents the matrix element of the Coulomb interaction U_{cv}^{cv}, and the cross-hatched four-point diagram represents the totality of higher-order diagrams which do not contain any horizontal two-particle intersections. All of the diagrams constituting this four-point diagram, for example the diagram shown in Fig. 1a, are different from zero only in the exciton phase and, in addition, they contain lines bearing the frequency ϵ that enters in Σ_{cv}. Thus, the two terms on the right hand side of Eq. (2) divide Σ_{cv} into two parts: $\Sigma_{cv} = \Sigma_0 + \Sigma_1$, where Σ_0 does not depend on the frequency and $\Sigma_1 \to 0$ for $\epsilon \gg d$, $p_z^2/2m$. Near the transition point, when Σ_{cv} is small and $\epsilon_0 \approx d$, it is obvious that $\Sigma_1 \approx \Sigma_0^3/d^2 \ll \Sigma_0$, but it does not contain the smallness associated with the order of the interaction. Precisely such diagrams for Σ_c and Σ_v as are shown, for example, in Fig. 1b are of the order of magnitude of Σ_0^2/d for an arbitrary number of interaction lines.

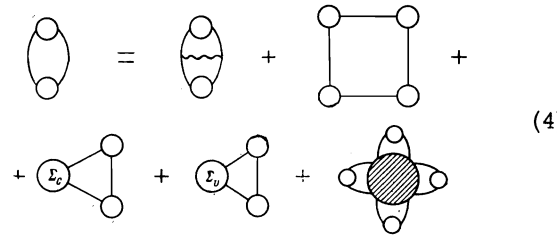
From what has been said above it follows that, to the first order in Σ_0, in the first term on the right hand side of Eq. (2) it is sufficient to restrict oneself to the expression $G_{cv} = G_c^{(0)} \Sigma_0 G_v^{(0)}$ for G_{cv}, and we then obtain an equation which has the same form as in the Hartree-Fock approximation:



(3)

where the circles denote Σ_0 and the thin lines denote the functions $G_c^{(0)}$ and $G_v^{(0)}$. After integrating over the frequencies and making the substitution $\Sigma_0 = (p_z^2/2m + d)\psi$, we obtain for ψ, in analogy to [6], the Schrödinger equation for the relative motion of an exciton with energy d in the momentum representation. A solution of this equation exists for $d < \epsilon_0$, where ϵ_0 is the maximum (with respect to the center of mass momentum $\mathbf{k}$) binding energy of the exciton. In the absence of any electric field this maximum is reached when $\mathbf{k} = 0$. Thus, $\psi = \eta \psi_{\mathbf{k}}(p_z)$ where $\psi_{\mathbf{k}}(p_z)$ denotes the wave function of an exciton with momentum $\mathbf{k}$ in the zero Landau band ($\psi_{\mathbf{k}}(0) = 1$), the constant η is determined upon taking account of the higher-order terms of the expansion in powers of Σ_0, and the vector $\mathbf{k}$ must then be determined from the condition that the thermodynamic potential be a minimum.

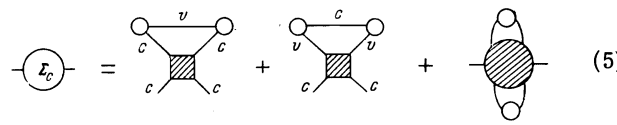
In order to determine the quantity η it is necessary to expand G_{cv} to third order in Σ_0, i.e., to also take the terms containing Σ_0^3, $\Sigma_c\Sigma_0$, $\Sigma_0\Sigma_v$, and Σ_1 into account. Only the terms corresponding to Σ_0^3 are usually taken into consideration[6] in the Hartree-Fock approximation. Since the terms of order Σ_0^3 determine the value of η without changing $\psi_{\mathbf{k}}(p_z)$, it will be more convenient for us to multiply both sides of the equation for Σ_0 by iG_{cv} and then integrate over frequency and momentum. Then, by carrying out the expansions indicated above we obtain the following equation:



(4)

(the circles correspond to Σ_0).

In the diagrams of fourth order in Σ_0, the interaction is eliminated with the aid of Eq. (3). The block diagram corresponding to Σ_c has the form



(5)

and a similar expression holds for Σ_v. (The indices c and v are written down explicitly only where ambiguities may arise.) Just as in the theory of Keldysh and Kozlov,[4] the eight-point and six-point vertices appearing in the last terms of Eqs. (4) and (5) correspond to the complete vertices for the scattering of two excitons and an electron by an exciton in the dielectric, which are not reducible to the antiparallel pair of lines $G_c^{(0)}$, $G_v^{(0)}$. The four-point vertices in Eq. (5) correspond to the complete vertices Γ_{cc}^{cc}, Γ_{vv}^{vv} describing the interaction of two electrons from the same band as well as the complete vertex Γ_{cv}^{cv} for the interaction of two electrons from different bands.

After substituting expression (5) into Eq. (4), let us denote the sum of the three terms containing the eight- and six-point vertices by $a\eta^4$, where in the absence of any external magnetic fields the quantity a would coincide, to within a coefficient, with the quantity λ from [4]. In the absence of an external field Eq. (4) coincides in analytic notation with the equation for the determination of the chemical potential μ of the excitons when $\epsilon_0 - d$ is interpreted as the chemical potential of the excitons and η is interpreted, to within a coefficient, as their density.

As was noted in [4], the exciton-exciton vertex should not contain a pole corresponding to the formation of a bi-exciton with negative total energy ξ_1. It follows from the work of Kadomtsev and Kurdyavtsev[8] that bi-excitons with $\xi_1 \approx -\epsilon_0$ exist in the presence of a strong magnetic field as $m_v \to \infty$. According to [8] the distance between holes turns out to be of the order of $R/L^{3/2}$ ($L = \ln(R^2/\lambda^2)$), that is, for a finite value of m_v the kinetic energy must be of the order of $L^3/m_v R^2 \approx \epsilon_0 L m_v/m_c$. Therefore, the bi-excitons should decay for sufficiently large values of H, when $L \gtrsim m_v/m_c$.

The first two diagrams on the right hand side of Eq. (4) correspond to the usual Hartree-Fock approximation. This approximation was used within the framework of the Green's function technique in the work by Kozlov and Maksimov.[6] The third and fourth diagrams also contain Hartree-Fock terms originating from the first two diagrams of Eq. (5). And, finally, the diagrams giving a contribution to the quantity a take the correlations of

the electrons with the exciton background into account. It will be shown below that all of the fourth-order diagrams give contributions of the same order of magnitude. Thus, it is impossible to regard the exciton background, which arises in the ground state of the excitonic insulator and which is responsible for the intermixing of states in the c and v bands, as an external self-consistent potential with a period corresponding to the distance between the extrema of the c and v bands. Near the transition point, this fact is essential only in connection with the determination of G_{cv}. However, for $\epsilon_0 - d \approx \epsilon_0$ in the range of energies of order ϵ_0, all matrix elements of the self-energy part of the Green's function become of the order of ϵ_0 and have the same scale of variation in frequency. Therefore, the elementary excitations in this domain are no longer single-particle excitations, but instead are closer to polarons with strong-coupling.

Now let us return to the first-order equation for Σ_0 and we shall now assume that $\mathbf{E} \neq 0$. Although the exciton's wave function $\psi_{\mathbf{k}}(z)$ has been obtained in the coordinate representation in article [5] (for $\mathbf{k}=0$), it will be more convenient for us to immediately determine $\Sigma_0(p_z, p_x, p_x')$ in the momentum representation. The integrals over the transverse momenta in Eq. (2) have the character of convolutions and are decomposed by the transformation

$$\Sigma_0(p_z, p_x, p_x + k_x) = \int \Sigma_{\mathbf{k}}(p_z) \exp\{i(p_x + k_x/2) k_y \lambda^2\}\, dk_y. \qquad (6)$$

The transformation (6), decomposing the integral with respect to p_x, is investigated in [9] in regard to the equation for the vertex part. This transformation achieves the transition from the Landau representation with a definite value of p_x to the representation of a definite two-dimensional transverse momentum, $\mathbf{k} = (k_x\ k_y)$, which is preserved in the c, v channels. Substituting (6) into (3) and integrating over the frequency, we obtain

$$\Sigma_{\mathbf{k}}(p_z) = \int \frac{U_{\mathbf{k}}(p_z' - p_z)\Sigma_{\mathbf{k}}(p_z')}{p_z'^2/2m + d_{\mathbf{k}}} \frac{dp_z'}{2\pi}, \qquad (7)$$

where $d_{\mathbf{k}} = d + \mathbf{V}\cdot\mathbf{k} - M\mathbf{V}^2/2$ is (according to [10]) the excitation energy of a noninteracting electron-hole pair with transverse momentum $\mathbf{k}$, and $M = m_{\perp c} + m_{\perp v}$. For $\lambda^2 k k_z \ll 1$ the interaction $U_{\mathbf{k}}(k_z)$ has a weak logarithmic dependence on the momenta (see [9]):

$$U_{\mathbf{k}}(k_z) \approx (e^2/\varkappa) \ln(\lambda |k_z| \max(\lambda k', 1)),$$

where[2)]

$$k' = |\mathbf{k}'|, \quad \mathbf{k}' = \mathbf{k} - M\mathbf{V}.$$

Therefore, to within logarithmic accuracy one can substitute $p_z' = (2m d_{\mathbf{k}})^{1/2} = R_{\mathbf{k}}^{-1}$ in the numerator of the integral (7). After integrating over p_z' we find that

$$\Sigma_{\mathbf{k}}(p_z) = \eta U_{\mathbf{k}}(\max\{p_z, R_{\mathbf{k}}^{-1}\}) / U_{\mathbf{k}}(R_{\mathbf{k}}^{-1})$$

for $d_{\mathbf{k}} = \epsilon_{\mathbf{k}}$, where $\epsilon_{\mathbf{k}} = me^4 L_{\mathbf{k}}^2/2\kappa^2$ ($L_{\mathbf{k}} = (\kappa/e^2) U_{\mathbf{k}}(R_{\mathbf{k}}^{-1}) \approx \ln(R_{\mathbf{k}}^2/\lambda^2)$ is the Coulomb logarithm).

The quantity $\Delta_{\mathbf{k}} = d_{\mathbf{k}} - \epsilon_{\mathbf{k}}$ represents the total ex- excitation energy of an exciton with momentum $\mathbf{k}$. In the absence of any electric field $V=0$, $d_{\mathbf{k}} = d$, and the minimum value of $\Delta_{\mathbf{k}}$ occurs at $\mathbf{k} = \mathbf{k}' = 0$. When $\mathbf{E} \neq 0$ the presence of the terms linear in $\mathbf{k}'$ leads to the result that the minimum value of $\Delta_{\mathbf{k}}$ is reached at a certain value $\mathbf{k} = \mathbf{k}_0 = \mathbf{k}_0' + M\mathbf{V}$ with $k_{x0}, k_{x0}' \neq 0$. Thus, the excitons condense into a state with nonvanishing momentum $\mathbf{k}_0$ and the distance between an electron and a hole is given by $\boldsymbol{\rho} = \lambda^2(\mathbf{k}_0' \times \mathbf{H})/H$. For $\eta > 0$ the quantity $\mathbf{k}(\eta)$ must be determined from the condition that the thermodynamic potential Ω be a minimum. However, the difference between $\mathbf{k}(\eta)$ and $\mathbf{k}(0) = \mathbf{k}_0$ is determined by terms of order η^6 in the expansion Ω and therefore this difference need not be taken into consideration in our approximation. In this connection, the subscript "zero" on $\mathbf{k}$ and $\mathbf{k}'$ will usually be omitted in what follows.

It is obvious that $\Sigma_{\mathbf{k}} = \Sigma_{\mathbf{k}_0}\delta(\mathbf{k} - \mathbf{k}_0)$ or in the Landau representation (with the fact that $k_y = 0$ taken into consideration) we have:

$$\Sigma_0(p_x, p_x') = \Sigma_{\mathbf{k}_0}\delta(p_x' - p_x - k_{x_0}). \qquad (8)$$

Then from Eqs. (1) we obtain the result that G_c and G_v are diagonal in p_x. This is explained by the fact that the exciton background with a definite $\mathbf{k}$ is described by a traveling wave, but not by a standing wave, and therefore does not violate the homogeneity of the crystal over large (in comparison with interatomic) distances.

In connection with the integration over frequencies in Eq. (4), it is necessary to make the substitution $\epsilon \to \epsilon + Vp_x$, which corresponds to a transformation to a coordinate system which is moving with the Hall velocity $\mathbf{V}$. The electric field vanishes in this coordinate system, and the dependence of the spectrum on p_x disappears. The dependence on k_x remains due to the presence of the δ-function (8), but the energies of the c-electrons acquire an additional constant term Vk_x. Furthermore, since the choice of the origin of the energy scale inside the forbidden gap is arbitrary, we obtain the same integrals as for the case when $\mathbf{E} = 0$, but the effective gap is given by $d_{\mathbf{k}} = d + \mathbf{V}\cdot\mathbf{k} - M_\perp V^2/2$, where $M_\perp = m_{\perp c} + m_{\perp v}$.

Now we see that all of the integrals over the longitudinal momenta converge in the region $p_z \approx (m d_{\mathbf{k}})^{1/2}$, just as in Eq. (7), and with the slow variation of $U_{\mathbf{k}}$ and $\Sigma_{\mathbf{k}}$ taken into consideration we obtain the result that the contribution of any diagram is proportional to $(\epsilon_{\mathbf{k}}/d_{\mathbf{k}})^n$, that is, all the diagrams of a given order in Σ_0 are of the same order of magnitude.

By evaluating the integrals in the first terms of Eq. (4), we obtain the following result term by term:

$$\eta^2 d_{\mathbf{k}} \left(\frac{m d_{\mathbf{k}}}{2}\right)^{1/2} = \eta^2 d_{\mathbf{k}} U_{\mathbf{k}} \frac{m}{2d_{\mathbf{k}}} - \eta^4 \left(\frac{3}{4} d_{\mathbf{k}} \left(\frac{m d_{\mathbf{k}}}{2}\right)^{1/2} + b + a\right). \qquad (9)$$

We shall denote the coefficient associated with $-\eta^4$ in Eq. (9) by $(2C)^{-1} d_{\mathbf{k}} (m k_{\mathbf{k}}/2)^{1/2}$, where C is a dimensionless coefficient. This coefficient consists of a positive term which does not explicitly contain the interaction term and two negative terms: b and a.

The quantity b corresponds to the diagrams containing only four-point vertices and arising from the first two terms on the right hand side of expression (5) for Σ_c and Σ_v. We note without presenting the proof that in the essential range of frequencies the total contribution of the four vertices Γ_{cc}^{cc}, Γ_{cv}^{cv}, Γ_{vc}^{vc}, and Γ_{vv}^{vv} is negative and does not exceed the value $2U_{cv}^{cv}$ in magnitude; whence one can easily show that $0 < b < \frac{1}{2} d (m d/2)^{1/2}$. We have not been able to make any estimates with regard to the value of a; therefore, as mentioned in [4] the quantity C may be both positive and negative, and also in general $C \approx 1$.

The negative sign for C gives a phase transition of the first kind, which occurs for a certain $d > \epsilon_0$. In this case the exciton phase corresponds to the formation of a drop of the excitonic liquid in the normal exciton gas. Thanks

to the appreciable anisotropy of the polarizability of the excitons, the formation of a partially ordered structure of the liquid crystal type is also probable in the presence of a strong magnetic field.

In connection with the variation of the external parameters (fields, temperature, pressure, impurity concentration) the sign of C may change, and the line of first-order phase transitions may turn into a line of second-order phase transitions and conversely. The gas approximation which we are using is, of course, not valid in the presence of a transition of the first kind; therefore, in what follows we shall assume that $C>0$. For $C>0$ we obtain $\eta^2=C(\epsilon_{\mathbf{k}}-d_{\mathbf{k}})/d_{\mathbf{k}}$ from Eq. (9).

In order to determine the corrections to the density of the thermodynamic potential Ω, it is necessary to calculate the average of the interaction Hamiltonian H_{int}. This average is given by

$$\langle H_{int}\rangle=-\frac{i}{2}\sum_{a,b}\Sigma_{ab}(\varepsilon,p_z)G_{ba}(\varepsilon,p_z)\frac{dp_z\,d\varepsilon}{(2\pi)^2}\frac{1}{2\pi\lambda^2},$$

where $1/2\pi\lambda^2$ is the degeneracy of the Landau level. In this expression the terms containing products of the diagonal elements of Σ and G vanish upon integrating over the frequency because the diagrams corresponding to these terms contain closed loops consisting of lines corresponding to Green's functions of a single type. Thus, $\langle H_{int}\rangle$ is determined to second order in η by the diagram shown on the left hand side of Eq. (4). The corresponding quantity in Eq. (9) must be multiplied by $-(2\pi\lambda^2)^{-1}$, and we then obtain

$$\langle H_{int}\rangle=-\eta^2 d_{\mathbf{k}}/V_{\mathbf{k}},$$

where $V_{\mathbf{k}}=(2\pi\lambda^2 R_{\mathbf{k}})^{-1}$ is the characteristic volume of an exciton with momentum $\mathbf{k}(R_{\mathbf{k}}=(2m\epsilon_{\mathbf{k}})^{-1})$. Integrating with respect to e^2, we obtain

$$\delta\Omega=-{}^1/_4 C^{-1}\eta^4 d_{\mathbf{k}}/V_{\mathbf{k}}.$$

The obtained expression for $\delta\Omega$ enables us to determine the thermodynamic characteristics of an excitonic insulator. By differentiating $\delta\Omega$ twice with respect to H, we obtain the following result for the abrupt change in the value of the magnetic susceptibility (when $\mathbf{E}=0$):

$$\delta\chi={}^1/_2 C(\mu^*)^2/d_{\mathbf{k}}V_{\mathbf{k}},$$
$$\mu^*=\partial(\varepsilon_0-d)/\partial H=-\partial d/\partial H+2\varepsilon_0/LH.$$

Furthermore, since the average distance $\rho_y=\lambda^2(k_x-MV)$ between the electrons and holes does not vanish in the new phase for $E\neq 0$, the transition is accompanied by the appearance of a dipole moment:

$$P_y=-(\partial\delta\Omega/\partial E_y)_{\mathbf{V}}=-ek_x\lambda^2\eta^2/2V_{\mathbf{k}}. \qquad (10)$$

The differentiation is carried out at constant $\mathbf{V}=(\partial\Omega/\partial\mathbf{k})_E$, that is, in our approximation for constant, zero velocity of the excitons. Therefore, it is natural that P_y is proportional to $-ek_x\lambda^2$, i.e., proportional to the exciton's dipole moment at constant velocity.[10] It is seen from expression (10) that the exciton density is equal to $\eta^2/2$. Differentiating P_y with respect to E_y, we obtain the following result for the discontinuity in the dielectric constant:

$$\delta\kappa=\frac{C}{2}\frac{(ek_x\lambda^2)^2}{V_{\mathbf{k}}d_{\mathbf{k}}}=\frac{C}{2\pi}(k_x\lambda)^2\kappa/L. \qquad (11)$$

Using the condition $d_{\mathbf{k}}=\epsilon_{\mathbf{k}}$ it is easy to show that $k_x\lambda=-eE_y mR^2\lambda/L$ at the transition point. The discontinuity of the dielectric constant is maximal when $\mathbf{k}$ is close to an inflection point of the function $\epsilon(\mathbf{k})$, that is, when $k\lambda\approx 1$. In this case $\delta\kappa/\kappa\approx 10^{-1}$ to 10^{-2} and the order of magnitude of the electric field is given by

$$E\approx L(mR^2\lambda e)^{-1}\sim 10^9 L\frac{m_0}{m}\frac{a^3}{R^2\lambda}\ \mathrm{V/cm}.$$

where $a_0=1/m_0e^2$ is the atomic size. We find that $E\approx 100$ V/cm for Bi_xSb_{1-x} alloys[11] with $H\approx 10^5$ Oe.

When $E\neq 0$ the phase transition point is displaced towards larger values of d. The magnitude of the critical electric field is determined by the expression

$$E_c=\frac{\varepsilon_0}{e\lambda}\left(\frac{d-\varepsilon_0}{\varepsilon_0 L}\right)^{1/3}\approx\frac{L^{-1/3}H_c}{mcR_{\mathbf{k}}}\left|\frac{\mu^*}{\mu}\frac{H_c-H_{c0}}{H_{c0}}\right|^{1/3},$$

where $\mu=e/mc$ and H_{c0} is the critical field for $E=0$.

We also note that anomalies should be observed in the acoustic phonon spectrum in the exciton phase. In fact, when $\eta\neq 0$ the renormalized states c contain impurity states v with amplitude η and conversely. Therefore, the forbidden band and the exciton excitations turn from indirect transitions into vertical transitions with a probability η^2. This makes it possible to have both direct optical transitions with a frequency of the order of $\epsilon_{\mathbf{k}}$ and the interaction of the exciton modes with the the acoustic modes. In contrast to the case of a dielectric, the exciton mode in an excitonic insulator does not have a gap (neglecting the matrix elements U_{cc}^{vv}[7]), and the spectrum, in analogy to the case of a Bose gas,[4] has the form

$$\varepsilon_{\mathbf{k}}=[\Delta k^2/M_{\mathbf{k}}+(k^2/2M_{\mathbf{k}})^2]^{1/2},$$

where $M_{\mathbf{k}}$ denotes the mass along $\mathbf{k}$ ($M_Z=m_c+m_v$, and according to[10] $M_\perp$ is given by $M_\perp=R^2/\lambda^2L$). Therefore, the interaction of the exciton and phonon spectra must have the form shown in Fig. 2. On this figure the dashed lines I and II indicate the noninteracting spectra of the phonons and excitons. The region of a strong interaction of the terms is determined from the condition $k=2M_{\mathbf{k}}s_{\mathbf{k}}$, where $s_{\mathbf{k}}$ denotes the speed of sound in the direction $\mathbf{k}$. In this region there are bound exciton-phonon vibrations which are analogous to a spin-acoustic resonance by the normal modes, and external sound radiation should experience strong attenuation. It is obvious that what has been said above only pertains to the case of a singlet ground state.

The theory discussed above can also be applied to the case of nonzero temperatures T, provided that $T\ll T_0$ where T_0 is the characteristic temperature above which the critical fluctuations of the exciton density become essential. In order to estimate T_0 let us calculate the contribution of the singular part of the vertex Γ_{cv}^{cv} in the second term of Eq. (5) for Σ_c. For $T=0$ this contribution vanished upon integrating over the frequency. For $T>0$ at zero frequency we have

$$\Gamma_{cv}^{cv}\approx\frac{\varepsilon_0}{V_0}\frac{1}{\Delta(T)+k^2/2M_{\mathbf{k}}}$$

where $\Delta(T)=\epsilon_0(T)-d$, and $\epsilon_0(T)=\epsilon_0(1-\alpha(T/d)^{1/2}e^{-d/T})$

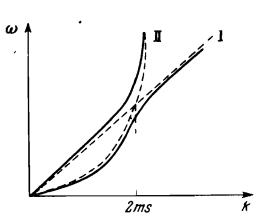


FIG. 2

denotes the exciton's binding energy at the temperature T ($\alpha \approx 1$). Integrating with respect to $\mathbf{k}$, we find that the residue of the regular contribution is given by $\delta\Sigma_c \approx LT/(\epsilon_0(\epsilon_0 d)^{1/2}$. This quantity is small in comparison with the terms of order $\eta^2 d \approx \Delta$ which we have taken into account provided $\eta \gg T/T_0$, where $T_0 = \epsilon_0/L$.

At temperatures $T \gtrsim T_0$ our expansion is not valid for any value of η. Nevertheless, for $\eta \ll 1$ the dependence of the exciton momentum on the electric field, which is determined by energies of the order of ϵ_0, is not modified. But instead of the abrupt change in the dielectric constant given by expression (11), there must be a singularity of the form $\delta\kappa/\kappa \approx A(k_x\lambda)^2(\Delta/\epsilon_0)^{-\alpha}$ with the same exponent α as for the singularity in $T_c - T$ in the expression for the specific heat of an imperfect Bose gas.

Note added in proof (20 December 1972). The probability for the excitation of an exciton by a long-wavelength field is proportional to the absolute value squared of the exciton's wave function, i.e., $V_0^{-1} = (2\pi\lambda^2 R_0)^{-1}$. Therefore, the probability of direct transitions is determined by the dimensionless parameter $\alpha = \eta^2 R_0^2/\lambda^2$. Here the large value of R_0^2/λ^2 may compensate for the smallness of η^2.

[1)] It was shown in [7] that the influence of this interaction is fundamental in nature. However, in view of the smallness of a_0/λ it is restricted to a narrow neighborhood of the phase transition point.

[2)] According to the article by Gor'kov and Dzyaloshinskiĭ, [10] the dependence of the matrix elements of the interaction and the binding energy on the transverse momentum, i.e., the lifting of the Landau degeneracy, is explained by the separation of the centers of the electron and hole orbits by the vector $\rho = \lambda^2(\mathbf{k}' \times \mathbf{H})/H$.

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78