Magnetic and charge neutralization of an electron beam injected into a magnetoactive plasma

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Magnetic and charge neutralization in cold magnetoactive plasma injected with a low-density electron beam is investigated. Analysis of long-period (along the beam) perturbations shows that the thickness of the tube in which are concentrated the fields and currents in the region of magnetic neutralization is the most sensitive to the external magnetic field strength. In particular, conditions are obtained that change the field in the magnetic neutralization region from a sheath configuration along the beam radius to a quasi-periodic configuration. The role of ions becomes significant in stronger external magnetic field. In this case both the skin depth and the diffusion length (distance from the beam front to the magnetic neutralization region) change (the latter decreases). Conditions for the absence of magnetic and charge neutralization are determined. Under these conditions currents in excess of the limit cannot pass through a plasma (or a gas).

1. INTRODUCTION

The reaction of unmagnetized plasma into which a low-density electron beam is injected was studied theoretically by a number of authors[1-6]. The first attempt to investigate the effect of an external magnetic field on beam-perturbed plasma is due to Lee and Sudan[6], who however neglected completely the effect of the ions. The present paper reports on a more detailed investigation of plasma behavior in response to the injection of an electron beam along the external magnetic field. Principal attention is paid to the problems of magnetic and charge neutralization.

An earlier paper[5] noted the important role of the concept of diffusion length, i.e., the distance from the beam front over which dissipative processes damp out the counter-streaming current induced in plasma and consequently eliminate the magnetic neutralization. In the presence of a relatively weak external magnetic field, where the ion effect is insignificant[6], the diffusion length is the same as in unmagnetized plasma. An external axial magnetic field merely increases the thickness of the circular layer in which the azimuthal magnetic field and the current are concentrated. The diffusion length changes in sufficiently strong fields, when it is necessary to take the ionic component of plasma into account.

Section 2 of this paper gives the initial expressions for the electromagnetic field, current, and charge density obtained in the asymptotic limit[1] when a low-density electron beam is injected into a magnetoactive plasma. Section 3 considers these expressions in the limit of a weak magnetic field ($\Omega_i \ll \nu_i$, where Ω_i, ν_i are the ionic gyrofrequency and the collision frequency), where the role of ions is still insignificant. Section 4 analyzes the case of a sufficiently strong magnetic field ($\Omega_i \gg \nu_i$), where the ionic effect is significant.

2. INITIAL EXPRESSIONS

The problem of magnetoactive plasma perturbed by a charged-particle beam is treated using the same assumptions and method as in[3]. The thermal spread of particles in plasma is neglected. Using the well-known dielectric permittivity tensor for a cold magnetoactive plasma

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_\perp & -ig & 0 \\ ig & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix}, \tag{2.1}$$

where

$$\varepsilon_\perp = 1 - \sum_{i,e} \frac{\omega_p^2(\omega + i\nu)}{\omega[(\omega+i\nu)^2 - \Omega^2]},$$

$$\varepsilon_\parallel = 1 - \sum_{i,e} \frac{\omega_p^2}{\omega(\omega + i\nu)}, \qquad g = \sum_{i,e} \frac{\omega_p^2 \Omega}{\omega[(\omega+i\nu)^2 - \Omega^2]},$$

we obtain in the asymptotic limit the following expressions for the fields and currents induced by the electron beam[1)]:

$$\mathbf{E}(t', r) = E_0 u \int_0^\infty \frac{dk_\perp}{2\pi} J_1(k_\perp r_0) \int_{-\infty}^\infty \frac{d\omega}{\omega^2 D} e^{i\omega t'} \{J_0(k_\perp r)[(\varepsilon_\perp - n^2)(\varepsilon_\perp - n_z^2) - g^2]\mathbf{e}_z + n_z n_\perp J_1(k_\perp r)[g\mathbf{e}_\varphi + i(n^2 - \varepsilon_\perp)\mathbf{e}_r]\},$$

$$\mathbf{B}(t', r) = B_0 c^2 \int_0^\infty \frac{dk_\perp}{2\pi} k_\perp J_1(k_\perp r_0) \int_{-\infty}^\infty \frac{d\omega}{\omega^3 D} e^{i\omega t'} \{iJ_1(k_\perp r)[\varepsilon_\perp(n^2 - \varepsilon_\perp) + g^2]\mathbf{e}_\varphi - n_z g[in_\perp J_0(k_\perp r)\mathbf{e}_z + n_z J_1(k_\perp r)\mathbf{e}_r]\}, \tag{2.2}$$

$$\mathbf{j}(t', r) = j_0 r_0 \int_0^\infty \frac{dk_\perp}{2\pi} J_1(k_\perp r_0) \int_{-\infty}^\infty \frac{d\omega}{\omega D} e^{i\omega t'} \{iJ_0(k_\perp r)(1 - \varepsilon_\parallel)[(\varepsilon_\perp - n^2) \cdot (\varepsilon_\perp - n_z^2) - g^2]\mathbf{e}_z + n_z n_\perp J_1(k_\perp r)([(n^2 - \varepsilon_\perp)(\varepsilon_\perp - 1) + g^2]\mathbf{e}_r + ig(1 - n^2)\mathbf{e}_\varphi)\},$$

$$\rho(t', r) = \rho_0 r_0 \int_0^\infty \frac{dk_\perp}{2\pi} J_1(k_\perp r_0) J_0(k_\perp r) \int_{-\infty}^\infty \frac{d\omega}{\omega D} e^{i\omega t'} [(\varepsilon_\perp - n^2)^2 - g^2].$$

Here $\mathbf{n} = \mathbf{k}c/\omega$, $k_z = \omega/u$, $t' = z'/u$, $z' = z - ut$ is the distance from the beam front to the point of observation,

$$E_0 = 2\pi e n_0 r_0, \quad B_0 = uE_0/c, \quad j_0 = \rho_0 u, \quad \rho_0 = e n_0,$$

$$D = n^4\left(\varepsilon_\perp \frac{k_\perp^2}{k^2} + \varepsilon_\parallel \frac{k_z^2}{k^2}\right) - n^2\left[\varepsilon_\perp\varepsilon_\parallel\left(1 + \frac{k_z^2}{k^2}\right) + \frac{k_\perp^2}{k^2}(\varepsilon_\perp^2 - g^2)\right] + \varepsilon_\parallel(\varepsilon_\perp^2 - g^2). \tag{2.3}$$

In (2.2) integration over ω is performed with the aid of the residues at the poles corresponding to the roots of the equation D = 0. In the general case this is an equation of eleventh degree in ω and it cannot be solved explicitly. We assume that the collision frequency of the plasma electrons is low: $\nu_e \ll \omega_{pe}$, Ω_e. High-frequency perturbations at $\omega \gtrsim \omega_{pe}$, Ω_e damp out with an increment $\sim\nu_e$ and therefore are exponentially small at a distance $|z'| > u/\nu_e$ from the beam front. Furthermore, in beams whose boundary diffusion is small enough, $L \gtrsim c/\omega_{pe}, \Omega_e$, where L is a characteristic dimension of the beam-current inhomogeneity, rapidly oscillating perturbations become small in comparison to low-frequency perturbations within the beam volume because of the averaging effect[5]. These short-period oscillations are of special interest to the problem of Cerenkov radiation of the electron beam at a large distance from its axis. In this paper, however, we merely consider plasma within the beam (along its radius) at a distance $|z'| \gg u/\omega_{pe}, \Omega_e$. At this distance the perturbation spectrum induced by the beam in plasma is mainly characterized by its low-frequency region $\omega \ll \omega_{pe}, \Omega_e$. In this region the analysis is conveniently performed as a function of the parameter ν_i/Ω_i (we assume that $\nu_i \sim \sqrt{m/M}\nu_e$).

3. THE CASE OF WEAK MAGNETIC FIELD ($\Omega_i \ll \nu_i$)

In the limit when $\Omega_i \ll \nu_i$ the ions contribute to the expression for D only within the frequency region $\omega \lesssim \nu_i \ll \nu_e$. This allows us to write the following approximate expression for D:

$$D \approx \frac{a(1+\varkappa^2)c^4}{\zeta\lambda^4\omega^5(\omega+i\nu_i)(\omega+i\nu_e)}\prod_{l=1}^{3}(\omega-\omega_l^{(1)}), \tag{3.1}$$

where

$$\varkappa = k_\perp\lambda, \quad \lambda = c/\omega_{pe}, \quad \zeta = \Omega_e^2/\omega_{pe}^2,$$
$$a = 1+\zeta f^2(1-\mu^2/\varkappa^2), \quad \mu = c/u\gamma, \quad f = \varkappa^2/(1+\varkappa^2),$$
$$\gamma^{-2} = 1 - u^2/c^2.$$

The roots of the equation D = 0 are approximately

$$\omega_1^{(1)} \approx -i\nu_e(b-\sqrt{d})/2a,$$
$$\omega_2^{(1)} \approx \begin{cases} -\frac{1}{2}i\{\nu_i+\varkappa^2\nu_e+[(\nu_i+\varkappa^2\nu_e)^2-4\nu_i\nu_e\varkappa^2(1+\Omega_g^2/\nu_g^2)]^{1/2}\}, & \varkappa^2 \lessdot \nu_i/\nu_e, \\ -i\nu_e(b+\sqrt{d})/2a, & \varkappa^2 \gg \nu_i/\nu_e, \end{cases} \tag{3.2}$$
$$\omega_3^{(1)} \approx -\frac{1}{2}i\{\nu_i+\varkappa^2\nu_e-[(\nu_i+\varkappa^2\nu_e)^2-4\nu_i\nu_e\varkappa^2(1+\Omega_g^2/\nu_g^2)]^{1/2}\}.$$

here

$$\Omega_g^2 = |\Omega_i\Omega_e|, \quad \nu_g^2 = \nu_i\nu_e, \quad b = 2f(1+\tfrac{1}{2}\zeta f),$$
$$d = \zeta f^4[(4+\mu)/\varkappa^2+\zeta+3].$$

We note that the roots $\omega_{1,2}^{(1)}$ are generalizations of the corresponding roots found in[6], where, in addition to neglecting the ions completely, they considered only two limiting cases: (a) $\zeta \sim 1$, $\gamma \gg 1$, and (b) $\zeta \gg 1$, $\gamma \sim 1$.[2)] Formulas (3.2) are valid for any γ and for $\zeta \ll (M/m)(\nu_i^2/\omega_{pi}^2)$ (which follows immediately from the condition $\Omega_i \ll \nu_i$).

Expressions (3.1) for D and (3.2) for the roots permit us to integrate over the frequencies in (2.2). Analysis of the remaining integrals in $k_\perp$ shows that at the distance $|z'| \ll u/\nu_e$ for a shallow skin depth $\lambda \ll r_0$, the region $(k_\perp\lambda)^2 \gg \nu_i/\nu_e$ makes the principal contribution to the integrals. We then can fully neglect the ionic effect and consider only the contribution from the poles $\omega_{1,2}^{(1)} \approx -i\nu_e(b \pm \sqrt{d})/2a$. To integrate over $k_\perp$ we must remember the following. The pole $\omega_1^{(1)}$ as a function of $k_\perp$ always lies in the lower half-plane of complex ω and consequently characterizes the field within the beam ($z' < 0$). The pole $\omega_2^{(1)}$ lies always (for any $k_\perp > 0$) in the lower half-plane if $b^2 > d$, which gives

$$\zeta < 4\gamma^4u^2/c^2. \tag{3.3}$$

Therefore the low-frequency perturbations considered here are contained inside the beam only for moderate fields. If condition (3.3) is not satisfied, perturbations appear ahead of the beam; these are due essentially to dissipative processes in the plasma and are exponentially small at distances $z' > u/\nu_e$. Since condition $\Omega_i \ll \nu_i$ limits ζ from above and the right-hand side of inequality (3.3) is large enough for relativistic beams, we consider (3.3) satisfied. Integration over $k_\perp$ in (2.2) for $|z'| \ll u/\nu_e$ offers no particular difficulties, since the integrals are reducible to the standard forms. To be brief, we merely show the results of integration for the azimuthal magnetic field B_φ and charge density ρ (these quantities are particularly interesting from the viewpoint of the magnetic and charge neutralization problem):

$$B_\varphi = B_0\lambda\,\mathrm{Im}\left\{\frac{\partial}{\partial r}\left[\Psi(w)\left(\frac{\zeta\mu^2(1+\zeta)}{2w_2^2}-i\right)\right]\right\}$$
$$\rho = -\rho_0\frac{r_0}{\lambda}\frac{u^2}{c^2}\frac{\zeta}{1+\zeta}\mathrm{Im}\left[\Psi(w)\frac{w^4}{w_2^2}\right], \tag{3.4}$$

where

$$w^2 = w_1^2 + iw_2^2, \quad w_1^2 = (1-\tfrac{1}{2}\zeta\mu^2)(1+\zeta)^{-1},$$
$$w_2^2 = (u/c)\sqrt{\zeta(1-\zeta\mu^2/4\gamma^2)}\,(1+\zeta)^{-1},$$

and the function $\Psi(w)$ is of the form

$$\Psi(w) = \begin{cases} -w^{-1}I_0(wr/\lambda)K_1(wr_0/\lambda), & r < r_0 \\ w^{-1}I_1(wr_0/\lambda)K_0(wr/\lambda), & r > r_0 \end{cases}$$

Equations (3.4) show that magnetic neutralization occurs only if $|w| = (1+\zeta)^{-1/4} \gg \lambda/r_0$. This is in agreement (with an accuracy to the exponent of $(1+\zeta)$) with the results of[6], which however were limited to the ultrarelativistic case ($\gamma \to \infty$). Under conditions of magnetic neutralization at the distance $|z'| < u/\nu_e$, the magnetic field is enclosed in a circular layer $\sim\lambda/\mathrm{Re}\,w$ thick near the lateral surface of the beam. If $\mathrm{Re}\,w < \mathrm{Im}\,w$ (or $\zeta < 2\gamma^2u^2/c^2$), the field has an oscillating configuration within that layer (with a period of $\sim\lambda/\mathrm{Im}\,w$). In the limit of $\zeta \to 4\gamma^4u^2/c^2$ the ring thickness increases and for $\zeta = 4\gamma^4u^2/c^2$ ($\mathrm{Re}\,w = 0$) the field is rigorously periodic along the radius with a period of $\lambda/|w|$. At this distance ($|z'| < u/\nu_e$) from the beam front charge neutralization is absent only if $|w| \lesssim \lambda/r_0$ and $\zeta \gtrsim 1$ (i.e., when there is no magnetic neutralization of the beam). This follows from the fact that the charge density is of the order of $\rho \approx \rho_0\zeta/(1+\zeta)$ for $|w| < r_0/\lambda$.

At a longer distance, $|z'| \gg u/\nu_e$, where the low-frequency region ($\omega \ll \nu_e$) makes the main contribution to the integrals with respect to ω, the fields $\mathbf{E}_z$ and $\mathbf{B}_\varphi$ and current density j_z differ little from the case of unmagnetized plasma (the residues at the poles $\omega_{2,3}^{(1)}$ are small for these values). Hence it follows that the diffusion length in a weakly magnetized plasma ($\Omega_i \ll \nu_i$) remains the same as in unmagnetized plasma ($z_d = (u/\nu_e)(r_0/\lambda)^2$). The charge density at the distance $|z'| < u/\nu_i$ is not exponentially small, as is the case with unmagnetized plasma, but is finite and, if $|z'| < u/\nu_i$, the contribution from the ions is low and ρ is approximately expressed by the formula

$$\rho(r,t') \approx \rho_0 r_0|t'|\zeta c^2\mu^{-2}\frac{\partial^2}{\partial t'\partial r_0}Q, \tag{3.5}$$

where

$$Q = \frac{1}{2|t'|\nu_e}I_0\left(\frac{rr_0}{2\lambda^2|t'|\nu_e}\right)\exp\left(-\frac{r^2+r_0^2}{4|t'|\nu_e\lambda^2}\right).$$

At still longer distances $|z'| \gg u/\nu_i$, the ions play a significant role. Here, if we assume for the sake of simplicity that $\Omega_g << \nu_g$, the functional dependence of ρ on r and t' remains practically the same (i.e., $\sim Q$) and only the coefficient of the function Q changes:

$$\rho(r,t') \approx \rho_0 \frac{\nu_e r_0}{\omega_{pi}^2}\left(\frac{\partial}{\partial t'} - \nu_i\right)\frac{\partial}{\partial r_0} Q. \tag{3.6}$$

It follows from (3.5) and (3.6) that the total charge density induced by the beam in a weakly magnetized plasma at the distance $|z'| \gg u/\nu_e$ is different from zero only inside a tube whose width is of the order of $r_0\sqrt{|z'|/z_d}$, i.e., it increases with the distance from the beam front. At the same time the charge density decreases in proportion to $\sqrt{z_d/|z'|}$; at the distance $|z'| > z_d$ the charge is small in comparison to the beam charge and is distributed throughout the volume.

4. THE CASE OF A STRONG MAGNETIC FIELD ($\Omega_i \gg \nu_i$)

We consider the effects of magnetic and charge neutralization of an electron beam injected into a strongly magnetized plasma. In addition to the condition $\Omega_i \gg \nu_i$ we assume that $\nu_e \ll \max(\Omega_i, \omega_{pi})$.

In this case the roots of the equation D = 0, lying in the low-frequency region ($\omega \ll \max(\Omega_i, \omega_{pi})$), can be written in the form

$$\omega_{1,2}^{(2)} = -i\frac{\nu_e}{2}\frac{\varkappa^2}{\varkappa^2+\alpha}\left\{1+\frac{\nu_i}{\nu_e\beta}(1+\varkappa^{-2}) \pm\left(\left[1+\frac{\nu_i}{\nu_e\beta}(1+\varkappa^{-2})\right]^2 - 4\frac{\nu_i}{\nu_e\beta}(1+\alpha\varkappa^{-2})\right)^{1/2}\right\}, \tag{4.1}$$

where $\alpha = (1 - v_A^2/\gamma^2 u^2)\beta^{-1}$, $\beta = 1 + v_A^2/c^2$, $v_A = c\Omega_i/\omega_{pi}$ is the Alfven velocity.

If the gyrofrequency of ions is lower than the ionic Langmuir frequency $\Omega_i^2 \ll \omega_{pi}^2$, then $\alpha \approx \beta \approx 1$ and the roots $\omega_{1,2}^{(2)}$ are written approximately in the form

$$\omega_1^{(2)} \approx -i\nu_e\frac{\varkappa^2}{\varkappa^2+1}, \qquad \omega_2^{(2)} \approx -i\frac{\nu_i}{u}. \tag{4.2}$$

The pole $\omega_1^{(2)}$ coincides with the diffusion pole occurring in the case of unmagnetized plasma[3] and makes the principal contribution to the components E_z, B_φ, and j_z. The remaining components and the charge ρ remain small quantities.

It is clear from the above that in magnetized plasma whose ionic gyrofrequency exceeds the ion collision frequency, the external magnetic field can affect the magnetic and charge neutralization (in particular, the significant difference in the behavior of the azimuthal magnetic field B_φ and the charge density ρ between magnetized and unmagnetized plasma perturbed by the beam) only if the external field is high enough.

Assuming that $\Omega_i^2 \gg \omega_{pi}^2$, we consider only the ultra-relativistic case ($\gamma^2 \gg 1$)[3]. Here the parameter α is small ($\alpha \ll 1$) and the roots $\omega_{1,2}^{(2)}$ are written approximately in the form

$$\omega_1^{(2)} \approx -i\nu_e\frac{a_\perp}{b_\perp}, \qquad \omega_2^{(2)} \approx -i\frac{\nu_i}{\beta}\frac{\varkappa^2}{a_\perp}, \tag{4.3}$$

where $a_\perp = \kappa^2 + \nu_i/\nu_e\beta$, $b_\perp = \kappa^2 + \alpha$.

The pole $\omega_2^{(2)}$ lies always in the lower half-plane of ω. The pole $\omega_1^{(2)}$ lies in the lower half-plane of ω for any $\kappa^2 > 0$ only if $\alpha > 0$, i.e., if $v_A < u\gamma$. This means that for $v_A < u\gamma$ there are no diffusion fields (which are largely due to particle collisions) ahead of the beam. On the other hand, if $\alpha < 0$, then for $\kappa^2 < |\alpha|$ the pole $\omega_1^{(2)}$ occurs in the upper half-plane. Consequently there are diffusion-type fields ahead of the beam. The contributions from the poles $\omega_1^{(2)}$ and $\omega_2^{(2)}$ are expressed respectively in terms of the integrals[4]

$$F_{nm} = \int_0^\infty \varkappa\, d\varkappa[\eta(-t')\eta(b_\perp) - \eta(t')\eta(-b_\perp)]J_m\left(\varkappa\frac{r}{\lambda}\right)J_m\left(\varkappa\frac{r_0}{\lambda}\right) \times\frac{\alpha^n}{a_\perp b_\perp^n}\exp\left(\frac{t'\nu_e a_\perp}{b_\perp}\right), \tag{4.4}$$

$$T_{nm} = \eta(-t')\left(\frac{\nu_e\beta}{\nu_i}\right)^{(m-1)/2}\int_0^\infty\frac{d\varkappa\varkappa^m}{a_\perp}J_n\left(\varkappa\frac{r}{\lambda}\right)J_1\left(\varkappa\frac{r_0}{\lambda}\right)\exp\left(\frac{t'\nu_i\varkappa^2}{\beta a_\perp}\right)$$

In this case the azimuthal magnetic field and charge density determined by low-frequency oscillations with spectra of $\omega_{1,2}^{(2)}$ are written in the form

$$B_\varphi = B_\varphi^{(0)} - B_0(T_{1,-1} + F_{11}), \quad \rho = \rho_0\left(\frac{r_0}{\lambda}\sqrt{\frac{\nu_i}{\nu_e\beta}}T_{02} + r_0\frac{\partial}{\partial r_0}F_{10}\right), \tag{4.5}$$

where

$$B_\varphi^{(0)} = {}^1/_2\eta(-t')B_0\begin{cases} r/r_0, & r<r_0 \\ r_0/r, & r>r_0 \end{cases}$$

is the self-field of the beam. The functions T_{nm} coincide with those derived in[3], apart from the coefficients, if the following substitutions are made in the derived functions: $c^2/\omega_{pe}^2 \to \lambda^2\beta\nu_e/\nu_i$, $\nu_e \to \nu_i/\beta$. Therefore β/ν_i and $t_d = z_d/u$ are the characteristic times $|t'|$ for these integrals.

To analyze the integrals F_{nm} we consider only moderate external fields (we assume that $v_A < u\gamma$, i.e., $\alpha > 0$). Then for the integrals F_{nm} the characteristic times are ν_e^{-1} and $t_d^* = \alpha t_d \ll t_d$. Proceeding as in[3], we can readily obtain approximate expressions for the integrals T_{nm} and F_{nm} in the corresponding time intervals t′. For the quantities in (4.5) we have ($T_{1,-1}$ was studied in[3]):

$$T_{02} \approx -\lambda\sqrt{\frac{\nu_e\beta}{\nu_i}}\frac{\partial}{\partial r_0}\begin{cases}\Psi_0(\sqrt{\nu_i/\nu_e\beta}), & |t'| \ll \beta/\nu_i \\ Q\beta\nu_e/\nu_i, & |t'| \gg \beta/\nu_i\end{cases}$$

$$F_{11} \approx \begin{cases}\Psi_1(\sqrt{\nu_i/\nu_e\beta}) - \Psi_1(\sqrt{\alpha}), & |t'| \lll \nu_e^{-1} \\ A_1 - A(|t'|\nu_e\alpha^{-1}, |r-r_0|\lambda^{-1}), & t_d^* \ggg |t'| \ggg \nu_e^{-1}; \\ B(|t'|\nu_i/\alpha\beta)t_d^*/8|t'|, & |t'| \ggg t_d^*\end{cases} \tag{4.6}$$

$$F_{10} \approx \begin{cases}\Psi_1(\sqrt{\nu_i/\nu_e\beta}), & |t'| \ll \nu_e^{-1} \\ F_{11}, & |t'| \gg \nu_e^{-1}\end{cases}.$$

Here we introduced the following notation:

$$\Psi_n(a) = \begin{cases} I_n\left(\frac{ar}{\lambda}\right)K_n\left(\frac{ar_0}{\lambda}\right), & r<r_0 \\ I_n\left(\frac{ar_0}{\lambda}\right)K_n\left(\frac{ar}{\lambda}\right), & r>r_0 \end{cases}; \qquad B(x) = 1 + xe^x\,\mathrm{Ei}(-x),$$

$$A_1 = \Psi_1\left(\sqrt{\frac{\nu_i}{\nu_e\beta}}\right)\exp\left(-\frac{|t'|\nu_i}{\alpha\beta}\right), \qquad A(x,y) = \frac{\lambda y}{4\sqrt{\pi r r_0}}\Gamma\left(-\frac{1}{2}\,\frac{y}{4\sqrt{x}}\right)$$

where Γ is an incomplete gamma-function and Ei is an integral exponential function. Equations (4.5) and (4.6) show that magnetic and charge neutralizations occur simultaneously only if

$$\lambda_2 = \lambda\sqrt{\beta\nu_e/\nu_i} \ll r_0. \tag{4.7}$$

The diffusion length is in this case the same as in the case of unmagnetized plasma. The magnetic field has a tubular structure at the distance $|z'| < z_d$. The thickness of the circular layer (or tube) varies depending on the distance to the beam front. It equals $\lambda_1 = \lambda/\sqrt{\alpha} \ll \lambda_2$ at the distance $|z'| \ll u/\nu_e$. The field B_φ in this current layer near the lateral beam surface is $\sim B_0\lambda_1/r_0$. As the distance is increased from u/ν to $u\beta\alpha/\nu_i$, the thickness of the tube increases reaching the value of $\sim\lambda_2$ and remains of the order of λ_2 up to the distance $u\beta/\nu_i$. In this layer of λ_2 the field $B_\varphi \sim B_0\lambda_2/r_0$. Finally, for $|z'| > u\beta/\nu_i$ the field B_φ assumes the same shape as in unmagnetized plasma[3] (at the corresponding distance $|z'| \gg u/\nu_e$).

If the condition $\lambda_1 \ll r_0 \ll \lambda_2$ is satisfied, magnetic neutralization occurs as before, but the diffusion length decreases down to $z_d^* = \alpha z_d \ll z_d$. The field B_φ at the distance of $|z'| < u/\nu_e$ is the same as under condition (4.7) at the corresponding distance. For $|z'| > u/\nu_e$ the layer that concentrates the field and current expands like $\sim r_0(|z'|/z_d^*)^{1/2}$. When $|z'|$ exceeds z_d^*, the self-field of the beam is restored. Finally, if $r_0 < \lambda_1$, the field B_φ throughout the beam volume is close to the beam self-field, i.e., the magnetic neutralization effect is absent.

As for the charge neutralization, it is effective throughout the beam volume under condition (4.7) and at the distance $|z'| < u/\nu_e$ the r dependence of charge density has a sheath configuration with a skin-layer thickness $\sim\lambda_2$ (the maximum density in the layer is low in comparison to ρ_0). On the other hand, if $r_0 < \lambda_2$, the beam charge is compensated only at the distance $|z'| > u\beta/\nu_i$; at a shorter distance the charge density in the main beam volume is not low: $\rho \approx \rho_0(1 - r/r_0)$ for $r < r_0$.

In the case of a stronger external field, when $v_A > u\gamma$ ($\alpha < 0$), the induced field B_φ at the distance $|z'| \gg u/\nu_e$ in the beam region $z' < 0$ (the functions F_{nm} are exponentially small here) is described by the same formulas as in[3], provided we substitute c/ω_{pe} by λ_2 and ν_e by ν_i/β. Hence it follows that only the thickness of the current layer changes (increases), while the diffusion length remains the same (z_d) as in the absence of the external magnetic field. Magnetic and charge neutralizations of the beam occur under the condition $r_0 \gg \lambda_2$ and the dependence of ρ on r and z' is the same as that of field B_φ. On the other hand, if $r_0 \lesssim \lambda_2$, there is no compensation of the beam magnetic field at the distance under consideration ($|z'| > u/\nu_e$) and the neutralization of the beam charge takes place as in the case of $\alpha > 0$ at a distance $|z'| > \beta u/\nu_i$ ($\rho \approx \rho_0$ for $|z'| < \beta u/\nu_i$).

Let us now say a few words about the perturbation of plasma in the region in front of the beam ($z' > 0$). In the case of a strong magnetic field $v_A > u\gamma$[5] non-exponentially small perturbation of plasma propagates to a considerable distance $z' \lesssim (u/\nu_i)(v_A^2/u^2\gamma^2 - 1)$ that increases with increasing magnetic field. The induced field B_φ at the distance $z' > u/\nu_e$ has either a tubular configuration (for $r_0 \gg \lambda_2$) or a volume configuration (for $r_0 \lesssim \lambda_2$) and, if $\lambda_1 \ll r_0 \lesssim \lambda_2$, the field is considerable ($B_\varphi \sim B_0$ for $r \lesssim r_0$) up to the distance of $z' \lesssim z_d^*$.

5. CONCLUSION

Thus, in the case of weakly magnetized plasma ($\Omega_i < \nu_i$) the condition for magnetic neutralization of the beam has the form $r_0 \gg \lambda(1 + \zeta)^{1/4}$ and the diffusion length remains the same (z_d) as in an unmagnetized plasma. The radial dependence of the azimuthal magnetic field changes with increasing external magnetic field intensity from a sheath configuration (with a skin-layer thickness of $-\lambda(1 + \zeta)^{1/4}$) to a periodic configuration. Charge neutralization can exist only at a distance of $|z'| < u/\nu_e$ if $\zeta \gtrsim 1$ and r_0^4/λ^4 (here magnetic neutralization is also absent).

A strong external magnetic field ($\Omega_i > \nu_i$) changes not only the conditions of magnetic and charge neutralization but also decreases (at $\lambda_1 \ll r_0 \ll \lambda_2$, $v_A < u\gamma$) the diffusion length ($z_d^* = \alpha z_d$). The condition for magnetic neutralization takes the form $r_0 \gg \lambda_1$ for $v_A < u\gamma$ or $r_0 \gg \lambda_2$ for $v_A > \gamma u$. Charge neutralization of the beam is absent if $r_0 \lesssim \lambda_2$ (regardless of the value of $v_A/u\gamma$) within a large volume ($|z'| \lesssim \beta u/\nu_i$).

The above analysis leads us to the following observation about the possibility of passing currents beyond cutoff through a plasma. In the case of weakly magnetized plasma ($\Omega_i < \nu_i$) the condition postulating a beam current larger than the limit ($I_b > I_A$, where $I_A = mc^2u\gamma/|e|$ is the Alfven current) practically coincides with the condition for the magnetic and charge neutralization of the beam. Therefore the flow of beyond-cutoff current through such plasma is possible. In strongly magnetized plasma ($\Omega_i > \nu_i$) the flow of beyond-cutoff current may turn out to be difficult because under the condition $r_0 \lesssim \lambda_2$ charge neutralization fails to extend to a sufficiently large beam volume ($|z'| \lesssim \beta u/\nu_i$). Therefore, when the field is strong enough so that $v_A \gtrsim r_0\omega_{pc}\sqrt{\nu_i/\nu_e}$, the beam is stopped by the space charge at the distance of $\sim(mc^3\gamma/2\pi e^2 n_0 u)^{1/2} \sim r_0\sqrt{I_A/I_b}$ from the injector[7].

We have obtained results for the case of a sharply bounded beam. However the current profile is usually diffuse in actual experiments with high-current electron beams. The corresponding results for such a beam can be obtained by averaging[5] the field, current, and charge induced in plasma by a sharply bounded beam over the volume of the diffuse beam. This changes the field (and current) structure in the plasma from a tubular into a volume configuration. The magnetic (current) neutralization takes place if the characteristic dimension L of the inhomogeneity introduced into the beam plasma is larger than the skin depth λ. For example, at the distance of $|z'| \lesssim u/\nu_e$ from the beam front, the total current through the beam cross section is $I \sim I_b(\lambda/L)^2$ instead of $I \sim I_b\lambda/r_0$ as in a beam with a sharp boundary. In the presence of an external magnetic field the quantity λ should be replaced in these expressions, depending on the field, by the corresponding skin depth $\lambda_{1,2}$ or $-\lambda(1 + \zeta)^{1/4}$.

We now compute the value of the magnetic field that can significantly affect the magnetic and charge neutralization of an electron beam in currently performed experiments with injecting high-current relativistic beams into a plasma (see for example the literature cited in[7]). We assume the following parameters for the beam: current I_b = 60 kA, electron energy E = 1 MeV ($\gamma \sim 3$), pulse length τ = 80 nsec, and beam radius $L \sim r_0 = 3$ cm. Under these conditions beam density $n_0 \sim 10^{11}$ cm^{-3} and the current exceeds the limit ($I_A \sim 50$ kA). The data for plasma are less specific. In the usual experiments with high-current beams plasma is formed when the beam itself ionizes a denser gas. Since beam beyond cutoff can propagate in gas only after the density of the newly formed plasma has exceeded the beam density, $n_p \gtrsim n_0$ (otherwise there is no neutralization of the self-fields of the beam that block its propagation), we can assume for our computation that $n_p \sim 10^{12}$ cm^{-3} (the theory requires that $n_p \gg n_0$). Then the parameter $\lambda/r_0 \sim \lambda/L \sim 0.2$ and the degree of neutralization of the beam current $\eta = (1 - I/I_b)100\%$ is approximately 96% (for the sharply bounded beam η would be ~80%) which from the experimental viewpoint is quite realistic for our parameters. For a plasma temperature of the order of several ev, the collision frequency of plasma electrons is $\nu_e \sim 10^7$ sec^{-1}. The density of neutral particles remaining in plasma is not high since we have neglected collisions of the beam electrons.

If the beam were neutralized in an unmagnetized plasma, i.e., if $\lambda \ll r_0$ (r_0 should be replaced by L from now on for the case of the diffuse beam) and $\omega_{pe} \gg \nu_e$, the beam would not be neutralized in magnetoactive plasma for $\Omega_e/\omega_{pe} \gtrsim (r_0/\lambda)^2 \gg 1$ in the case of $\Omega_i \ll \nu_i$ when the ionic effect is insignificant [6]. Rewriting this inequality in the form

$$\Omega_i/\nu_i \geqslant (\omega_{pe}/\nu_e)\sqrt{m/M}(r_0/\lambda)^2,$$

we conclude that the axial magnetic field that precludes neutralization of the self-field of the beam should actually be much larger ($\Omega_i \gg \nu_i$). According to the research performed here for fields $\Omega_i \gg \nu_i$, the minimum external magnetic field intensity, in which the diffusion length (this length limits the longitudinal dimension of the beyond-cutoff beam) can shorten and the charge or magnetic (current) neutralization can be absent, is determined by the relation ($\lambda_2 \gtrsim r_0$, $\Omega_i \gtrsim \omega_{pi}$)

$$B_{0\,min} \sim \max\{10 r_0^{-1}(r_0/\lambda)^2,\ 10^{-4}\sqrt{n_p}\}\ \text{kG},$$

where r_0 and λ are in cm and n_p is in cm^{-3}. Consequently the compensation of a beam with a radius of the order of several cm and under the conditions of $r_0^2 \sim L^2 \gg \lambda^2$, $n_p \gg n_0$, $\omega_{pe} \gg \nu_e$ (otherwise neutralization would be absent even without the external magnetic field) can be degraded only by quite high external fields measuring hundreds of kilogauss and more. The experiments have so far been performed only with fields of the order of tens of kilogauss and less; they naturally failed to reveal any significant effect of such fields on the neutralization of high-current relativistic electron beams.

We finally note that in addition to the low-frequency diffusion type perturbations due to plasma particle collisions, investigated here, there are also Cerenkov radiation fields at the distance of $|z'| < u/\nu_i$ from the beam front (and also in the beam itself). The minimum frequency of these fields is Ω_i. Therefore if the characteristic current rise time τ_0 exceeds Ω_i^{-1} all oscillatory perturbations are averaged over the beam volume [5] (incoherent radiation) and make a small contribution to the diffusion fields in the beam region. In the high-current experiments τ_0 is of the order of tens of nsec so that for external fields over tens of kilogauss the condition $\tau_0\Omega_i > 1$ is satisfied and the effect of high-frequency fields is not significant to beam neutralization.

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1) For the sake of simplicity we assume an infinite injection time. The generalization to the case of a finite injection time is obvious [4].

2) The corresponding roots $\chi_{7,8}$ (and also $\chi_{5,6}$) found in [6] for case (a) are incorrectly stated for large k; that however is immaterial to the results obtained there.

3) The results obtained below are qualitatively valid also for $\Omega_i \sim \omega_{pi}$ and $\gamma \sim 1$. However the effect of the external magnetic field under these conditions is still not very significant.

4) The resonance case $\alpha \to 0$ is not considered on the assumption that $|1 - v_A^2/u^2\gamma^2| > \nu_i/\nu_e$.

5) At $v_A < u\gamma$, the plasma perturbation ahead of the beam propagates to a much shorter distance $z' \lesssim u/\Omega_i$, and is not due to particle collisions.

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