Optical frequency modulation of electrons diffracted by a crystal

V. V. Fedorov, K. E. Kir'yanov, and A. I. Smirnov
Leningrad Institute of Nuclear Physics, USSR Academy of Sciences
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The mechanism of modulation of an electron beam passing through a single crystal plate that diffracts it is considered. The possibility of photon emission (or absorption) by an electron is due in this case to distortion of the electron dispersion surface. The frequency and direction of polarization of the modulating radiation are determined by the frequency and direction of electron vibrations in the crystal (Pendellösung effect). The frequencies may lie in a broad region, from infrared to soft x-rays. Estimates show that the modulation index may reach several percent.

1. The modulation of an electron beam passing through a thin dielectric film illuminated by a laser beam is due, as shown in [1,2], to the interference between states of different energies, resulting from stimulated absorption (emission) of n quanta $h\nu$ ($n = 0, \pm 1, \pm 2$) by the electron. The cited papers considered the case when the absorption of the photons by the electrons is due to the jump of the dielectric constant near the film boundaries. The modulation depth depends in this case periodically on the film thickness. A more effective modulation mechanism, based on the inverse Cerenkov effect, is considered in [3,4]. In this case the photons are absorbed in the entire volume of the dielectric, so that the depth of modulation increases with the film thickness[4].

2. We consider in the present paper the possibility of modulating an electron beam as it passes through a single-crystal film under diffraction conditions. The emission (absorption) of photons can occur in this case in the entire thickness of the crystal, and is allowed by the energy-momentum conservation laws, owing to distortions of the electron dispersion surface (which is an equal-energy surface in momentum space) by the diffraction[1)].

Such a modulation mechanism has the following feature. The frequency and direction of the polarization of the modulating light are determined by the so-called pendulum (Pendellösung) effect[5], which is produced when electrons are diffracted by a sufficiently thick film $D \gtrsim \xi_g$ (ξ_g is the extinction length), i.e., they are connected with the frequency and direction of the vibrations of the electron that is diffracted in the crystal (see (6) and (9)).

3. We consider for simplicity one system of crystallographic planes in a crystal. This system is characterized by a reciprocal-lattice vector $\mathbf{g}$ ($|\mathbf{g}| = 1/d$, where d is the distance between planes). Let electrons of energy $\mathscr{E}_0$ ($\mathscr{E}_0/mc^2 \ll 1$) be incident only at the Bragg angle θ to these planes, as indicated in the figure. In this case, as is known when the dynamic theory of diffraction (see, e.g., [6]), the propagation of the electron in a crystal is described by a superposition of Bloch wave functions

$$\psi_0 = \frac{1}{\sqrt{2\Omega}}[b^{(1)}(\mathbf{k}_0^{(1)}, \mathbf{r}) + b^{(2)}(\mathbf{k}_0^{(2)}, \mathbf{r})]\exp\left(-\frac{2\pi i}{h}\mathscr{E}_0 t\right) \quad (1)$$

where

$$b^{(1)}(\mathbf{k}_0^{(1)}, \mathbf{r}) = i\sqrt{2}\sin(\pi \mathbf{g}\mathbf{r})\exp[2\pi i(\mathbf{k}_0^{(1)} + {}^1/_2\mathbf{g})\mathbf{r}],$$
$$b^{(2)}(\mathbf{k}_0^{(2)}, \mathbf{r}) = \sqrt{2}\cos(\pi \mathbf{g}\mathbf{r})\exp[2\pi i(\mathbf{k}_0^{(2)} + {}^1/_2\mathbf{g})\mathbf{r}], \quad (2)$$

$h\mathbf{k}_0^{(1,2)}$ is the quasimomentum of the electron in the crystal, and Ω is the normalization volume. The subscripts 1 and 2 pertain to the corresponding branches of the dispersion surface of the electron. Its equation is given by

$$(k_0 - K_0)(|\mathbf{k}_0 + \mathbf{g}| - K_0) = U_g^2/4K_0^2. \quad (3)$$

Here

$$K_0 = (2m\mathscr{E}_0/h^2 + U_0)^{1/2} = (2m(\mathscr{E}_0 + V_0)/h^2)^{1/2},$$

V_0 is the average potential in the crystal, $V_g = h^2U_g/2m$ is the amplitude of the first harmonic of the periodic potential of the lattice. If the Bragg condition

$$|\mathbf{k}_0 + \mathbf{g}| = k_0 \quad (4)$$

is satisfied, Eq. (3) takes the simpler form

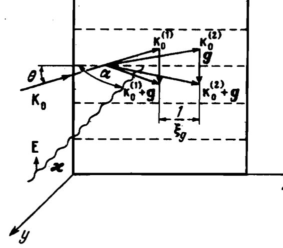
$$k_0^{(1,2)} - K_0 = \mp U_g/2K_0. \quad (5)$$

The minus and plus signs in the right-hand side of (5) pertain to the first and second branches of the dispersion surface, respectively.

Let the photons propagate at an angle α to the z direction (see the figure). If $h\nu \ll \mathscr{E}_0$, then the emission or absorption of a photon by an electron hardly violates the Bragg condition. It follows then from (5) and from the energy-momentum conservation laws that a transition with emission (absorption) of a quantum $h\nu$ is possible from branch 1 (2) of the dispersion surface corresponding to the energy $\mathscr{E}_0$, to the branch 2 (1) of the surface of lower (higher) energy. This determines uniquely the photon wavelength[2)]:

$$\lambda = \frac{mc}{hK_0\cos\theta}\xi_g\left(1 + \frac{hK_0\cos\theta}{mc}\cos\alpha\right) \approx \frac{c}{v\cos\theta}\xi_g\left(1 + \frac{v\cos\theta}{c}\cos\alpha\right) \quad (6)$$

$\xi_g = K_0\cos\theta/U_g$ is the extinction length. Expression (6)



Electrons with momentum hK_0 are incident at the Bragg angle on reflecting planes that are parallel to the (yz) plane. The vectors $\mathbf{K}_0$, $\mathbf{k}_0^{(i)}$, and $\mathbf{k}_0^{(i)} + \mathbf{g}$ lie in the (xz) plane. The maximum depth of modulation is produced when $\boldsymbol{\kappa}$ lies in a plane parallel to the crystallographic planes, while $\mathbf{E}$ is parallel to $\mathbf{g}$.

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was obtained under the assumption that $d \ll \lambda$ and $\lambda_e \ll \xi_g$ (λ_e is the de Broglie wavelength of the electron). It has a simple physical meaning. An electron propagating in the crystal in an average direction parallel to the z axis and at a velocity $v \cos\theta$, changes its direction of motion with a spatial period ξ_g as a result of beats between the waves (Pendellösung effect), i.e., it oscillates in the direction of g with frequency $\nu_e = v\cos\theta/\xi_g$. The wavelength λ of the radiated light is determined, depending on the direction, by the Doppler effect:

$$\lambda = \frac{c}{v\cos\theta}\xi_g\left(1+\frac{v\cos\theta}{c}\cos\alpha\right),$$

which coincides with (6). From the foregoing reasoning it follows also that the light will be polarized along the vector **g** (see (9)). The extinction lengths of electrons with energies, say, 10—100 keV lie in the wide range from 100 to 10,000 Å, depending on the crystal and on the system of reflecting planes[6]. Consequently, the frequency of the modulating radiation can be chosen in the range from the infrared to the soft x-ray part of the spectrum.

4. The depth of modulation γ of the electron beam at the frequency ν, on going through a film placed in the field of an electromagnetic wave, is determined by the amplitude $a_{0\pm\nu}$ of the stimulated absorption (emission) of a quantum $h\nu$ by the electron[1,2]:

$$\gamma = 2|a_{0\pm\nu}|. \tag{7}$$

In first-order perturbation theory, the amplitude of the stimulated absorption of a photon by an electron passing through a crystal of thickness D under diffraction conditions is

$$a_{0\pm\nu} = -\frac{2\pi i}{h}\int\limits_0^{D/v\cos\theta}\langle\psi_{\pm\nu}|w|\psi_0\rangle dt, \tag{8}$$

where

$$w = -\frac{e}{mc}(\mathbf{A}\mathbf{p}), \quad \mathbf{A} = A_0\mathbf{u}\cos 2\pi(\varkappa\mathbf{r} - \nu t)$$

A is the vector potential of the modulating field and **u** is a unit polarization vector. Calculating (8) and expressing A_0 in terms of the amplitude of the electric field intensity E_0 in the electromagnetic wave, we obtain

$$\gamma = \frac{1}{4}\frac{c}{v\cos\theta}\frac{eE_0\lambda}{mc^2}\frac{D}{d}\cos(\widehat{\mathbf{ug}}). \tag{9}$$

For electrons with energy 50 keV ($\theta \approx 0.05$), in the region of the visible light, say λ = 6000 Å (which corresponds to an extinction length $\xi_g \approx 1500$ Å) we have $\gamma = 2\%$ at E_0 = 200 cgs esu and D = 2000 Å. This is of the same order as obtained for absorption at the film boundary[1,2]. In the present case, however, a larger depth of modulation can be obtained by increasing in the interaction time $D/v\cos\theta$.

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1)In the cases of[1-4], the energy-momentum hinderance is lifted by distortion of the dispersion surface of the photon on the vacuum-dielectric boundary and inside the dielectric, respectively.

2)Since usually $V_0 \ll \mathscr{E}_0$, it follows that $hK_0 \approx mv$.

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