Stability of nonequilibrium Fermi distributions with respect to Cooper pairing

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The stability of normal and superconducting nonequilibrium Fermi systems relative to Cooper pairing and Cooper pair decay, respectively, is investigated. If the inequality $\hbar / \tau\Delta \ll 1$ is obeyed, where Δ is the superconducting gap thickness and τ is the electron-phonon collision time, the stability condition does not depend explicitly on the method whereby the imbalance is produced, and is expressed only in terms of the superconducting excitation distribution function $m(\epsilon)$. It is shown that if the sign of the difference $1 - 2m(\epsilon)$ is constant throughout the complete energy region ϵ, the superconducting state will be stable. The stability condition obtained in the paper is used to analyze a number of concrete cases.

1. INTRODUCTION

The question of the possibility of Cooper pairing of electrons under strongly nonequilibrium conditions has recently evoked considerable interest from the experimental point of view[1,2] as well as from the point of view of theoretical developments.[3-12]

As Éliashberg has shown,[6] in a number of interesting cases even in nonequilibrium systems the gap parameter Δ is determined from the BCS equation, in which it is only necessary to replace the equilibrium distribution function of the superconducting excitations by the nonequilibrium one:

$$\Delta = -\lambda N(0) \int_{\Delta}^{\omega_D} \Delta \frac{n(-\varepsilon) - n(\varepsilon)}{(\varepsilon^2 - \Delta^2)^{1/2}} d\varepsilon, \tag{1.1}$$

where λ is the effective coupling constant of the electron-electron interaction, N(0) is the density of electron states near the Fermi surface, ω_D is the limiting frequency of the Debye phonons, and $n(\epsilon)$ is the (nonequilibrium) distribution function of the superconducting excitations. It is determined by the type of external perturbation creating the nonequilibrium state, and it also depends on Δ as a parameter. If the distribution function is anisotropic, then $n(\epsilon)$ should be understood as the corresponding average value over a surface of constant energy. In the so-called symmetric case, which we shall mainly be occupied with in what follows, one has

$$n(-\varepsilon) = 1 - n(\varepsilon). \tag{1.2}$$

Let Eq. (1.1) have a nontrivial solution. For the sake of definiteness, we first assume this solution to be unique. In addition, a solution with $\Delta = 0$ also exists. Thus arises the necessity of comparing these two solutions and ascertaining which of them will actually be realized.

In the equilibrium case this question can be resolved on the basis of simple energy considerations. The solution with $\Delta \neq 0$ would correspond to a smaller energy of the ground state (or, at finite temperatures below the critical temperature—it would correspond to a smaller value of the free energy) than the solution with $\Delta = 0$. In nonequilibrium cases simple energy considerations do not permit us to compare these two states since the energy of the superconductor is not conserved: it interacts with the external system creating the nonequilibrium. Therefore, in order to solve the problem which has been posed, it is natural to investigate the stability of these two states: the state with $\Delta = 0$ with regard to Cooper pairing, and the state with $\Delta \neq 0$ with regard to the destruction of the pairs. In order to do this it is necessary to calculate the two-particle retarded Green's function of the electrons in a conductor for small total momentum and energy of the electrons. If the Green's function, regarded as a function of the total energy, has a pole in the upper half-plane, this implies instability. The instability of a normal Fermi system with attractive interactions for T = 0 is established in such a manner.[13a] The stability of the normal, nonequilibrium Fermi distribution was first investigated by Batyev[4] in the same way, using Keldysh's diagram technique.[14]

The stability of the state with $\Delta \neq 0$ with respect to small fluctuations of the order parameter Δ is investigated in the present article. The solution of the stability problem is greatly facilitated owing to the fact that here the nonequilibrium distribution function of the excitations can be regarded as fixed. The point is that in order to investigate the stability of the system with regard to pairing, it is sufficient to trace its evolution during a time interval of order $\hbar/\Delta$. This time interval is assumed to be much smaller than the characteristic electron-phonon collision time τ, during which pronounced changes occur in the average values of the quasi-particle distribution functions. We substantially utilize the inequality $\hbar/\tau\Delta \ll 1$ in the construction of the theory. It is found that the superconducting nonequilibrium state is stable in all cases when Eq. (1.1) has a nontrivial solution and the difference $n(-\epsilon) - n(\epsilon)$ keeps the same sign for all $\epsilon > 0$.

However, if the sign of the difference $n(-\epsilon) - n(\epsilon)$ is not constant, then the system generally possesses a number of nontrivial properties. Thus, additional modes of collective oscillations, which are not present in the equilibrium superconductor, may exist in it. Furthermore, the system may turn out to be unstable with respect to fluctuations of the order parameter Δ, where the instability manifests itself as an increase in the amplitude of these collective oscillations. An example of such an instability is considered in Sec. 4.

For a system in an external field, the investigation of stability is valid only when the momentum acquired by a Cooper pair during the field's period $1/\Omega$ (or, in the case of a constant field, over the length of the sample) is small in comparison with the coherence length ξ:[1)]

$$eE / \Omega \ll \hbar / \xi = \hbar m\Delta / p_F,$$

where p_F is the "Fermi momentum," that is, the momentum corresponding to the middle of the energy gap. This criterion turns out to be substantially more stringent than the requirement that $e^2E^2/m\,\Omega^2$ be small in comparison with Δ; it characterizes the accuracy of our theory in determining the critical value Δ_c, corresponding to the boundary of the stability region.

In the case of a normal conductor one can show that the influence of the field can be neglected if its work on a pair during one period of the field is small in comparison with the increment of growth of the amplitude of the Cooper pair wave function. The theory can actually be applied to the case when the electron distribution function is noticeably perturbed near the Fermi surface whereas the influence of the field on the dynamics of the electrons is negligible—for example, if the nonequilibrium distribution appeared in connection with the illumination of a thin metal film.

Hitherto the question has only been the dynamical stability of the system with respect to Cooper pairing. However, since the topic of discussion is strongly nonequilibrium systems it is necessary in general to also investigate their stability with respect to perturbations of the quasi-particle distribution functions. This investigation is carried out mainly by standard methods with the help of the kinetic equation. It is only necessary to keep the following point in mind. It turns out to be possible to investigate the dynamical stability of the system for a fixed quasi-particle distribution function. On the other hand, it is impossible to investigate the stability of the distribution function for a fixed spectrum because the magnitude of the gap Δ and together with it the transition probabilities as well depend on the shape of the distribution function. Therefore, a perturbation of the distribution function must lead to a perturbation of the order parameter, and therefore also leads to a perturbation of the transition probabilities.

We shall use the results of the general theory in order to investigate the stability in a number of specific cases. In particular, one unusual example is considered—the so-called inverse Fermi distribution. We want to explain[2)] the fact (which appears to be paradoxical at first glance) that Eq. (1.1) has a nonvanishing solution Δ for $\lambda > 0$ (repulsion) for the case when the distribution function $n(\epsilon)$ has the following form in the energy range of interest to us: $n(\epsilon) = 1$ for $\epsilon > 0$ and $n(\epsilon) = 0$ for $\epsilon < 0$. In this case the solution with $\Delta = 0$ turns out to be unstable with respect to pairing, and the solution with $\Delta \neq 0$ turns out to be stable.

One might describe the physics of this bound state in the following way. The inverse distribution of Fermi particles is equivalent to an ordinary distribution of Fermi holes with a negative effective mass. But, as is clear from an analysis of the corresponding Schrödinger equation, particles with negative effective mass form bound states in the presence of repulsion. The case of the bielectron[15] is a well-known example of this.

The bound states, which arise in the present case, form a Bose condensate, a gap appears in the spectrum of the single-particle excitations, and the total energy of the system increases upon pairing. One can verify this, in particular, by calculating the energy of the system as a function of Δ. The energy has a maximum at the value of Δ determined from condition (1.1).

Thus, in the present example we verify that energy considerations in general do not permit us to judge the dynamical stability of a nonequilibrium system.

It should be noted that it is necessary to consider the inverse Fermi distribution as a purely model example because of the following reason. If it is assumed that, upon a further increase of the energy ϵ within the limits of the same band, the function $n(\epsilon)$ again vanishes, then even though such a scheme is stable with respect to the generation of plasmons and other longitudinal oscillations,[13b] the characteristic electron-electron collision time τ_e in it turns out to be so small that the parameter $\hbar/\tau_e\Delta$ turns out to be of the order of unity in the best case. However, if $n(\epsilon) = 1$ right up to the edge of the forbidden band, then the corresponding system is unstable with respect to the generation of plasmons and other longitudinal oscillations.

We shall use the diagram technique developed by Keldysh.[14] It permits us to determine simultaneously the energy spectrum of a nonequilibrium system and also its kinetic characteristics, that is, in our case we can determine the nonequilibrium distribution function of the superconducting excitations. In particular, an expression for the operator describing the collisions of the excitations with phonons is obtained by such a method. The corresponding expression was previously obtained by Éliashberg.[6] [3)] However, our expression (2.18) is applicable to any anisotropic, nonequilibrium distribution function, and is not averaged over angles, as was done in[6].

2. DERIVATION OF THE KINETIC EQUATION

Following Keldysh,[14] we define the normal Green's function of the electrons in the following way:

$$\hat{G}^{\alpha\beta}(x,x')=\begin{pmatrix} G_c^{\alpha\beta}(x,x') & G_+^{\alpha\beta}(x,x') \\ G_-^{\alpha\beta}(x,x') & \tilde{G}_c^{\alpha\beta}(x,x')\end{pmatrix}=\begin{pmatrix} G_{11}^{\alpha\beta} & G_{12}^{\alpha\beta} \\ G_{21}^{\alpha\beta} & G_{22}^{\alpha\beta}\end{pmatrix}$$
$$=\begin{pmatrix} -i\langle T\psi_\alpha(x)\psi_\beta^+(x')\rangle, & i\langle\psi_\beta^+(x')\psi_\alpha(x)\rangle \\ -i\langle\psi_\alpha(x)\psi_\beta^+(x')\rangle, & -i\langle\tilde{T}\psi_\alpha(x)\psi_\beta^+(x')\rangle\end{pmatrix}. \tag{2.1}$$

Here α and β are spin indices, T and $\tilde{T}$ denote T-products and anti-T-products, respectively, and the angular brackets denote averages with respect to the unperturbed Hamiltonian.

In analogous fashion we introduce the matrix functions $\hat{F}$ and $^+\hat{F}$:

$$F^{\alpha\beta}(x,x')=\begin{pmatrix} \langle T\psi_\alpha(x)\psi_\beta(x')\rangle, & -\langle\psi_\beta(x')\psi_\alpha(x)\rangle \\ \langle\psi_\alpha(x)\psi_\beta(x')\rangle, & \langle\tilde{T}\psi_\alpha(x)\psi_\beta(x')\rangle\end{pmatrix}$$
$$=\begin{pmatrix} F_c^{\alpha\beta}(x,x') & F_+^{\alpha\beta}(x,x') \\ F_-^{\alpha\beta}(x,x') & \tilde{F}_c^{\alpha\beta}(x,x')\end{pmatrix}. \tag{2.2}$$

The function $^+\hat{F}$ is obtained from $\hat{F}$ by replacing all of the operators ψ in Eq. (2.2) by ψ^+.

As is usually done, let us assume that the interaction does not depend on the spins. Then

$$\hat{G}^{\alpha\beta}(x,x')=\hat{G}(x,x')\delta_{\alpha\beta},\quad \hat{F}^{\alpha\beta}(x,x')=-\hat{F}(x,x')I_{\alpha\beta},$$
$$\hat{I}=\begin{pmatrix} 0 & 1 \\ -1 & 0\end{pmatrix}i\sigma_y.$$

Similarly we find:

$$^+F^{\alpha\beta}={}^+FI_{\alpha\beta}.$$

The phonon Green's function is given by

$$\hat{D}=\begin{pmatrix} D_c D_+ \\ D_- \tilde{D}_c\end{pmatrix}=\begin{pmatrix} -i\langle T\varphi(x)\varphi(x')\rangle, & -i\langle\varphi(x')\varphi(x)\rangle \\ -i\langle\varphi(x)\varphi(x')\rangle, & -i\langle\tilde{T}\varphi(x)\varphi(x')\rangle\end{pmatrix}. \tag{2.3}$$

Later we shall be interested in the case of a spatially homogeneous system, where the Green's functions depend on the difference between the spatial coordinates. Then, assuming that the Green's functions are slowly varying functions of the total time t, and evaluating the Fourier component with respect to the difference of coordinates and times, we obtain the following result in the zero-order approximation (furthermore, we take $\hbar = 1$):

$$\hat{G}_0^{-1} = \left(\frac{i}{2}\frac{\partial}{\partial t} + \varepsilon - \xi_p\right)\hat{\sigma}_z, \tag{2.4}$$

where σ_z is a matrix which acts on the lower indices (not the spin indices) of the Green's function, $\xi_p = (p^2/2m) - \mu$, μ denotes the origin with respect to which the energy is measured; later on we choose this to coincide with the middle of the superconducting energy gap.

The Feynman diagram technique can be used for the functions $\hat{G}$ and $\hat{F}$, and consequently the corresponding system of Dyson equations has the form

$$\left(\frac{i}{2}\frac{\partial}{\partial t} + \varepsilon - \xi_p\right)\sigma_z\hat{G}(p,t) = \hat{1} + i\hat{\Sigma}(p,t)\hat{G}(p,t) + i\hat{\Delta}(p,t)^+\hat{F}(p,t),$$
$$\left(\frac{i}{2}\frac{\partial}{\partial t} - \varepsilon - \xi_p\right)\sigma_z{}^+\hat{F}(p,t) = i\hat{\Sigma}^T(-p,t)^+\hat{F}(p,t) + i^+\hat{\Delta}(p,t)\hat{G}(p,t). \tag{2.5}$$

One can isolate the system of equations for $G_c(p, t)$ and $^+F_c(p, t)$ from the system (2.5) by taking the 11-component in the matrix equations. In this connection we discard the time derivative and also all terms which contain the functions G_+, D_+, and $^+F_+$. The latter step is permissible if the parameter $1/\tau\Delta$ is small, where τ is the characteristic time for electron-phonon collisions.

As an example let us present the expression for the self-energy part Δ_c, which appears in the right-hand side of the equation

$$\Delta_c(\varepsilon, \mathbf{p}) = \int \frac{d^3q}{(2\pi)^3}\int \frac{d\omega}{2\pi}|c_\mathbf{q}|^2 D_c(-\mathbf{q}, -\omega)F_c(\mathbf{p}-\mathbf{q}, \varepsilon-\omega), \tag{2.6}$$

where $c_\mathbf{q}$ is the electron-phonon coupling constant. Just as in the equilibrium case, the dispersion $\Delta_c(\epsilon, \mathbf{p}, t)$ arises for $\epsilon \sim \omega_D$. For our purposes, however, as will be clear from what follows below, energies $\epsilon \ll \omega_D$ are important. In addition, just like in the equilibrium case, frequencies $\omega \approx \Delta(0, p_F) \ll \omega_D$ are essential in Eq. (2.6). Taking these inequalities into account, we obtain, just as in the equilibrium case, the following equation for $\Delta_c(0, p_F)$:

$$\Delta = -\frac{\lambda}{2}\int \frac{d^3p}{(2\pi)^3}\int \frac{d\varepsilon}{2\pi}F_c(\varepsilon, \mathbf{p}), \tag{2.7}$$

where λ is a constant.

For values of $\epsilon \ll \omega_D$ the solution of the system of equations (2.5) for the functions G_c and F_c has the form

$$G_c(\varepsilon, \mathbf{p}) = u_p^2\left(\frac{m_\mathbf{p}}{\varepsilon - \varepsilon_\mathbf{p} - i\delta} + \frac{1 - m_\mathbf{p}}{\varepsilon - \varepsilon_\mathbf{p} + i\delta}\right) + v_p^2\left(\frac{m_{-\mathbf{p}}}{\varepsilon + \varepsilon_\mathbf{p} + i\delta} + \frac{1 - m_{-\mathbf{p}}}{\varepsilon + \varepsilon_\mathbf{p} - i\delta}\right),$$
$$^+F_c(\varepsilon, \mathbf{p}) = F_c(\varepsilon, \mathbf{p}) = -iu_pv_p\left(\frac{m_\mathbf{p}}{\varepsilon - \varepsilon_\mathbf{p} - i\delta} + \frac{1 - m_\mathbf{p}}{\varepsilon - \varepsilon_\mathbf{p} + i\delta} - \frac{m_{-\mathbf{p}}}{\varepsilon + \varepsilon_\mathbf{p} + i\delta} - \frac{1 - m_{-\mathbf{p}}}{\varepsilon + \varepsilon_\mathbf{p} - i\delta}\right), \tag{2.8}$$
$$u_p^2 = \frac{1}{2}\left(1 + \frac{\xi_p}{\varepsilon_p}\right), \quad v_p^2 = \frac{1}{2}\left(1 - \frac{\xi_p}{\varepsilon_p}\right), \quad \varepsilon_p = (\Delta^2 + \xi_p^2)^{1/2}.$$

Here $m_\mathbf{p}$ is an undetermined (for the time being) positive function whose absolute value does not exceed unity. We shall call it the distribution function of the excitations, and we shall write down a kinetic equation for it below. One can verify that it is related to the distribution function of the quasi-electrons by the following equations:

$$m_\mathbf{p} = n_\mathbf{p} \quad \text{for} \quad \xi_\mathbf{p} > 0,$$
$$m_\mathbf{p} = 1 - n_\mathbf{p} \quad \text{for} \quad \xi_\mathbf{p} < 0. \tag{2.9}$$

By substituting expressions (2.8) into Eq. (2.7) we obtain Eq. (1.1) of the BCS theory. Using the definitions of the Green's functions, it is not difficult to obtain the following spectral representations for the causal functions:

$$G_c(\mathbf{p}, \varepsilon) = \int_{-\infty}^{\infty}\frac{d\varepsilon'}{2\pi i}\left[\frac{G_+(\varepsilon', \mathbf{p})}{\varepsilon - \varepsilon' - i\delta} - \frac{G_-(\varepsilon', \mathbf{p})}{\varepsilon - \varepsilon' + i\delta}\right],$$
$$^+F_c(\mathbf{p}, \varepsilon) = \int_{-\infty}^{\infty}\frac{d\varepsilon'}{2\pi i}\left[\frac{^+F_-(\varepsilon', p)}{\varepsilon - \varepsilon' - i\delta} - \frac{^+F_+(\varepsilon', \mathbf{p})}{\varepsilon - \varepsilon' + i\delta}\right]. \tag{2.10}$$

One can write down a similar relationship for $F_c(\epsilon, \mathbf{p})$. The relationships for the anti-causal functions are obtained if we interchange the positions of the functions with plus and minus subscripts on the right-hand side of (2.10).

Comparing expressions (2.10) with (2.8), we find

$$G_+(\varepsilon, \mathbf{p}) = 2\pi i[u_p^2 m_\mathbf{p}\delta(\varepsilon - \varepsilon_\mathbf{p}) + v_p^2(1 - m_{-\mathbf{p}})\delta(\varepsilon + \varepsilon_\mathbf{p})],$$
$$G_-(\varepsilon, \mathbf{p}) = -2\pi i[u_p^2(1 - m_\mathbf{p})\delta(\varepsilon - \varepsilon_\mathbf{p}) + v_p^2 m_{-\mathbf{p}}\delta(\varepsilon + \varepsilon_\mathbf{p})], \tag{2.11}$$

$$^+F_+(\varepsilon, \mathbf{p}) = {}^+F_-(-\varepsilon, -\mathbf{p}) = F_+(\varepsilon, \mathbf{p})$$
$$= -2\pi u_pv_p[(1 - m_\mathbf{p})\delta(\varepsilon - \varepsilon_\mathbf{p}) - m_{-\mathbf{p}}\delta(\varepsilon + \varepsilon_\mathbf{p})]. \tag{2.12}$$

It is clear from (2.11) that)

$$m_\mathbf{p} = \int_0^{\infty}\frac{d\varepsilon}{2\pi i}[G_+(p) - G_-(-p)]. \tag{2.13}$$

Thus, in order to derive the kinetic equation for the function $m_\mathbf{p}$, it is necessary to obtain equations for $G_+(p)$ and $G_-(p)$. Since these functions are purely imaginary, by taking the real part of the matrix component 12 of the first equation of (2.5) we obtain

$$\frac{\partial}{\partial t}\operatorname{Im} G_+ = \operatorname{Im}\{\Sigma_+G_- - \Sigma_-G_+ + \Delta_+{}^+F_- - \Delta_-{}^+F_+\}. \tag{2.14}$$

Here, in connection with the transformation of the right-hand part, it is convenient to use the Lehmann representations for the self-energy parts, which have the form

$$\Sigma_c(\mathbf{p}, \varepsilon) = -\int \frac{d\varepsilon'}{2\pi i}\left[\frac{\Sigma_+(\mathbf{p}, \varepsilon')}{\varepsilon - \varepsilon' - i\delta} - \frac{\Sigma_-(\mathbf{p}, \varepsilon')}{\varepsilon - \varepsilon' + i\delta}\right]. \tag{2.15}$$

This can easily be checked in lowest-order perturbation theory, which is the approximation of interest to us.

As an example, let us write down the perturbation theory expression for one of the self-energy parts, namely Δ_-:

$$\Delta_-(\varepsilon, \mathbf{p}) = \int \frac{d^3q}{(2\pi)^3}\int \frac{d\omega}{2\pi}|c_\mathbf{q}|^2 D_-(\omega, \mathbf{q})F_-(\varepsilon - \omega, \mathbf{p} - \mathbf{q}). \tag{2.16}$$

Here $D_-(\omega, \mathbf{q})$ is the phonon propagator. In the lowest-order approximation we find

$$D_\pm(\omega, \mathbf{q}) = -2\pi i[(N_{\mp\mathbf{q}} + 1)\delta(\omega \pm \omega_\mathbf{q}) + N_{\pm\mathbf{q}}\delta(\omega \mp \omega_\mathbf{q})], \tag{2.17}$$

where $N_\mathbf{q}$ is the phonon distribution function.

A similar relationship is also obtained for $G_-(-p)$. Subtracting one from the other, substituting the explicit expressions for the propagators and the self-energy parts into the right-hand side, and then integrating over ϵ in the semi-infinite limits, we obtain the following expression for the collision integral:

$$\left(\frac{\partial m_\mathbf{p}}{\partial t}\right)_{\text{coll}} = 2\pi\sum_\mathbf{q}|c_\mathbf{q}|^2\{(1 - m_\mathbf{p})m_{\mathbf{p}-\mathbf{q}}(u_pu_{\mathbf{p}-\mathbf{q}} - v_pv_{\mathbf{p}-\mathbf{q}})^2$$

$$\times[(1+N_{-q})\delta(\varepsilon_p-\varepsilon_{p-q}+\omega_q)+N_q\delta(\varepsilon_p-\varepsilon_{p-q}-\omega_q)]-m_p(1-m_{p-q})$$
$$\times(u_pu_{p-q}-v_pv_{p-q})^2[N_{-q}\delta(\varepsilon_p-\varepsilon_{p-q}+\omega_q)+(1+N_q)\delta(\varepsilon_p-\varepsilon_{p-q}-\omega_q)]$$
$$+(1-m_p)(1-m_{-p+q})(u_pv_{p-q}+u_{p-q}v_p)^2[(1+N_{-q})\delta(\varepsilon_p+\varepsilon_{p-q}+\omega_q)$$
$$+N_q\delta(\varepsilon_p+\varepsilon_{p-q}-\omega_q)]-m_pm_{-p+q}(u_pv_{p-q}+u_{p-q}v_p)^2$$
$$\times[N_q\delta(\varepsilon_p+\varepsilon_{p-q}+\omega_q)+(1+N_{-q})\delta(\varepsilon_p+\varepsilon_{p-q}-\omega_q)]\}. \quad (2.18)$$

The first two terms on the right-hand side describe processes which conserve the number of excitations, that is, the scattering of excitations accompanied by the absorption or emission of phonons. The third and fourth terms describe the creation of a pair of quasi-particles from the condensate with the absorption of a phonon and the annilation of a pair with the emission of a phonon. The terms containing δ-functions of the form $\delta(\epsilon_p + \epsilon_{p-q} + \omega_p)$ vanish identically since in the quasi-particle representation we are using $\epsilon_p > 0$. However, instead of quasi-electrons above the gap and quasi-holes below the gap, as we are now doing, one can write down the kinetic equation in another representation used by us earlier,[10,11] which operates by using the quasi-electron distribution function n_p both above and below the gap.[4)] In this representation the form of the collision operator remains the same, but since for $\xi_p < 0$ the energy of the quasi-electrons in this representation is negative, all of the δ-functions give a contribution to the collision operator.

3. THE TWO-PARTICLE GREEN'S FUNCTION AND THE CONDITIONS FOR STABILITY

In order to investigate stability, it is necessary to determine the two-particle retarded Green's function associated with a small total momentum q and total frequency ω. Here we shall use an effective four-fermion Hamiltonian with an effective coupling constant λ.

In the normal metal the problem reduces to a summation of the series which is graphically shown in Fig. 1 (see[13a]). The vertex part

$$\hat\Gamma_{\alpha\beta,\gamma\delta}(q)=\hat\Gamma(q)(\delta_{\alpha\gamma}\delta_{\beta\delta}-\delta_{\alpha\delta}\delta_{\beta\gamma})$$

satisfies the following matrix equation:

$$\hat\Gamma(q)=\hat\Gamma_0+i\hat\Gamma_0\hat S(q)\hat\Gamma(q), \quad (3.1)$$

where $\hat\Gamma_0 = \lambda\hat\sigma_z$. The quantity $\hat\Gamma$ has the properties of a boson Green's function and is a 2×2 matrix, and $\hat S$ is the self-energy part, which is given by

$$\hat S=\begin{pmatrix} S_c & S_+\\ S_- & \tilde S_c\end{pmatrix}$$
$$=\begin{pmatrix} \int\frac{d^4p}{(2\pi)^4}G_c(p)G_c(q-p), & \int\frac{d^4p}{(2\pi)^4}G_+(p)G_+(q-p)\\ \int\frac{d^4p}{(2\pi)^4}G_-(p)G_-(q-p), & \int\frac{d^4p}{(2\pi)^4}\tilde G_c(p)\tilde G_c(q-p)\end{pmatrix}. \quad (3.2)$$

In order to obtain the retarded Green's function Γ_r we make a unitary transformation of Eq. (3.1) with the help of the matrix $U = (1 + i\sigma_y)/\sqrt{2}$. Finally we have

$$U^{-1}\hat\Gamma U=\begin{pmatrix}0 & \Gamma_a\\ \Gamma_r & \Gamma_d\end{pmatrix},\quad \Gamma_d=\Gamma_++\Gamma_-,\quad \Gamma_{r,a}=\Gamma_c-\Gamma_\pm. \quad (3.3)$$

Furthermore, we find

$$U^{-1}\hat S U=\begin{pmatrix}S_d & S_r\\ S_a & 0\end{pmatrix},\quad S_d=S_++S_-,\quad S_{r,a}=S_c+S_\pm. \quad (3.4)$$

In deriving these formulas we have used the identities[14]

$$\Gamma_c+\tilde\Gamma_c=\Gamma_++\Gamma_-, \quad (3.5)$$
$$S_c+\tilde S_c+S_++S_-=0. \quad (3.6)$$

Hence

$$\Gamma_r=\lambda+i\lambda S_r\Gamma_r. \quad (3.7)$$

Thus, the constructive method of evaluating Γ_r consists in writing down the graphical equation Fig. 1 directly for the retarded Green's function Γ_r. One can evaluate the self-energy part S_r appearing in it with the aid of the relationship (3.4).

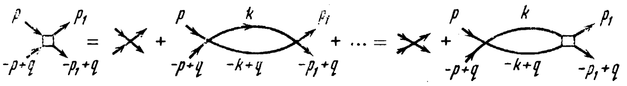

FIG. 1

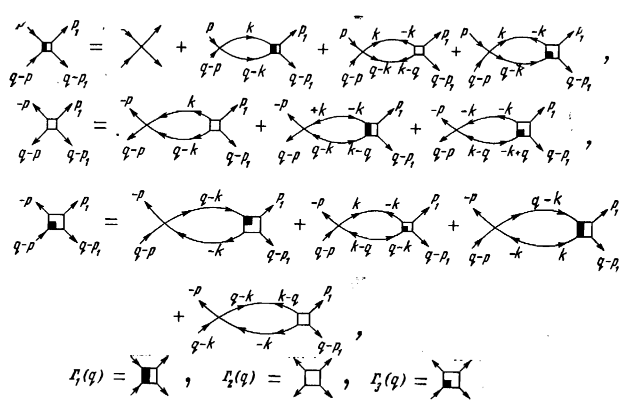

FIG. 2

Anomalous electron Green's functions exist in the case of a superconductor. Therefore, in calculating the self-energy parts, the system of equations, which is represented graphically in Fig. 2, arises for the quantities Γ_1, Γ_2 and Γ_3:

$$(\hat 1-i\lambda\hat S_r)\begin{pmatrix}\Gamma_{1r}\\ \Gamma_{2r}\\ \Gamma_{3r}\end{pmatrix}=\begin{pmatrix}\lambda\\ 0\\ 0\end{pmatrix}. \quad (3.8)$$

Here the elements of the matrix $\hat S_c$ are given by

$$S_{11}(q)=S_{22}(-q)=\int\frac{d^4k}{(2\pi)^4}G_c(k)G_c(q-k),$$
$$S_{12}(q)=S_{21}(q)=\int\frac{d^4k}{(2\pi)^4}F_c(k)F_c(q-k),$$
$$S_{13}(q)=S_{31}(q)=S_{23}(-q)=S_{32}(-q)=-\int\frac{d^4k}{(2\pi)^4}F_c(k)G_c(q-k), \quad (3.9)$$
$$S_{33}(q)=-\int\frac{d^4k}{(2\pi)^4}[G_c(k)G_c(k+q)+F_c(k)^+F_c(k+q)].$$

By doing the calculation and then analytically continuing the answer, we obtain the following results (the subscript r is omitted):

$$S_{11}(q)=\sum_{\mathbf k}[u^2_{\mathbf k+q/2}u^2_{\mathbf k-q/2}f_q(\mathbf k)+v^2_{\mathbf k+q/2}v^2_{\mathbf k-q/2}f_{-q}(\mathbf k)+2u^2_{\mathbf k+q/2}v^2_{\mathbf k-q/2}\varphi_q(\mathbf k)] \quad (3.10)$$
$$S_{12}(q)=-\sum_{\mathbf k}u_{\mathbf k+q/2}v_{\mathbf k+q/2}u_{\mathbf k-q/2}v_{\mathbf k-q/2}[f_q(\mathbf k)+f_{-q}(\mathbf k)-2\varphi_q(\mathbf k)],$$
$$S_{13}(q)=-i\sum_{\mathbf k}u_{\mathbf k+q/2}v_{\mathbf k-q/2}\{u_{\mathbf k+q/2}u_{\mathbf k-q/2}[\varphi_q(\mathbf k)-f_q(\mathbf k)]$$
$$-v_{\mathbf k+q/2}v_{\mathbf k-q/2}[\varphi_q(\mathbf k)-f_{-q}(\mathbf k)]\},$$
$$S_{33}(q)=-S_{12}(q)+\sum_{\mathbf k}[(u^2_{\mathbf k-q/2}u^2_{\mathbf k+q/2}+v^2_{\mathbf k-q/2}v^2_{\mathbf k+q/2})\varphi_q(\mathbf k)$$
$$+u^2_{\mathbf k+q/2}v^2_{\mathbf k-q/2}[f_q(\mathbf k)+f_{-q}(\mathbf k)]],$$
$$f_q(\mathbf k)=i\frac{m_{\mathbf k+q/2}+m_{-\mathbf k+q/2}-1}{\omega-\varepsilon_{\mathbf k+q/2}-\varepsilon_{\mathbf k-q/2}+i\delta}=f_q(-\mathbf k), \quad (3.11)$$

$$\varphi_q(\mathbf{k}) = i\frac{m_{\mathbf{k}+\mathbf{q}/2} - m_{\mathbf{k}-\mathbf{q}/2}}{\omega - \varepsilon_{\mathbf{k}+\mathbf{q}/2} + \varepsilon_{\mathbf{k}-\mathbf{q}/2} + i\delta} = \varphi_{-q}(\mathbf{k}). \tag{3.12}$$

We obtain the dispersion equation by setting the determinant of the system of equations (3.8) equal to zero:

$$\|\hat{1} - i\lambda\hat{S}_r(q)\| = 0. \tag{3.13}$$

It follows from Eq. (3.13) that the dispersion equation is invariant under the substitutions $\lambda \to -\lambda$ and $m_{\mathbf{k}} \to 1 - m_{\mathbf{k}}$, that is, the spectrum of the collective oscillations and of the two-particle excitations in a system having the usual Fermi distribution and attractive interactions turns out to be the same as in a system with the inverse Fermi distribution and repulsion.

Let us start with an analysis of the stability conditions for the normal metal. For $\Delta = 0$ and $\mathbf{q} = 0$ the dispersion equation (3.13) gives

$$1 = \lambda N(0) \int_{-\omega_D}^{\omega_D} d\xi \frac{1 - 2n(\xi)}{\omega - 2\xi + i\delta}. \tag{3.14}$$

For the given form of the distribution function $n(\xi)$, this equation has complex roots. If the roots are located in the lower half-plane, then the state is stable. If the function $n(\xi)$ depends on some external parameter (and in the equilibrium case, it would depend on the temperature), then a root may intersect the real axis for a certain value of this parameter. This is therefore the stability boundary of the normal state. It is determined from the condition that Eq. (3.14) has a real root. In this case, by separating out the imaginary part in Eq. (3.14), we find

$$n(\omega/2) = 0. \tag{3.15a}$$

By choosing the origin with respect to which the energy ξ is measured, and hence the quantity $\omega/2$, to coincide with the point at which $n(\xi) = 1/2$, we obtain the condition $\omega = 0$ from Eq. (3.15a). By isolating the real part in Eq. (3.14) we obtain the following equation for the determination of the stability boundary:

$$1 = -\frac{1}{2}\lambda N(0) \int_{-\omega_D}^{\omega_D} \frac{d\xi}{\xi}[1 - 2n(\xi)]. \tag{3.15b}$$

Here $n(\xi)$ is the average of $n_{\mathbf{k}}$ over the corresponding constant-energy surface. In the case of equilibrium, Eq. (3.15b) goes over into the well-known equation for the determination of the critical temperature T_c. In so far as we are able to judge, Eqs. (3.15) differ somewhat from the corresponding stability conditions derived by Batyev.[4]

Let us proceed to the investigation of a superconductor. When $\mathbf{q} = 0$ Eq. (3.13) takes the form

$$[\gamma + (4\Delta^2 - \omega^2)Z][\gamma - (4\Delta^2\gamma + \omega^2)Z + 2\Delta^2\omega^2 Z^2] = 0, \tag{3.16}$$

$$Z(\omega) = \lambda N(0) \int_{\Delta}^{\infty} \frac{1 - 2m(\varepsilon)}{(\varepsilon^2 - \Delta^2)^{1/2}} \frac{d\varepsilon}{\omega^2 - 4\varepsilon^2 + i\omega\delta}, \tag{3.17}$$

$$\gamma = 1 + \lambda N(0) \int_{\Delta}^{\omega_D} \frac{d\varepsilon}{(\varepsilon^2 - \Delta^2)^{1/2}}[1 - 2m(\varepsilon)]. \tag{3.18}$$

So that the stationary value, Δ_s, of the parameter Δ is determined from the equation $\gamma = 0$. The relationship $n(-\xi) = 1 - n(\xi)$ was used in the derivation of Eq. (3.16).

When $\gamma = 0$ the left part of Eq. (3.16) breaks up into a product of factors, two of which give the real roots $\omega^2 = 0$ and $\omega = \pm 2\Delta_s$, and the other two factors lead to the equations

$$Z(\omega) = 0, \tag{3.19}$$

$$Z(\omega) = +1/2\Delta_s^2. \tag{3.20}$$

Equation (3.20) doesn't have any solutions. In fact, one can rewrite it in the form

$$\frac{\lambda N(0)}{2} \int_1^{\infty} dx \frac{1 - 2m(x\Delta)}{(x^2 - 1)^{1/2}(x^2 - \omega^2/4\Delta_s^2 - i\omega\delta)} = 1.$$

The small factor $\lambda N(0)$ appears to the left; in order of magnitude this factor is equal to $\ln^{-1}(\omega_D/\Delta_0)$, where Δ_0 denotes the equilibrium gap for $T = 0$. As one can easily verify, the smallness of this factor can not be compensated by the integral, whose order of magnitude does not exceed unity.

Thus, the stability boundary of the superconducting state is determined by Eq. (3.19). In this equation we set $\omega = \omega' - i\omega''$. The stability boundaries are determined from the condition that ω'' becomes an infinitesimal negative quantity. By isolating the imaginary part in Eq. (3.19), we find that this condition is satisfied provided that one of the following two conditions is fulfilled:

$$\omega' = 0, \tag{3.21a}$$

$$m(\omega_i'/2) = 1/2. \tag{3.22a}$$

These are also those values of the frequency at which an instability may begin.

We note that Eq. (3.22a) may, in general, have several solutions, in which case we shall distinguish them by the subscript i. The real part of Eq. (3.19) together with conditions (3.21a) or (3.22a) determine those values of the external parameters, characterizing the non-equilibrium nature of the distribution, at which the instability begins. The corresponding equation for case (3.21a) has the form

$$\int_{\Delta_s}^{\infty} \frac{d\varepsilon}{\varepsilon^2} \frac{1 - 2m(\varepsilon)}{(\varepsilon^2 - \Delta_s^2)^{1/2}} = 0 \tag{3.21b}$$

and for case (3.22a)

$$\int_{\Delta_s}^{\infty} \frac{d\varepsilon}{(\varepsilon^2 - \Delta_s^2)^{1/2}} \frac{1 - 2m(\varepsilon)}{\varepsilon^2 - \omega_i'^2/4} = 0. \tag{3.22b}$$

We see that in order for Eqs. (3.21) and (3.22) to have a solution, the function $1 - 2m(\xi)$ must be alternating in sign.[5)]

Thus, if the function $m(\epsilon)$ in the important range of variation of ϵ does not pass through the value 1/2, then the superconducting state is stable.

Furthermore, if the function $1 - 2m(\epsilon)$ has only one change of sign, then Eq. (3.22) doesn't have a solution. In this case the stability boundary is determined by Eq. (3.21b). Undamped, collective oscillations with frequencies $\omega^2 < 4\Delta_s^2$ exist in a system in its region of stability; these are determined by the following dispersion equation:

$$\int_{\Delta_s}^{\infty} \frac{d\varepsilon}{(\varepsilon^2 - \Delta_s^2)^{1/2}} \frac{1 - 2m(\varepsilon)}{\varepsilon^2 - \omega^2/4} = 0. \tag{3.23}$$

Thus, we see that in this case the instability is related to the vanishing of the frequency of one of these undamped, collective modes.

However, if the function $1 - 2m(\epsilon)$ changes sign more than once, then a solution of Eq. (3.22) may also exist. And the actual boundary of the stability region is determined by the most stringent condition out of all the conditions which are obtained in this manner. In this case weakly-damped modes with frequencies, whose dispersion equation has the form $Z(\omega) = 0$, may also exist near the boundary of the stability region. The instability may appear as a change in the sign of the attenuation coefficient for one of these modes.

Now let us explain the principle used in selecting the diagrams associated with the derivation of the system of equations (3.8). When $\gamma = 0$ the dispersion equation (3.16) has, in particular, the roots $\omega = \pm 2\Delta_S$. But, as $\omega \to \pm 2\Delta_S$ each of the self-energy parts S_{ik} diverges logarithmically, and the problem reduces to the summation of an infinite sequence of diagrams, that is, it reduces to the solution of the system of equations shown in Fig. 2.

On the other hand, in order to obtain the pole associated with small values of ω and $\mathbf{q}$, that is, the acoustic mode, it is sufficient (to within terms of order $\lambda N(0) \ll 1$) to sum the same sequence of diagrams. In connection with satisfying the self-consistency condition $\gamma = 0$ the principal terms of the series, which do not depend on ω and $\mathbf{q}$, cancel each other, and consequently a pole appears as $\omega \to 0$ and $\mathbf{q} \to 0$.

The summation of the sequence of diagrams, permitting us to obtain the roots $\omega = 0$ and $\omega = \pm 2\Delta$, has been carried out in the articles by Migdal and Maleev[17] for the equilibrium superconductor at T = 0.

Now let us investigate the acoustic solution of the dispersion equation, which tends to zero as $\mathbf{q} \to 0$[18,19], for values of $\mathbf{q} \neq 0$. Carrying out an expansion of the dispersion equation (3.13) correct to terms of order q^2, we obtain the following result for an isotropic distribution:

$$\omega^2 = c^2 q^2, \tag{3.24}$$

At T = 0 and for the equilibrium case,[19] one finds

$$c^2 = v_F^2 / 3. \tag{3.25}$$

Expression (3.25) is calculated to the zero-order approximation with respect to the small parameter $\lambda N(0)$. The accuracy of our calculation permits us, in principle, to also derive the next approximation in powers of this parameter. There is no attenuation of sound as long as $\omega^2(q) < 4\Delta_S^2$. We emphasize that this result also pertains to the inverse Fermi distribution.

4. EXAMPLES OF THE INVESTIGATION OF STABILITY

Now let us discuss the stability of the nonequilibrium superconducting states which were previously considered in articles[5,7,8-12]. The nonequilibrium distribution function $m(\epsilon)$ in the presence of an ultrahigh-frequency field is found in the article by Ivlev and Éliashberg.[7] The state is stable whenever the corresponding equation for the gap has a solution. Under the same external conditions, i.e., at the same temperature and for the same power of the high-frequency field, the normal state is also stable (in any event, if the temperature is above the superconducting transition point in equilibrium). Thus, one of these two states must be metastable.

A completely analogous situation occurs in the case when the nonequilibrium state of the superconductor can be described with the aid of a quasifield, analyzed by the authors in[9].

A different situation is considered in[8], that of the strongly nonequilibrium state which appears when a conductor is illuminated by light of a definite spectral composition. Here the Fermi step becomes more abrupt than in equilibrium at the given temperature, and the

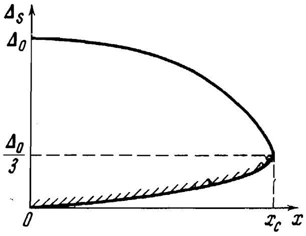

FIG. 3

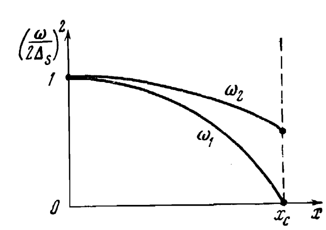

FIG. 4

FIG. 3. The dependence of Δ_s on the concentration of excitations, $x (x_c = 1/3\sqrt{3})$.

FIG. 4. The dependence of the frequencies of the collective modes on the concentration of excitations ($x_c = 1/3\sqrt{3}$).

corresponding superconducting state is stable, since the difference $1 - 2n(\epsilon)$ has a constant sign for $\epsilon > 0$. However, the normal state is unstable in the presence of sufficiently intense illumination. The easiest way to see this is to note that the distribution which appears differs very little from the equilibrium distribution at T = 0. Thus, there is an important distinction between the present case and the situations which we were talking about above.

The superconducting state with an inverse Fermi distribution (with a repulsive interaction between the particles) is also stable, because $n(\epsilon) > 1/2$ in the essential range of positive energies. However, the normal state associated with the inverse Fermi distribution is unstable.

Owen and Scalapino[12] have recently studied the nonequilibrium state of a superconductor having an excess (in comparison with its equilibrium value) concentration of superconducting excitations.[6] They confine their discussion to the stability in this model at T = 0. Introducing ν, the chemical potential of the excitations, we may write the equation of state (1.1) in the following form:

$$\nu + (\nu^2 - \Delta_s^2)^{1/2} = (\Delta_0 \Delta_s)^{1/2}. \tag{4.1}$$

Here Δ_0 denotes the half-width of the equilibrium gap, that is, the half-width for zero concentration of the excitations. It is convenient to rewrite expression (4.1) by introducing the dimensionless concentration of excitations:

$$x = \frac{1}{2\Delta_0 N(0)} \sum_p n_p = \frac{1}{\Delta_0}(\nu^2 - \Delta_s^2)^{1/2}. \tag{4.2}$$

Then Eq. (4.1) can be written down in the form[7]

$$\Delta_s(\Delta_s - \Delta_0)^2 = 4x^2 \Delta_0^3. \tag{4.3}$$

The dependence of Δ_S on x is schematically shown in Fig. 3.

The stability criterion (3.25) gives the following result for the critical value of the quantity ν:

$$\nu_c = 2\Delta_0 / 3\sqrt{3}, \tag{4.4}$$

which corresponds to the following result for the critical concentration of excitations

$$x_c = 1/3\sqrt{3} \tag{4.5}$$

and for the gap

$$\Delta_c = \Delta_0 / 3. \tag{4.6}$$

In order to understand which branch of the curve shown in Fig. 3 is unstable, let us consider the dispersion equation (3.19). Its solution has the form

$$\omega_1^2 = 4\Delta_s^2 [1 - x^2 (\Delta_0 / \Delta_s)^3], \tag{4.7a}$$

$$\omega_2^2 = 4\Delta_s^2 \{1 - x^2 (\Delta_0/\Delta_s)^3 [1 - 2x(\Delta_0/\Delta_s)^{1/2}]^2\}. \tag{4.7b}$$

For $x = x_c$ and $\Delta_s = \Delta_c$ the solution ω_1^2 tends to zero like $(x_c - x)^{1/2}$, and $\omega_2^2 = 32\Delta_c^2/9$. Using the equation of state (4.3), we see that $\omega_1^2 < 0$ for $x < x_c$ and $\Delta_s < \Delta_c$, and therefore this branch of the solution is unstable. The dependences of ω_1^2 and ω_2^2 on the concentration in the stable region are schematically shown in Fig. 4.

Thus, two collective modes exist in the strongly nonequilibrium state in a superconductor, and for $\mathbf{q} = 0$ the frequencies of these modes are determined by formulas (4.7). Such modes are not present in equilibrium. The instability which we have been considering—this is an instability of that collective mode which appears in the nonequilibrium state. Since the given state is a state of incomplete equilibrium, it is clear that the criterion that the energy be a minimum should also define, and actually does define in this case, the stability region of the superconducting state.

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1)We thank L. P. Gor'kov for calling our attention to this fact.

2)We have learned that D. A. Kirzhnits and Yu. V. Kopaev are also studying a similar problem.

3)A similar formalism has also been used by Elesin [16] in order to solve the problem of a semiconductor in the field of a strong electromagnetic wave.

4)In this representation the factors u_p and v_p are determined by other expressions (see [11]). We take this opportunity to mention that in [11] in expression (2.18) for the collision operator the last two terms were erroneously omitted. Taking these terms into account leads to a change of the numerical coefficients in the final formulas, namely, it leads to the replacement in formula (43) of the coefficient associated with Δ/kT by $1/2 + \pi/4$ and it also leads to the addition of the term $(1 - \ln 2)/2$ in the expression for ln C.

5)This implies, as one can easily see, a "negative temperature" of the superconducting excitations.

6)This state has subsequently been studied in detail experimentally. [20]

7)Equation (4.3), which we have derived, differs slightly from the corresponding equation in [12].

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