A possible mechanism for the peak effect in hard superconductors

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The interaction between the vortex-filament lattice and a periodic distribution of dislocations is investigated. The mean pinning force has a peak if the periodicity of the vortex-filament lattice is equal to that of the dislocation array. The results agree with the experimental findings.

1. INTRODUCTION

The experimental dependence of the critical current I_c in hard superconductors on the external magnetic field reveals in many experiments that the maximum of I_c is reached shortly before the field reaches the critical value H_{c2} (see, e.g., [1–7]). Within the framework of modern concepts, one can name many causes of this peak effect[8–10]. Since the ability of hard superconductors to carry current without losses up to the critical current can be attributed to the pinning of the vortex filaments by the lattice defects, attempts are also made to relate the appearance of the peak effect with the pinning mechanism. In this paper we wish to propose and discuss the theory of one of the possible mechanisms.

2. CALCULATION OF THE AVERAGE PINNING FORCE

To calculate the density of the pinning force, we use the Peach-Koehler formula[11]

$$k_i=\varepsilon_{ijk}\sigma_{km}\alpha_{jm} \tag{1}$$

where k_i is the density of the forces with which the mechanical stresses σ_{km} act on the dislocation distribution α_{jm}. For α_{jm} one can introduce a quasidislocation density, with the aid of which we can describe the point defects, precipitates, etc.[12].

The dislocation distribution is a property of the investigated samples and is assumed specified. The stresses σ_{km} in our model are the proper stresses in the superconducting samples, due to the lattice of the vortex filaments. We start with the calculation of these proper stresses, assuming for simplicity that the sample is infinite.

The starting point of our calculations are the Ginzburg-Landau equations[13–16], which are connected with the equations of linear elasticity theory and take the following form:

$$\frac{1}{2m}\left(-i\hbar\partial_i-\frac{2e}{c}A_i\right)^2\Psi+\alpha_0(1+a_{ln}(\mathrm{Ink}\,\chi)_{ln}+{}^1/{}_2a_{ijkl}\varepsilon_{ij}^E\varepsilon_{kl}^E)\Psi, \tag{2}$$

$$\beta_0(1+b_{ln}(\mathrm{Ink}\,\chi)_{ln}+{}^1/{}_2b_{ijkl}\varepsilon_{ij}^E\varepsilon_{kl}^E)|\Psi|^2\Psi=0,$$

$$j_i^{(s)}=\frac{c}{4\pi}\varepsilon_{ijk}\varepsilon_{kmn}A_{n,m,j}=-\frac{i\hbar e}{m}(\Psi^*\Psi_{,i}-\Psi\Psi_{,i}^*)-\frac{4e^2}{mc}A_i|\Psi|^2, \tag{3}$$

$$(C_{ijkl}+\delta C_{ijkl})\varepsilon_{kl}^E=(\mathrm{Ink}\,\chi)_{ij}, \tag{4}$$

$$\mathrm{Ink}(\varepsilon_{ij}^E+\delta\varepsilon_{ij}+\varepsilon_{ij}^P)=0. \tag{5}$$

Here ϵ_{ij}^E and ϵ_{ij}^P are the elastic and plastic deformations, and the symbol Ink denotes alternation of indexes of the form

$$(\mathrm{Ink}\,\chi)_{ln}=-\varepsilon_{lki}\varepsilon_{nmj}\chi_{ij,k,m}, \tag{6}$$

χ_{ij} is the stress function of elasticity theory[16,19], ϵ_{ijk} is a unit antisymmetrical tensor, and the symbol $\Psi_{,k}$ denotes a derivative with respect to the k-th coordinate. In the derivation of (2)–(5) we used the following expressions for the changes in the elastic constants δC_{ijkl} and for the spontaneous deformation $\delta\epsilon_{ij}$:

$$\delta C_{ijkl}=\alpha_0a_{ijkl}|\Psi|^2+{}^1/{}_2\beta_0b_{ijkl}|\Psi|^4, \tag{7}$$

$$\delta\varepsilon_{ij}=-\alpha_0a_{ij}|\Psi|^2-{}^1/{}_2\beta_0b_{ij}|\Psi|^4, \tag{8}$$

with α_0 and β_0 the known Ginzburg-Landau coefficients[13]. a_{ij}, a_{ijkl}, b_{ij}, and b_{ijkl} are material tensors describing the connection between elastic properties and the superconductivity properties.

The system (2)–(5) was solved approximately in analogy with the Kammerer solution[16]. As the zeroth approximation we assume a state in which there is a lattice of vortex filaments[17,18] and in which there are no deformations. Then, in first-order approximation, we solve the mechanical equations (4) and (5) connected with the field of the order parameter Ψ. Since we wish to calculate only those proper stresses which are due to the vortex-filament lattice itself, we put $\epsilon_{ij}^P=0$. We obtain

$$\mathrm{Ink}((C+\delta C)^{-1}\mathrm{Ink}\,\chi)=-\mathrm{Ink}\,\delta\varepsilon. \tag{9}$$

This equation for the stress functions χ_{ij} can be solved, according to Kammerer[16], by successive approximations, and for our model we can again confine ourselves to the first-order approximation ($\delta C = 0$). The remaining equation for the stress function

$$\mathrm{Ink}\,C^{-1}\mathrm{Ink}\,\chi^{(c)}=-\mathrm{Ink}\,\delta\varepsilon \tag{10}$$

can be solved by the method described by Kroner[19]. If isotropy is assumed

$$C_{ijkl}^{-1}=\frac{1}{2G}\delta_{ik}\delta_{jl}-\frac{1}{2G}\frac{1}{m+1}\delta_{ij}\delta_{kl},\quad a_{ln}=a\delta_{ln},\quad b_{ln}=b\delta_{ln} \tag{11}$$

(G is the shear modulus, m is the reciprocal of the Poisson coefficient, and a and b determine the relative volume change $\delta\epsilon_{ii}$ due to the quantity $|\Psi|$[16]), then Eq. (9) goes over, after making the transformation

$$\chi_{ij}^{(0)\prime}=\frac{1}{2G}\left(\chi_{ij}^{(0)}-\frac{1}{m+2}\chi_{kk}^{(0)}\delta_{ij}\right),\quad \chi_{ij,i}^{(0)\prime}=0 \tag{12}$$

into the equation of the double potential

$$\Delta\Delta\chi_{ij}^{(0)\prime}=-(\alpha_0a|\Psi|^2+{}^1/{}_2\beta_0b|\Psi|^4)_{,k,k}\delta_{ij}+(\alpha_0a|\Psi|^2+{}^1/{}_2\beta_0b|\Psi|^4)_{,i,j}, \tag{13}$$

where we substitute for Ψ the well known Abrikosov solution[17,18]

$$|\Psi|^2=\Psi_0^2|c_0|^2 3^{-1/4}\sum_{mn}(-1)^{mn+m+n}\cdot \times\exp\left\{-\frac{\pi}{\sqrt{3}}(m^2+n^2-mn)+\frac{2\pi i}{d}\left(mx+\frac{2n-m}{\sqrt{3}}y\right)\right\}. \tag{14}$$

Here d is the distance between the vortex filaments.

The stress functions $\epsilon_{ij}^{(0)\prime}$ are written in the form

$$\chi_{ij}^{(0)\prime}=\sum_{mn}\chi_{ij}^{(0)\prime mn}\exp\left\{\frac{2\pi i}{d}\left(mx+\frac{2n-m}{\sqrt{3}}y\right)\right\}, \tag{15}$$

and the right hand side of (13) is expanded in the corresponding Fourier seires. This yields $\chi_{ij}^{(0)\prime mn}$ and then, according to (12) $\chi_{ij}^{(0)mn}$. We now calculate the stresses with the aid of (4). We obtain

$$\sigma_{ij}=\sum_{mn}\sigma_{ij}^{mn}\exp\left\{\frac{2\pi i}{d}\left(mx+\frac{2n-m}{\sqrt{3}}y\right)\right\}. \tag{16}$$

The stresses due to the vortex-filament lattice have the same periodicity as the vortex filament lattice itself. The coefficients σ_{ij}^{mn} are given in the Appendix. The convergence of the series (16) is ensured by the convergence of (14).

For the average density of the pinning force we can write

$$\bar{k}_i=\frac{1}{(2L)^2}\cdot\int_{-L}^{L}\int_{-L}^{L}\varepsilon_{ijk}\sum_{mn}\sigma_{kl}^{mn}\exp\left\{\frac{2\pi i}{d}\left(mx+\frac{2n-m}{\sqrt{3}}y\right)\right\}\alpha_{jl}(x,y)\,dx\,dy. \tag{17}$$

This formula will be used in the next section for a periodic distribution of the dislocations.

3. PEAK EFFECT AS THE RESULT OF A PERIODIC DISTRIBUTION OF THE DISLOCATIONS

By way of the simplest example of a periodic dislocation distribution, we consider the arrangement shown in the figure[20]. In the shaded regions, the dislocation density has a constant value $\bar{\alpha}_{ij}$, and in the remaining regions it is equal to zero. If we expand this distribution in a Fourier series, we obtain[20]

$$\alpha_{ij}=\sum_{rs}\alpha_{ij}^{rs}\exp\left\{\frac{2\pi i}{l}(rx+sy)\right\}, \tag{18}$$

$$\alpha_{ij}^{rs}=\frac{\bar{\alpha}_{ij}}{\pi^2}\frac{1}{rs}\sin\frac{\pi c}{l}r\sin\frac{\pi c}{l}s. \tag{19}$$

We now confine ourselves for $\bar{\alpha}_{ij}$ to the special case of edge dislocations, which lie parallel to the z axis and whose Burgers vectors are directed along the x axis (i.e., only the component $\bar{\alpha}_{31}$ differs from zero). In addition, we take into consideration the fact that the stress components σ_{13} and σ_{23} are equal to zero. Integration yields in formula (17)

$$\bar{k}_1=-\sum_{mn}\sigma_{12}^{mn}\sum_{rs}\alpha_{31}^{rs}\frac{\sin 2\pi(m/d+r/l)L}{2\pi(m/d+r/l)L}\sin 2\pi\left(\frac{2n-m}{d\sqrt{3}}+\frac{s}{l}\right)L\times\left[2\pi\left(\frac{2n-m}{d\sqrt{3}}+\frac{s}{l}\right)L\right]^{-1}, \tag{20}$$

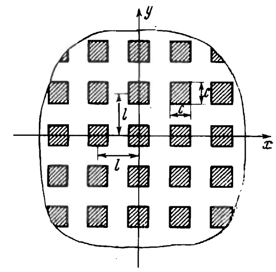
$$\bar{k}_2=\sum_{mn}\sigma_{11}^{mn}\sum_{rs}\alpha_{31}^{rs}\frac{\sin 2\pi(m/d+r/l)L}{2\pi\left(\frac{m}{d}+\frac{r}{l}\right)L}\sin 2\pi\left(\frac{2n-m}{d\sqrt{3}}+\frac{s}{l}\right)L\times\left[2\pi\left(\frac{2n-m}{d\sqrt{3}}+\frac{s}{l}\right)L\right]^{-1}. \tag{21}$$

The average quantities $\bar{k}_1$ and $\bar{k}_2$ are now functions of the period d of the vortex-filament lattice. The quantity d^2 itself is proportional to the magnetic induction:

$$d^2=\frac{2}{\sqrt{3}}\frac{\Phi_0}{B}, \tag{22}$$

where Φ_0 is the flux quantum.

The value of d can vary under the influence of an external magnetic field. Since the function (sin xL)/xL has a clearly pronounced maximum for large values of L at x = 0, and in the remaining regions differs little from



Dislocation distribution

zero, the only terms that contribute to the expressions for $\bar{k}_1$ and $\bar{k}_2$ are those in which the arguments of both sign functions are simultaneously equal to zero. Therefore in double sum over m and n there remain only the terms for which m = 2n or m = 0. If we now take into account the expression for σ_{ij}^{mn} (see the Appendix), we obtain

$$\bar{k}_1=0, \tag{23}$$

$$\begin{aligned}\bar{k}_2=\sigma_{11}^{01}&\sum_s\alpha_{31}^{0s}\sin 2\pi\left(\frac{2}{d\sqrt{3}}+\frac{s}{l}\right)L/2\pi\left(\frac{2}{d\sqrt{3}}+\frac{s}{l}\right)L\\+\sigma_{11}^{0-1}&\sum_s\alpha_{31}^{0s}\sin 2\pi\left(\frac{2}{d\sqrt{3}}-\frac{s}{l}\right)L/2\pi\left(\frac{2}{d\sqrt{3}}-\frac{s}{l}\right)L\\+\sigma_{11}^{02}&\sum_s\alpha_{31}^{0s}\sin 2\pi\left(\frac{4}{d\sqrt{3}}+\frac{s}{l}\right)L/2\pi\left(\frac{4}{d\sqrt{3}}+\frac{s}{l}\right)L\\+\sigma_{11}^{0-2}&\sum_s\alpha_{31}^{0s}\sin 2\pi\left(\frac{4}{d\sqrt{3}}-\frac{s}{l}\right)L/2\pi\left(\frac{4}{d\sqrt{3}}-\frac{s}{l}\right)L.\end{aligned} \tag{24}$$

The first two sums in (24) differ from zero only at discrete values of l:

$$2l=d\sqrt{3}|s|,\quad s=\pm1,\ \pm2,\ \ldots \tag{25}$$

The remaining sums make a finite contribution at

$$4l=d\sqrt{3}|s|,\quad s=\pm1,\ \pm2,\ \ldots \tag{26}$$

If the lattice period d of the vortex-filament lattice is connected by one of these two relations with the dislocation-distribution period, then a sharp maximum appears in the average density of the force $\bar{k}_2$. This peak of the average density of the pinning force manifests itself in experiment as a peak of the critical current. The case s = 0 corresponds to d → ∞ and will therefore not be considered.

4. DISCUSSION

The theory developed here is confirmed in the experimental results described in the literature. Schlump et al.[5] observed the peak effect precisely in the case when the distance between the vortex filaments coincides with the dislocation-lattice period. Similar results were obtained by Koch and Carpenter[6], who observed the peak when the distance between the vortex filaments was equal to the distance between the precipitations. Petermann[7] obtained a peak at a distance between precipitates equal to four distances between the vortex filaments.

Actually, the peak is not as clearly pronounced as in our model. This can be attributed to the fact that the dislocation structure in the samples does not always have the same periodicity, and this causes a broadening of the peak. The fact that the sample has finite dimensions also contributes to the broadening of the peak. With the aid of more realistic models of the dislocation distribution it will be possible to determine which of the many peaks that are possible on the basis of (25) and (26) is actually produced in a given sample. In our special case, the largest peak pertains to s = 1, since α_{31}^{0s} is proportional to 1/s.

In conclusion we make one more remark concerning the temperature dependence of the magnetic field at which the peak occurs. The magnetic induction **B** is connected with the field **H** in the vicinity of $\mathbf{H}_{c2}$ by the relation[17,18]

$$\mathbf{B}-\mathbf{H}=\frac{\mathbf{H}-\mathbf{H}_{c2}}{1.16(2\varkappa^2-1)}. \tag{27}$$

We denote the fields pertaining to the peak by the subscript p, and obtain for the external magnetic field $\mathbf{H}_p$ using the well known temperature dependence of $\mathbf{H}_{c2}$.

$$\mathbf{H}_p=\frac{1.16(2\varkappa^2-1)\mathbf{B}_p}{1+1.16(2\varkappa^2-1)}+\frac{\sqrt{2}\,\varkappa\mathbf{H}_c}{1+1.16(2\varkappa^2-1)}\left[1-\left(\frac{T}{T_c}\right)^2\right] \tag{28}$$

The known experimental data do not make it possible to verify this relation.

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APPENDIX

We present here expressions for the coefficients of Eq. (16) corresponding to the frequently used approximate form of Abrikosov's solution.

$$|\Psi|^2=\Psi_0^2|c_0|^2 3^{-1/4}\left\{1-2\exp\left(-\frac{\pi}{\sqrt{3}}\right)\left(\cos\frac{2\pi}{d}\left(x-\frac{1}{\sqrt{3}}y\right)+\cos\frac{2\pi}{d}\frac{2}{\sqrt{3}}y+\cos\frac{2\pi}{d}\left(x+\frac{1}{\sqrt{3}}y\right)\right)\right\}.$$

If the vortex filaments are directed along the z axis, then all the components σ_{13}^{mn}, σ_{23}^{mn}, and also the coefficients σ_{11}^{21}, σ_{11}^{-2-1}, σ_{22}^{01}, σ_{22}^{0-1}, σ_{22}^{02}, σ_{22}^{0-2}, σ_{12}^{01}, σ_{12}^{0-1} vanish. The nonzero components are given by the formulas

$$\begin{aligned}
&{}^1/_4\sigma_{11}^{01}={}^1/_4\sigma_{11}^{0-1}=\sigma_{11}^{10}=\sigma_{11}^{-10}=\sigma_{11}^{11}=\sigma_{11}^{-1-1}={}^1/_3\sigma_{22}^{10}\\
&={}^1/_3\sigma_{22}^{-10}={}^1/_3\sigma_{22}^{11}={}^1/_3\sigma_{22}^{-1-1}={}^1/_4\sigma_{33}^{01}={}^1/_4\sigma_{33}^{0-1}={}^1/_4\sigma_{33}^{10}={}^1/_3\sigma_{33}^{-10}\\
&={}^1/_4\sigma_{33}^{11}={}^1/_4\sigma_{33}^{-1-1}=-3^{-1/2}\sigma_{12}^{10}=-3^{-1/2}\sigma_{12}^{-10}=3^{-1/2}\sigma_{12}^{11}=3^{-1/2}\sigma_{11}^{-1-1}={}^1/_4A,\\
&{}^1/_6\sigma_{11}^{12}={}^1/_6\sigma_{11}^{-1-2}={}^1/_6\sigma_{11}^{1-1}={}^1/_6\sigma_{11}^{-11}={}^1/_3\sigma_{11}^{20}\\
&={}^1/_3\sigma_{11}^{-20}={}^1/_3\sigma_{11}^{22}={}^1/_3\sigma_{11}^{-2-2}={}^1/_4\sigma_{11}^{02}={}^1/_4\sigma_{11}^{0-2}\\
&={}^1/_2\sigma_{22}^{12}={}^1/_2\sigma_{22}^{-1-2}={}^1/_2\sigma_{22}^{1-1}={}^1/_2\sigma_{22}^{-11}={}^1/_8\sigma_{22}^{21}={}^1/_8\sigma_{22}^{-2-1}\\
&=\sigma_{22}^{20}=\sigma_{22}^{-20}=\sigma_{22}^{22}=\sigma_{22}^{-2-2}={}^1/_8\sigma_{33}^{12}={}^1/_8\sigma_{33}^{-1-2}={}^1/_8\sigma_{33}^{1-1}={}^1/_8\sigma_{33}^{-11}\\
&={}^1/_8\sigma_{33}^{21}={}^1/_8\sigma_{33}^{-2-1}={}^1/_4\sigma_{33}^{20}={}^1/_4\sigma_{33}^{-20}={}^1/_4\sigma_{33}^{22}={}^1/_4\sigma_{33}^{-2-2}={}^1/_4\sigma_{33}^{02}={}^1/_4\sigma_{33}^{0-2}\\
&={}^1/_2 3^{-1/2}\sigma_{12}^{12}={}^1/_2 3^{-1/2}\sigma_{12}^{-1-2}=-{}^1/_2 3^{-1/2}\sigma_{12}^{1-1}=-{}^1/_2 3^{-1/2}\sigma_{12}^{-11}=-3^{-1/2}\sigma_{12}^{20}\\
&=-3^{-1/2}\sigma_{12}^{-20}=3^{-1/2}\sigma_{12}^{22}=3^{-1/2}\sigma_{12}^{-2-2}={}^1/_4B.
\end{aligned}$$

Here

$$A=2G\frac{m}{m-1}\frac{H_c^2}{4\pi}(\bar{a}-2\bar{b})\exp\left(-\frac{\pi}{\sqrt{3}}\right)+2\bar{b}\exp\left(-\frac{2\pi}{\sqrt{3}}\right),$$

$$B=2G\frac{m}{m-1}\frac{H_c^2}{4\pi}\bar{b}\exp\left(-\frac{2\pi}{\sqrt{3}}\right),$$

$$\bar{a}=a|c_0|^2 3^{-1/4},\quad \bar{b}={}^1/_2 b(|c_0|^2 3^{-1/4})^2.$$

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