# Rate of penetration of a magnetic flux into type-II superconductors

B. V. Petukhov and V. R. Chechetkin

*L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences*
*Crystallography Institute, USSR Academy of Sciences*
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The time required for a vortex to surmount the surface barrier in type-II superconductors is calculated. The dependence of the activation energy needed for formation of a vortex nucleus on the magnetic field strength is determined. The pre-exponential factor in the transition probability near $H_{c1}$ is found from the Fokker-Planck equation.

## 1. THE ACTIVATION ENERGY

In type II superconductors, the mixed state, first considered by Abrikosov [1], becomes energywise favored at magnetic fields above $H_{c1}$. The magnetic flux penetrates into the sample from the surface in the form of vortex filaments. The interaction of the vortex with the surface leads to the presence of an energy barrier, which has been calculated for an ideally smooth surface in [2,3]. The energy of a rectilinear vortex V(y) parallel to the surface is shown as a function of the distance from the surface in Fig. 1.

Penetration of the vortex through the barrier takes place as a result of fluctuations. Since the energy of the vortex is proportional to its length, the flipping of an entire rectilinear vortex is less probable than the initial transition of a small part of the vortex and its further expansion. The energy of a curved vortex has the form

$$U\{y(x),z(x)\}=\int\left\{\frac{\Phi_0 H_{c1}}{4\pi}[(1+y'^2+z'^2)^{1/2}-1]+V(y)\right\}dx, \tag{1}$$

where $\Phi_0$ is the flux quantum, $H_{c1}$ the lower critical field. The field H is directed along the surface on the x axis. In the potential relief $U\{y(x), z(x)\}$, there is a saddle point to which corresponds a saddle configuration of the vortex $y_0(x)$, $z = 0$. The configuration $y_0(x)$ can be determined from the equation

$$\delta U\{y(x)\}/\delta y=0. \tag{2}$$

Inasmuch as the integrand in (1) does not depend on x, we can immediately write down the first integral of Eq. (2):

$$\frac{\Phi_0 H_{c1}}{4\pi}\left[\frac{y_0'^2}{(1+y_0'^2)^{1/2}}-(1+y_0'^2)^{1/2}+1\right]-V(y_0)=\text{const}. \tag{3}$$

Since $V(y) = 0$, $y' = 0$ on the surface of the sample (the line of force cannot have a kink), we have const = 0. The activation energy computed with the help of Eq. (3) has the form

$$U_0=2\int_0^{y_m}\sqrt{V(y)\left(\frac{\Phi_0 H_{c1}}{2\pi}-V(y)\right)}\,dy, \tag{4}$$

where $y_m$ is determined from the equation $V(y_m) = 0$. The principal contribution to the integral (4) is made by the region $y \sim \delta$, where $\delta$ is the penetration depth of the field. In this region, V(y) has the form

$$V(y)=\frac{\Phi_0}{4\pi}He^{-y/\delta}-\frac{\Phi_0}{4\pi}(H-H_{c1}). \tag{5}$$

(It can be established that the term which corresponds to the interaction of the filament with its image, is important only at distances of the order of $\delta/\ln\kappa$ and therefore can be omitted with accuracy $\sim 1/\ln\kappa$.) Substituting V(y) in (4), we obtain

$$U_0=\frac{\Phi_0 H\delta}{2\pi}\left\{\frac{H_{c1}}{H}-\left[\frac{H^2-H_{c1}^2}{H^2}\right]^{1/2}\left(\arcsin\frac{H_{c1}}{H}+\frac{\pi}{2}\right)+\frac{\pi}{2}\right\}; \tag{6}$$

$$U_0=\frac{\pi+2}{4\pi}\Phi_0 H_{c1}\delta \quad \text{as } H\to H_{c1}, \tag{6a}$$

$$U_0=\Phi_0 H_{c1}^2\delta/8H \quad \text{for } H_c\gg H\gg H_{c1}. \tag{6b}$$

The latter limiting case has been considered by Galaĭko. [4]

## 2. TIME OF SURMOUNTING THE BARRIER

The transition through the barrier takes place as a result of thermal fluctuations and the probability of transition is proportional to $\exp\{-U_0/T\}$. In the limiting case $H \to H_{c1}$, we can determine the dependence of the preexponential factor on the field in a manner similar to what has been done by Petukhov and Pokrovskiĭ. [3] The problem reduces to the solution of the Fokker-Planck equation.

In the vicinity of the saddle point, $U\{\mathbf{y}(x)\}$ can be represented as a quadratic form:

$$U\{\mathbf{y}_0+\delta\mathbf{y}\}=U_0+\frac{1}{2}\iint\frac{\delta^2 U}{\delta\mathbf{y}(x)\delta\mathbf{y}(x')}\delta\mathbf{y}(x)\delta\mathbf{y}(x')\,dx\,dx', \tag{7}$$

where

$$\delta\mathbf{y}=(\delta y, \delta z). \tag{8}$$

This quadratic form can be diagonalized:

$$U=U_0+\frac{1}{2}\sum\mu_n\eta_n^2. \tag{9}$$

Among the eigenvalues of the quadratic form (9) there are one negative one, which corresponds to motion along the saddle and two zero ones corresponding to shifts of the entire configuration along the surface. The remaining eigenvalues are positive. As will be seen from the following, the most important region in the calculation of the eigenvalues is the one in which the vortex deformation is weak; therefore, we can use a simplified expression for the energy:

$$U\{\mathbf{y}(x)\}=\frac{\Phi_0}{4\pi}\int\left\{H_{c1}\frac{y'^2}{2}+H_{c1}\frac{z'^2}{2}+\left(He^{-y/\delta}-H+H_{c1}-\frac{1}{2}h(2y)\right)\right\}dx \tag{10}$$

where the field h(2y) describes the interaction of the

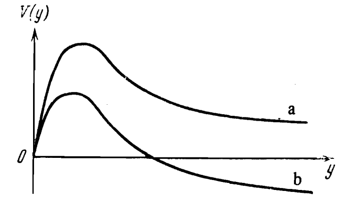


FIG. 1. Energy of a rectilinear vortex V(y) as a function of distance from the surface: a – for $H < H_{c1}$, b – for $H > H_{c1}$.

vortex with its image and is equal to $h(2y) = (\Phi_0/2\pi\delta^2)K_0(2y/\delta)$.

In this case, the problem of determining the coefficients $\mu_n$ of the quadratic form (9) reduces to estimates of the eigenvalues of the Schrodinger equation:

$$-\frac{\Phi_0}{4\pi}H_{c1}\frac{d^2\varphi_y}{dx^2}+\frac{d^2V}{dy^2}\varphi_y=\mu_{ny}\varphi_y, \tag{11a}$$

$$-\frac{\Phi_0}{4\pi}H_{c1}\frac{d^2\varphi_z}{dx^2}=\mu_{nz}\varphi_z. \tag{11b}$$

The form of the potential $V''(y_0(x))$ is shown in Fig. 2. The zero eigenvalue $\mu_x = 0$ corresponds to the eigenfunction $\varphi = dy_0/dx$, as is easily established, by differentiating (3) with respect to x. This eigenvalue exists even when the wells are moved out to an infinitely great distance. In this case, the zero level will be doubly degenerate. As the wells move together, a splitting of the zero eigenvalue takes place. The amount of the splitting can be found in much the same way as the ground state of the $H_2^+$ ion is determined,[6] and turns out to be proportional to $[(H - H_{c1})/H_{c1}]^{3/2}$. In what follows, the value of this eigenvalue will be found by another method.

Among the $\mu_{nz}$ there is also a zero eigenvalue $\mu_{0z} = 0$, which corresponds to translational symmetry along the z axis. All the remaining eigenvalues $\mu_{ny}$, $\mu_{nz}$ are positive and, since the configuration $y_0(x)$ is bent essentially only near the ends at distances of the order of $\delta$ that are small in comparison with the total length of the vortex $l_0$, correspond to free oscillations:

$$\mu_{ny}\approx\mu_{nz}\approx\frac{\Phi_0H_{c1}}{4\pi}\frac{\pi^2n^2}{l_0^2}=\frac{\Phi_0}{8\pi}\frac{n^2}{\delta^2}(H-H_{c1}). \tag{12}$$

Inasmuch as the eigenvalue corresponding to motion along the saddle is much smaller than the other (nonzero) eigenvalues, we can assume that the motion along the saddle is the slowest, and that the nonequilibrium shape of the vortex can be characterized by a single parameter, assuming that equilibrium has been established in the other degrees of freedom.

Since the vortex is weakly curved on the principal part of its length, replacement of the vortex energy by the free energy is done in a manner similar to such a replacement for the rectilinear vortex, and reduces to renormalization of $H_{c1}$, which will be assumed to have been done in what follows. As a saddle-point parameter, it is convenient to choose the distance $l$ between the ends of the vortex filament. Thus, for example, the length $l_0$ between the ends of the filament in the saddle-point configuration is equal to

$$l_0=\int dx=2\int\frac{1}{\sqrt{2}}\left[e^{-y/\delta}-\frac{H-H_{c1}}{H_{c1}}\right]^{-1/2}dy=\sqrt{2}\,\pi\delta\left(\frac{H_{c1}}{H-H_{c1}}\right)^{1/2}. \tag{13}$$

To determine the form of the energy $U(l)$ as a function of $l$, we must vary the functional (1) with the added condition $\int dx = l$. The calculations are entirely similar to those which were carried out for the determination of the activation energy and lead to the dependence

$$U(l)=U_0+\frac{\pi\Phi_0H_{c1}\delta^2}{l_0}-\frac{\pi\Phi_0H_{c1}\delta^2}{2l}-\frac{\Phi_0}{4\pi}(H-H_{c1})l, \tag{14}$$

where $U_0$ is determined by Eq. (6). Near the saddle point, $U(l)$ has the form

$$U(l)=U_0+{}^1/_2U''(l_0)(l-l_0)^2,$$

where

$$U''(l_0)=-\frac{\Phi_0H_{c1}}{\sqrt{2}\,\pi^2\delta}\left(\frac{H-H_{c1}}{H_{c1}}\right)^{3/2}. \tag{15}$$

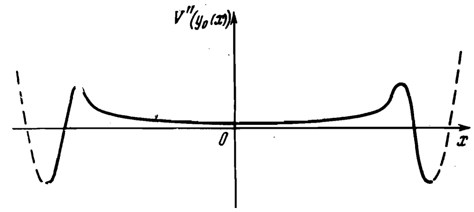

FIG. 2. Potential in Eq. 11a. The dashed part of the curve corresponds to distances at which destruction of the core of the vortex takes place.

The Fokker-Planck equation for the distribution function of the vortices has the form[7]

$$\frac{\partial f}{\partial t}=\frac{\partial}{\partial l}\left\{\frac{1}{\eta_l}\left[\frac{\partial U}{\partial l}f+T\frac{\partial f}{\partial l}\right]\right\}. \tag{16}$$

The coefficient of viscosity $\eta_l$ is connected with the viscosity coefficient per unit length $\eta$ by the relation

$$\eta_l=\eta\int\left(\frac{\partial y}{\partial x}\right)^2dx. \tag{17}$$

The coefficient of viscosity $\eta$ was found by Gor'kov and Kopnin[8] and is equal to $\eta = 6\gamma\sigma\Phi_0H_{c1}/c^2$, where $\gamma$ is a numerical factor of the order of unity.

Solving Eq. (20) in the quasistationary approximation, we determine the vortex flux from the surface of the sample per unit time:

$$\frac{1}{\tau}\sim\frac{S}{\sqrt{T}\,\eta\delta^4}\left(\frac{\Phi_0H_{c1}}{\delta}\right)^{1/2}\left(\frac{H-H_{c1}}{H}\right)^{3/4}\exp\left\{-\frac{U_0}{T}\right\}, \tag{18}$$

S is the area of the surface of the sample.

## 3. CONCLUSION

An estimate of the argument of the exponential in Eq. (18) gives a value of the order of $10^5$. This shows that in the case of an ideally smooth surface, penetration of the vortex into the sample from the surface at $H \sim H_{c1}$ is practically impossible. Evidently, the most probable mechanism for establishment of the mixed state is the creation and penetration of vortices close to roughnesses of the surface. In the case of sharply delineated roughnesses, the field near them can reach values of the order of $H_c$ and the energy barrier is absent in this case. For such a mechanism, the vortex filaments would exist near the roughnesses and at fields below $H_{c1}$. This fact can be tested experimentally. Equation (18) determines the interval in which the lifetime of the metastable state can vary depending on the treatment of the surface. The field dependence described by Eq. (18) could also exist when a film of a superconductor with a low value of the field $H_c$ is sputtered onto the sample.

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