# Cosmological consequences of a spontaneous breakdown of a discrete symmetry

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In theories involving spontaneous symmetry breakdown one may expect a domain structure of the vacuum. Such a structure does not exist near a cosmolgical singularity, when the temperature is above the Curie point, but this structure must appear later during the cosmological expansion and cooling down. We discuss the properties of the domain interfaces and of the space with domains in the large, the law of cosmological expansion in the presence of domains, and the influence of domains of the homogeneity of the Universe at a late stage.

## 1. INTRODUCTION

Recently field-theoretical models with spontaneous symmetry breakdown have been used for the construction of a unified theory of weak and electromagnetic interactions[1, 2]. Within the framework of these models attempts have been made, in particular, to understand such phenomena as the violation of CP-invariance[3, 4]. Below we shall discuss possible cosmological and astrophysical consequences of such models on the example of a simple model of violation of CP-invariance.

We recall that the violation of CP-invariance, discovered in 1964 in the decay of $K_L^0$-mesons, has so far not shown up in the decay of any other particle. The nature of the interaction which violates the CP-invariance remains mysterious. All experimental data do not contradict the assumption of a superweak interaction (with dimension-less coupling constant of the order of $10^{-13}$), although other possibilities are not excluded, e.g., a milliweak interaction (with a coupling constant of the order of $\sim 10^{-9}$). The model which we are going to discuss belongs to the type of milliweak interactions. In this model the initial Lagrangian is CP-invariant, and the CP-violation is a consequence of the fact that there are two minimal-energy states, each of which has its own nonvanishing classical field (Condensate). The interactions of these fields with the particles is CP-noninvariant. Such a mechanism is called a mechanism of spontaneous symmetry breakdown.

The problem of cosmological consequences of a spontaneous symmetry breakdown has been considered earlier by several authors (cf. the review article by Grib, Damaskinskiĭ and Maksimov[6]). Bogolyubov[7] has remarked, in particular, that in this case it is natural to expect the existence of vacuum domains. Kirzhnits[8, 9] has called attention to the fact that during the early stages of the hot (big-bang) Universe the spontaneously broken symmetry is reestablished and the intermediate Bosons in the Weinberg Model become massless. T. D. Lee has noted that the sign of the condensate, which determines the signs of the CP-noninvariant phases in observable phenomena, turns out to take on one or the other of its values in a random manner, becoming fixed during the early stages of cooling down of the Universe[5].

## 2. THE LAGRANGIAN AND THE TWO VACUA

We discuss the problem of establishing the sign of the CP-noninvariant phases, and the domain structure of the vacuum which appears as a consequence, on the example of the model Lagrangian[5, 11]

$$\mathscr{L}=\frac{1}{2}(\partial\varphi)^2-\lambda^2(\varphi^2-\eta^2)^2+\bar\psi(i\hat\partial-m-ig\gamma_5\varphi)\psi.$$

Here $\varphi$ is a pseudoscalar field, $\psi$ is the baryon field and $\lambda$ and $\eta$ are real parameters.

The Lagrangian $\mathscr{L}$ is CP-invariant. However, the vacuum is characterized by one of the two classical solutions, each by itself being CP-noninvariant: $\langle\varphi\rangle = \sigma\eta$, where $\sigma = \pm 1$ (the brackets denote the vacuum expectation value). This leads to a violation of the CP-invariance of each of the two possible Lagrangians which result when one goes over to a stable vacuum. Indeed, substituting $\varphi = \chi + \sigma\eta$ and carrying out the rotation

$$\psi\to\exp(i\alpha\gamma_5)\psi,\quad \operatorname{tg}2\alpha=-g\sigma\eta/m,$$

we have

$$\mathscr{L}=\frac{1}{2}(\partial\chi)^2-\frac{1}{2}m_\chi^2\chi^2-4\sigma\lambda^2\eta\chi^3-\lambda^2\chi^4+\bar\psi\left(i\hat\partial-M-\frac{igm}{M}\gamma_5\chi-\frac{g^2\sigma\eta}{M}\chi\right)\psi,$$

where

$$M=\sqrt{m^2+g^2\eta^2},\quad m_\chi=2\sqrt{2}\lambda\eta.$$

The simultaneous presence in the Lagrangian of the two last terms which have opposite CP-symmetry exhibits the CP-violation, for which the sign of the phase depends on the sign $\sigma$. Thus, a $\chi$-meson exchange between baryons leads to violation of CP- and T-invariance in baryon-baryon scattering. Our model Lagrangian contains no strange particles. However, it is not difficult to include them and to derive CP-violating effects in the decays of K mesons.

We now discuss the parameters $\lambda$ and $\eta$ which occur in the Lagrangian. The first is dimensionless and characterizes the degree of nonlinearity of the theory; the second has the dimension of mass and characterizes the amplitude of the classical condensate. The quantity $m_\chi = 2\sqrt{2}\lambda\eta$ can hardly be much smaller than several GeV. In particular the possibility of existence of light $\chi$ mesons, e.g., with mass $m_\chi \lesssim 1$ MeV, is excluded. The existence of such light $\chi$ particles would lead to the result that the emission of real $\chi$ particles would be much more probable than the observed CP-noninvariant processes between the usual particles, where $\chi$ mesons participate only as virtual particles. Such a possibility is excluded, e.g., by the data from a search for the mode $K^+ \to \pi$ + light neutral particle. The upper bound for the rate of this mode is smaller than $4 \times 10^{-6}(K^+)$ (cf.[10]).

As regards the parameter $\lambda$, it is reasonable to consider $\lambda \lesssim 1$ within the framework of our approach. The occurence of $\lambda \gg 1$ in the renormalized Lagrangian seems unlikely. At any rate, the tree approximation becomes illegitimate for $\lambda \gg 1$ and one must take account the contribution from loops. Taking this into account we come to the conclusion that $\eta \gtrsim 1$.

In the simplest version of the theory the quantity $\sigma$ is random and depends on the evolution history. One may then assume that there are regions with $\sigma = +1$ and $\sigma = -1$, i.e., with opposite signs of the CP-violation. The signs of all the CP-noninvariant effects in these regions are opposite to one another and, in particular so is the sign of the parameter $\epsilon$ in the $K_L^0$ decay.

## 3. PROPERTIES OF THE DOMAIN WALLS

We consider the boundary between two such regions. A large energy density must appear here. Minimizing it, we find that the thickness of a boundary layer at rest (we shall call this layer a wall) is $\Delta \sim 1/m_\chi \sim 1/\lambda\eta$, and its surface density is $\mu \sim \lambda^{3}$ [2].

The classical wave equation for the field $\varphi$ in vacuo, in the stationary case ($\partial\varphi/\partial t = 0$) has the form

$$\Delta\varphi = \frac{\partial}{\partial\varphi}\lambda^2(\varphi^2-\eta^2)^2.$$

This equation has the trivial, coordinate-independent solutions $\varphi = 0$, $\varphi = -\eta$, $\varphi = +\eta$, of which the first is unstable. It is easy to find the solution describing the transition layer between $\varphi = -\eta$ and $\varphi = +\eta$, namely:

$$\varphi = \eta \operatorname{th} \sqrt{2}\lambda\eta x$$

for a layer situated in the yz-plane. The Lagrangian $\mathscr{L}$ has been chosen in such a manner that in the regions $\varphi = -\eta$ and $\varphi = \eta$ the energy density vanishes identically. In the transition layer there is some additional energy proportional to the area, with superficial density $\mu$

$$\mu = \int \left[\frac{1}{2}(\nabla\varphi)^2 + \lambda^2(\varphi^2-\eta^2)^2\right] dx.$$

Substituting the solution, we obtain

$$\mu = \sqrt{2}\lambda\eta^3 \int_{-\infty}^{+\infty} (\operatorname{ch}\xi)^{-4}\, d\xi = \frac{4\sqrt{2}}{3}\lambda\eta^3.$$

One can verify that, in agreement with the general principles, the solution minimizes the integral which expresses $\mu$ for given values of $\varphi = \pm\eta$ at $x = \pm\infty$.

The quantity $\mu$ could be called surface tension. In a generally covariant notation $\int T_{00} dV = \mu S$, where S is the area of the separation surface which falls into the integration region. Without a formal proof we list the consequences of this. If the boundary is situated in the x = const. plane, then

$$\int T_{yy}\, dV = \int T_{zz}\, dV = -\mu S.$$

The other components of the stress tensor vanish, in particular $R_{xx} = 0$. The integral $\int T_{tt} dn = -\mu$ (t is a tangent direction and n is the normal to the surface) is obviously the total tension on the surface: force per unit length.

The separation surface is invariant with respect to tangential motion. Let, in the rest system of the surface,

$$\mu = \int T_{00}\, dn = -\int T_{tt}\, dn, \quad \int T_{0t}\, dn = 0.$$

Under a Lorentz boost along the surface with speed v:

$$\left(\int T_{00}\, dn\right)' = \frac{1}{1-v^2}\left[\int T_{00}\, dn + v^2 \int T_{tt}\, dn\right] = \int T_{00}\, dn,$$

$$\left(\int T_{0t}\, dn\right)' = \frac{v}{1-v^2}\left[\int T_{00}\, dn + \int T_{tt}\, dn\right] = 0 = \int T_{0t}\, dn,$$

$$\left(\int T_{tt}\, dn\right)' = \frac{1}{1-v^2}\left[v^2 \int T_{00}\, dn + \int T_{tt}\, dn\right] = \int T_{tt}\, dn.$$

It follows from these equations that one cannot distinguish any single rest frame in the separation plane. However, a motion along the normal to the surface (speed w) is quite observable. Carrying out the appropriate Lorentz transformation we obtain

$$\left(\int T_{00}\, dn\right)'' = \mu/\sqrt{1-w^2}, \quad \left(\int T_{0n}\, dn\right)'' = \mu w/\sqrt{1-w^2},$$

$$\left(\int T_{nn}\, dn\right)'' = \mu w^2/\sqrt{1-w^2}.$$

The other (tangential) components do not change. Curved portions of the surface undergo a normal acceleration, equal to (in units with c = 1) the mean curvature

$$d^2x/dt^2 = 1/R_{1} + 1/R_{2}$$

according to the intuitive picture of the motion of a stretched film. Along the film curvature waves $x \sim e^{iky}$ can propagate with propagation velocity exactly equal to the speed of light.

## 4. THE DYNAMICS OF THE WALLS

We consider a volume containing a large number of chaotically oriented surfaces at rest. We denote by s($cm^2/cm^3$) the average area per unit volume; it is obvious that s is of the order of the reciprocal mean distance L between two surfaces, i.e., of the reciprocal thickness of the vacuum domain of one sign.

Averaging over the volume the energy and stress densities, we obtain

$$\varepsilon = \mu s, \quad T_{ik} = \delta_{ik} p, \quad p = -2\mu s/3 = -2\varepsilon/3.$$

Such an "equation of state" of a multi-domain vacuum agrees with intuitive considerations. We consider a volume V and apply to it the first law of thermodynamics:

$$E = \varepsilon V, \quad dE = -p dV, \quad V d\varepsilon + \varepsilon dV = 2\varepsilon dV/3,$$
$$E \sim V^{2/3}, \quad d\varepsilon/\varepsilon = -dV/3V, \quad \varepsilon \sim V^{-1/3}.$$

If the domains are scaled up conformally, $V \sim L^3$, the area $\propto L^2 \sim V^{2/3}$. It is necessary to underline the fact that this result is valid only for conformally expanding domains (i.e., remaining similar to themselves).

V. I. Zakharov has remarked that in the general case a chaotic normal motion of the surfaces leads to the appearance of positive components of the tensor $T_{ik}$, so that $p > -2\epsilon/3$; in particular, for a definite intensity of the motion it is possible that $p > 0$.[3]

If the motion itself is caused by a tension on initially stationary walls, the motion leads to a decrease of the area. We disregard the general cosmological expansion, so that V = const. Then obviously E = const., $\epsilon$ = const. For constant $\epsilon$ the decrease of S is accompanied by an increase in p.

We consider the "collapse of a bubble" of condensate with $\sigma = -1$ surrounded by condensate with $\sigma = +1$. This collapse occurs under the action of the tension of the curved wall. Taking into account energy conservation we have $\gamma = R_0^2/R^2(t)$, where $\gamma = (1 - w^2)^{-1/2}$ and $R_0$ is the initial radius of the bubble with walls at rest. Substituting w = dR/dt and solving the equation we obtain that the collapse time of the bubble is $t \approx 1.3R_0$, i.e., the bubble contracts with almost the speed of light. During the final stage the walls get destroyed emitting $\chi$ mesons and other particles.

In a cosmological situation the expansion of the Universe diminishes all the momenta and counteracts the increase of p. On the other hand, when a collision of surfaces occurs under a finite angle in the process of motion (process analogous to the formation of caustics in optics), there must appear oscillations of the $\chi$ field, or, using a different language, there appear quanta of the $\chi$ field and possibly other particles.

The detailed dynamics of the process has not been studied until now.[4)] The limiting case of a conformal stretching $p = -2\epsilon/3$ is logically possible. For example, one could consider a cubic lattice with alternating regions with $\varphi = +\eta$ and $\varphi = -\eta$. The plane boundaries guarantee the basence of accelerations[5)].

## 5. LAW OF COSMOLOGICAL EXPANSION OF THE DOMAIN STRUCTURE

We consider on a macroscopic level the cosmological expansion of a cellular medium with equation of state $p = -2\epsilon/3$, and the corresponding scale-dependence of p and $\epsilon$: $p \sim \epsilon = k/b$, where k is a coefficient determined by the structure of the cells. For simplicity we consider a spatially flat Universe[6)] with the metric

$$ds^2=dt^2-b^2(t)\,[dx^2+dy^2+dz^2].$$

The Einstein equation is of the form (cf.[11])

$$\frac{1}{2}\left(\frac{db}{dt}\right)^2=\frac{4\pi}{3}G\varepsilon b^2=\frac{4\pi}{3}Gkb.$$

Its solution is

$$b=b_1t^2,\quad \varepsilon=3/2\pi Gt^2,\quad k=3b_1/2\pi G$$

(the constant $b_1$ is arbitrary and cancels out from all physical results). Prescribing the surface density of the wall $\mu = m^3$, we obtain the average distance between the walls[7)]: $L = Gm^3t^2$ (we omit dimensionless factors). The domains are formed at the time $t_0$:

$$T=m,\quad L=m^{-1},\quad \varepsilon=T^4=m^4,\quad t_0=1/m^2\sqrt{G}=t_n^2/t_G,$$

where $t_n \sim 10^{-24}$ sec is the "nuclear" time, $t_G \sim 10^{-43}$ sec is the "Planck gravitational time" $G^{1/2}$ so that $t_0 \sim 10^{-5}$ sec.

The instant $t_1$ is defined by the condition $L = t_1$, hence

$$t_1=1/Gm^3=t_n^3/t_G^2=t_0t_n/t_G\sim10^{14}\text{ sec}.$$

If the boundaries of the domains are in a state of chaotic relativistic motion, then, as for a relativistic gas, $p = \epsilon/3$. In this case the energy density of the boundaries decreases like $b^{-4}$, where b is the distance between two arbitrarily chosen points of the expanding Universe. Obviously, this is possible only if the boundaries are not "stretched" and the corresponding average energy density $\gamma m^3/L$ is much larger than its minimal value. The latter is defined by the fact that the maximal dimension of a domain does not exceed t, and consequently $\epsilon \geq m^3/t$ (we assume that the superficial density of walls at rest is $\mu \sim m^3$, where $m \sim m_p$ for $\lambda \sim 1$). In the case of rapid chaotic motions of the boundaries there is a large excess area of the walls, the empty volumes increase as $b^3$ for unchanged area of the walls, and the motions decay according to the law $\gamma \sim 1/b$. At the same time $\epsilon$ decays as $1/Gt^2$. Such a regime is possible up to $\epsilon = 1/Gt^2 \gtrsim m^3/t$, i.e., up to the time $t_1$. After this time the walls get stretched and $\epsilon$ decreases according to the law $1/b$ and $L \sim Gm^3t^2$ for $t \gtrsim t_1$.

If the walls of the domains were stretched from the time $t_0$ (e.g., in the case of the above mentioned cubic structure), the law $L \sim Gm^3t^2$ would start not at $t \sim t_1$, but at $t \sim t_0$

The present observational data relative to the isotropy and homogeneity of the Universe and the total energy density in the whole Universe, require that domains with heavy walls disappear at a quite early stage of the evolution (we denote the corresponding instant by $t_2$). In order to show this we consider the domain structure in terms of newtonian gravitation theory. At the separation surfaces the potential $\Phi$ has a discontinuity in its derivative equal to $G\mu$, so that the change of potential within a cell of size L is of the order[8)] $\delta\Phi = G\mu L$. The Universe is filled with background radiation (temperature 2.7°K). Observations show that this radiation does not depend on the direction ($\delta T/T < 10^{-3}$). The gravitational redshift should yield $\delta T/T \sim \alpha\delta\Phi$. Here the dimensionless $\alpha = 0$ only in the extremely unlikely case of spherical symmetry of the separating surfaces relative to the observation point. In general $\alpha < 1$, but probably $\alpha > 0.05$, e.g., for a cubic lattice.

Thus it is necessary that $G\mu L \ll 1$. Probably a detailed analysis of the Hubble redshift of the galaxies (the relation log z vs. log m for given M, where m is the apparent magnitude and M is the absolute magnitude of the galaxy) will lead to the same conclusion: $G\mu L < 1$, albeit with a weaker inequality.

Substituting $\epsilon = \mu/L = 1/Gt^2$, $L = Gt^2$, we arrive at the conclusion that $t_2 < 1/\mu G = t_1$.

The observable picture of the Universe (its visible isotropy and homogeneity) is incompatible with the assumption of a coarse domain structure with domains having dimensions of the order of or larger than today's t. Either the parameters of the walls are such that at the present time (i.e., for an average matter density of $10^{-29} - 10^{-30}$ g/cm$^3$) one can place a large number of walls over a distance of $ct = 2 \times 10^{28}$ cm$^9$ (for this it is necessary that the superficial density of walls be $\mu < 0.1$ g/cm$^2$, which seems unrealistic from the viewpoint of the theory of elementary particles, since it corresponds to $\lambda \gtrsim 10^3$ for $m_\chi \sim m_p$), or (and below it is this variant we will discuss) the domains disappeared at some time $t_2 < t_1$, sufficiently early so that the homogeneity does not get noticeably violated; in addition, for an early $t_2$ (before the recombination of the electrons and the protons) the Compton scattering evens out the anisotropy of the background radiation. The disappearance of the domain walls means a transformation of the surface energy of the walls into other energy forms–the energy of massive quanta of the $\chi$ field or into the general equilibrium radiation. A consideration of these consequences leads to restrictions on $t_2$.

## 6. A CAUSAL COUPLING BETWEEN REMOTE OBJECTS

In the model where the domain structure and the equation of state $p = -2\epsilon/3$ is restricted to the time interval $t_0 < t < t_2$, we consider the propagation of disturbances with the speed of light in two stages: from $t_0$ to $t_2$ and from $t_2$ to $t_4$ of today. We take into account that the second stage is subdivided into: 2a) $p = \epsilon/3$, $t_2 < t < t_3$ and 2b) $p = 0$, $t_3 < t < t_4$.

Accordingly we have in the metric

$$1)\ b=ft^2,\quad 2a)\ b=gt^{1/2},\quad 2б)\ b=ht^{2/3}$$

with the matching conditions

$$ft_2^2=gt_2^{1/2},\quad gt_3^{1/2}=ht_3^{2/3}.$$

Following Misner[13], we compare the region encompassed by present-day observations, P (present), and the region which is causally connected by signal propagation to the singularity, C (causality). The region P can be determined by considering the rays passing through the coordinate origin (X = 0) at the instant $t_4$. The region C is determined by considering the rays emerging from the origin at time $t_0$. An invariant measure of the region, independent of the arbitrariness in the selection of the scale factor b(t), is its present size $X = b(t_4)x$, where x is a comoving coordinate (cf. the metric). A simple computation yields for $X_P$

$$x_P = \int^{t_4} \frac{dt}{b(t)} = 3h^{-1}t_4^{1/3}, \qquad X_P = 3t_4.$$

For the determination of the C-region it is essential to use the matching conditions

$$x_C = \int_{t_0} \frac{dt}{b(t)} = f^{-1}t_0^{-1},$$

$$X_C = t_0^{-1}t_2^{3/2}t_3^{-1/6}t_4^{2/3} = t_4(t_2/t_0)(t_2/t_3)^{1/2}(t_3/t_4)^{1/3}.$$

Owing to the peculiar expansion law during the initial (domain) stage it is quite possible that $X_C \gg X_P$. Indeed assume, for instance, that $t_0 = 10^{-5}$ sec, $t_2 = t_3 = 10^{12}$ sec, $t_4 = 6 \times 10^{17}$ sec. In units of length

$$X_P = c\cdot 3t_4 = 6\cdot 10^{28}\ \text{cm} \quad X_C = ct_4\cdot 10^{17}\cdot 10^{-2} = 3\cdot 10^{14}X_P = 2\cdot 10^{43}\ \text{cm}$$

The inequality $X_C \gg X_P$ remains true under more modest assumptions, e.g., for $t_0 = 10^{-5}$ sec, $t_2 = 10^5$ sec ($z = 3 \times 10^7$), $t_3 = 10^{13}$ sec, and $t_4 = 6 \times 10^{17}$ sec, we obtain $X_C = 10^2 X_P$.[10)] But it is just such a situation which Misner considered necessary, and for its sake he considered preferable an anisotropic closed model of the Universe. The condition $X_C \gg X_P$ denotes the possibility in principle that the conditions in the accessible part of the Universe are evening out. In the domain theory this condition is compatible with a flat or with an open homogeneous and isotropic Friedmann cosmological solution!

## 7. CONCLUSIONS AND SOME UNSOLVED PROBLEMS

We can thus draw three fundamental conclusions:

1. Spontaneous violation of CP-invariance leads to the formation of a domain structure of the vacuum. This occurs because during the cooling down of causally disjointed points of the Universe the signs of the condensate appear in a random fashion.

2. For $\lambda < 1$ the walls of the domains are so heavy that their existence would lead to a radical change of the cosmological evolution of the Universe.

3. If there is no mechanism that leads to the disappearance of domains at a sufficiently early stage of the evolution of the Universe, the domains would lead to conclusions which are in contradiction with experiment.

Thus, either the model of spontaneous breakdown of CP-symmetry discussed by us is false, or there must exist mechanisms which facilitate the disappearance of the domains.

How early should one remove the domain walls and make the whole Universe into a single domain? For this one obviously must first violate the symmetry of the two solutions $\varphi = \pm\eta$. On the other hand, the symmetry breakdown should not be contained in the Lagrangian. This would make the idea of spontaneous symmetry breakdown devoid of content. The idea appears to relate the symmetry breakdown to the material content of the Universe, and more concretely, to the excess of baryons, or maybe even with the fact that the Universe is expanding, i.e., the fact that one considers a solution which is not symmetric under the interchange of t and $-t$. It would be very interesting to establish a relation between the local violation of CP-invariance and the charge asymmetry of the Universe as a whole. This observed asymmetry consists in an excess of baryons over the antibaryons; there is possibly an excess of neutrinos over antineutrinos. Unfortunately we have not found yet a concrete interaction which satisfies these conditions.

A "skewing" of the initial Lagrangian could be caused by a term of the type $\bar{\psi}\psi\varphi$, but such a term is forbidden by the CP-invariance of the Lagrangian. A term of the form $\bar{\psi}\gamma_5\psi\varphi$ which is permitted by the CP-invariance, vanishes when averaged over the spins and momenta of the baryon excess.

If the skewness in favor of one of the solutions $\sigma = +1$ or $\sigma = -1$ could be determined by the expansion of the Universe, then this would lead to a connection between the direction of the arrow of time in the microcosm and the macrocosm. The choice of the sign of the T-noninvariant phases in all CP-noninvariant phenomena would in this case occur during the early stages of the evolution of the Universe due to its expansion.

Of particular interest is the problem of interaction of particles with the walls and in particular, the problem of transparency of walls consisting of neutral matter with respect to photons of various wavelengths.

We have not found a realistic answer to the question of what observations one could use for the determination of the sign of in some remote region of space. The reason for this is that all known CP-noninvariant effects are very small and specific. If we were dealing with a spontaneous breakdown of P-parity, then the neutrinos would furnish us information about the sign of the condensate, since the polarization of the neutrinos produced in different domains would be opposite[11)]. This would be so if, e.g., the weak current would contain terms of the form $\bar{e}\gamma_a(1 + \gamma_5\varphi/\eta)\nu$ in distinction from models considered until now, such a theory would be unrenormalizable).

In the case of a discrete symmetry the walls cannot become wide and light owing to the joint action of two terms: the kinetic one $(\partial\varphi)^2$ and the potential one $\lambda^2(\varphi^2 - \eta^2)^2$. For the case of continuous symmetries of the gauge type the potential contains $\lambda^2(|\varphi|^2 - \eta^2)^2$ and the transition from the domain with $\varphi = \eta \exp(i\alpha_1)$ to a domain with $\varphi = \eta \exp(i\alpha_2)$ occurs on account of a change of the phase $\alpha$ at constant modulus $|\varphi|$, so that the potential energy in the transition layer does not increase. This leads to the result that in the case of gauge symmetry the walls will flow apart and there will appear longer and longer wavelength changes of the phase of the condensate. We hope to return later to this question.

Finally, we dwell on the unsolved problem of renormalization of the vacuum energy, i.e., the cosmological constant. This question exists even without connection to models of spontaneous symmetry breakdown, but in these models it is more acute. To date it is not clear what mechanism is responsible for the form of the renormalized Lagrangian such that the minimum of the energy corresponds to a value zero of the energy. If this were not so, the condensate would yield a cosmological term, which depending on the sign would either prevent or enhance the expansion of the Universe.

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[1)]In quantum theory $\mathscr{L}$ is the renormalized Lagrangian.

[2)]We utilize units with $\hbar = c = 1$.

[3)]We are indebted to V. I. Zakharov for useful discussions of the question of the cosmological role of domains, and in particular, for pointing out that rapidly moving chaotically oriented walls are similar to a relativistic gas (cf. infra).

[4)]In some special cases of nonlinear equations solitons, particularly solitons of the type of a $\theta$-function and not only those of the $\delta$-function type, do not convert into a set of weak waves due to collisions. Therefore care is needed in conclusions about the disappearance of the walls.

[5)]The problem of stability of edges and corners remains open.

[6)]It is curious that in the case $p = -2\epsilon/3$, as in the problem with the cosmological constant, it is possible to combine the spatial closure of the universe with unbounded expansion in time. For $p > 0$ only an open universe can expand.

[7)]An accelerated (and not retarded, as usual) expansion follows from the fact that in general relativity Newton's equation for $\dot{b}$ involves $(\epsilon + 3p)/c^2$ in place of $\rho$. For $p = -2\epsilon/3$ we have $\epsilon + 3p = -\epsilon < 0$.

[8)]In CGS units the potential has the dimension of the square of a velocity. Using units with $c = \hbar = 1$ we consider the potential $(\Phi/c^2)$ as dimensionless.

[9)]In comparing with the paper of Sokolov and Shvartsman [12] on the possibility of partitioning the Universe one must keep in mind that they consider the topology of the Universe without violating the homogeneity; in our case the walls really exist and have finite density.

[10)]The theory of cosmological nucleosynthesis can yield more stringent restrictions on $t_2$ ($t_2 < 1s$). Then the inequality $X_C \gg X_P$ is possible for $m_\chi \gg m_p$.

[11)]Could the results of the Davis experiment indicate that the Sun is situated in a different domain than the Earth?

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